Efficient Insurance in a Market Theory of Payroll and Self-Employment*

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Work in Progress, Comments Welcome

Abstract

We propose a theory of self-employment which, contrary to earlier studies, does not rely on ex-ante heterogeneity across agents. Households face a choice between self-employment and searching for a job at a firm. Both labor and product markets have search frictions. In the mixed equilibrium households trade the risk of not meeting a customer when self-employed against the risk of unemployment and the rent extraction in a guaranteed wage contract. The equilibrium is inefficient under risk-aversion due to excessive firm entry and self-insurance in the pricing by self-employed that attracts too many customers. Type-dependent differentiated taxes and unemployment insurance benefits can restore efficiency under a balanced budget.

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1 Introduction

In this paper we propose a theory of the composition of employment that focuses on the key distinction between self- and payroll employment: exposition to the risk of not selling production versus exposition to the risk of not finding a job. The self-employed are the sole claimants of the fruits of their labor, but bear the risk of not selling production fully on their own. The wage contract limits this risk for the employee, but requires sharing the surplus of a match with the firm. Moreover, not all of those looking for a wage contract are able to find one, so that some become unemployed. In order to highlight these observations, we explicitly model the problems of finding a job in the labor market and selling production in the goods market.

The motivation for this paper is that over the last two decades the majority of the developed economies have experienced a shift of composition of employment towards more own-account work and freelancing. As a result, self-employment added up to 14% of the labor force in the European Union in 2012. For comparison, the unemployment rate in the EU that year was 10.4%. There is significant variation of the self-employment rate in the cross-section as well, even if one considers developed economies only. These facts, illustrated in Table 1, are difficult to reconcile with existing theories of self-employment that emphasize individual heterogeneity of skills, preferences, or cognitive biases (Parker, 2012), which we don't expect to vary much over time and across countries.

Country	1993	2013
UK	9.1	11.7
Netherlands	6.9	11.8
Germany	4.0	6.0
Italy	11.8	16.4
Sweden	7.8	6.4
Denmark	4.7	5.4

Table 1: Share of own account workers in total employment. Source: Key Indicators of the Labor Market, ILO.

In addition, Denmark and Sweden have recently introduced unemployment insurance schemes that are designed specifically for the self-employed. However, in the majority of developed countries this type of insurance is still absent. On the one hand, it is widely believed that being self-employed is riskier than being an employee. On the other hand, if self-employment is driven by the desire to be one's own boss or by higher tolerance for risks, insurance for the self-employed is hard to justify.

We aim to make two contributions. First, we propose a novel parsimonious theory of self-employment that does not rely on individual-level differences between people but focuses on trade-offs between labor and goods market frictions. Changes in these markets, as well as in technology, provide a natural explanation for the long run behavior of self-employment rates.

Second, we show that there should be insurance benefits for the self-employed that fail to sell, solely for the purpose of maximizing the volume of goods traded with customers. Insurance for the self-employed eliminates a business stealing externality exerted by risk-averse self-employed who have an incentive to set their prices too low from the societal point of view, to reduce the risk of not selling their production. This paper therefore provides a rationale for the UI policies of Denmark and Sweden that is not only based on risk sharing, but also on efficiency.

We consider an economy represented on Figure 1. It is inhabited by homogenous individuals producing an indivisible good. The good cannot be consumed by these individuals but can be exchanged with buyers for a divisible endowment that the individuals can use to pay for their consumption. The individuals face a career choice problem. They can either become self-employed, producing and trying to sell on their own, or seek a job at a firm, which tries to sell the goods produced by the individual it employs.

Individuals entering the labor market cannot coordinate their job applications to firms that post wages, which results in involuntary unemployment. Similarly, buyers cannot coordinate their visits to firms and self-employed that post prices, resulting in unsold inventories. An employee is guaranteed the wage even if the firm fails to sell the goods. However, an employee has to share the expected surplus with the firm. The self-employed face the risk of not selling on their own, but they forego the risk of unemployment.

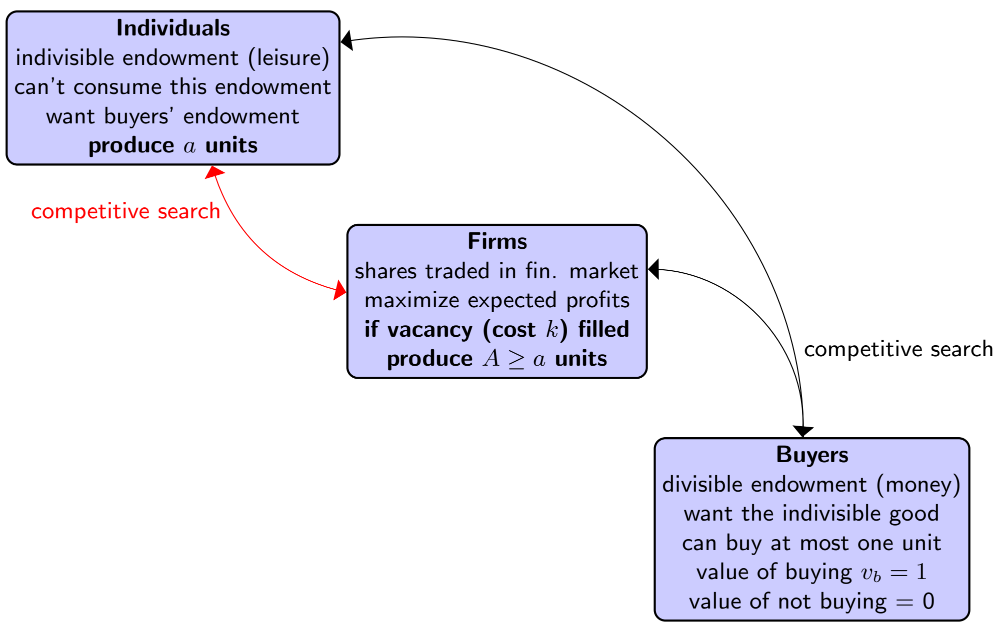

Figure 1: A snapshot of the model economy

Unlike in the standard Mortensen-Pissarides framework, in our model a match with a firm is thus not necessary to generate income. Our model endogenously determines firm entry and a self-employment rate depending on the existence of productivity gains (net of entry costs) to firm formation. One can think of those gains as resulting from additional capital, training or knowledge the firm has at its disposal, or - in a richer framework with intermediate inputs - from economizing on internal transaction costs as in Coase (1937). Firms in our model are both an intermediary between the employee and the buyers, and a vehicle of production that cannot exist without any form of competitive advantage over independent production by the self-employed. Yet, the trade-off between the frictions in the goods and labor market leads to the coexistence of firm employment and self-employment in equilibrium.

The presence of firms and self-employed implies the goods market consists of two different types of sellers. If individuals are risk-averse, these two different types of sellers have different objectives, creating inefficiencies that can be potentially corrected by policy. Risk-averse self-employed, unlike risk-neutral firms, have an incentive to self-insure via their pricing decision. By decreasing prices they attract on average more buyers so that their selling probability increases. However, the lowering of prices by the self-employed exerts a business stealing externality on firms. Other things kept equal, firms' expected profits fall. Consequently, fewer firms enter and the economy benefits to a lesser extent from their competitive advantage in production. As a result, the volume of the goods traded drops.

On top of that, the ability to self-insure makes a career in self-employment relatively more attractive than entering the labor market. This effect is countered by the conventional effect that firms post lower wages to increase the job finding rate of risk-averse job seekers. The latter, however, comes at the cost of excessive vacancy creation. Generally, the employment composition is different than the composition that would maximize the volume of goods traded net of firm entry costs. For some parameter values, the ability to self-insure in self-employment can dominate the market insurance offered by firms. As a result, the self-employment rate may *increase* in risk aversion.

We find that the combination of four instruments under a balanced budget can maximize the production sold net of entry costs, while offering insurance to risk-averse individuals. This optimal policy mix consists of differentiated taxes and unemployment insurance benefits for both workers and self-employed. Optimal UI benefits for the self-employed eliminate business stealing, while optimal UI benefits for job seekers raise wages and stop excessive firm entry. Differentiated taxes then balance the budget while ensuring an optimal employment allocation via the career choice of individuals. We show that whenever the job finding probability exceeds the selling probability (so that the self-employment income can be considered riskier), the UI benefits for the self-employed should be more generous than the UI benefits for employees.

Related literature. Our paper is related to three strands of the literature: on the causes of self-employment, on frictional goods markets and intermediation, and on optimal unemployment insurance. Below we describe our contribution to those papers.

There is a variety of theories explaining self-selection into self-employment. A big fraction of this literature puts individual characteristics and heterogeneity as a reason for self-employment. We list a limited selection of those papers, whereas Parker (2004) offers an extensive survey. Lucas (1978), Jovanovic (1982) and Poschke (2013) assume that being an entrepreneur/self-employed requires a separate skill, potentially different than a skill needed to be an employee. Kihlstrom and Laffont (1979) postulate differences in risk aversion that lead to undertaking entrepreneurial activities. De Meza and Southey (1996) find that self-employed entrepreneurs are significantly more optimistic than employees. Lindquist et al. (2015) document the importance of family background. We complement this literature, because in our model self-employment is an equilibrium outcome that does not require any ex ante individual heterogeneity. Besides, Rissman (2003, 2007) assumes returns to self-employment are drawn from an exogenous distribution riskier than the wage distribution. We offer a model that endogenously generates those risk differentials. Finally, our model is complementary to papers that explain self-employment from financing frictions (Buera, 2009, Evans and Jovanovic, 1989), because a frictional financial market could be introduced as an additional stage in the career choice game for those who choose self-employment.

To the best of our knowledge, self-employment has not been introduced into models with frictions in the goods market. Existing papers study the macroeconomic consequences of goods market frictions (See e.g. Branch et al. (2014), Kaplan and Menzio (2013), Michaillat and Saez (2013), Petrosky-Nadeau and Wasmer (2015)), or characteristics of firms that operate in a frictional goods market. Most closely related are Shi (2002), who explains the size-wage differential in the labor market by a sufficiently large size-revenue differential in the goods market, and Godenhielm and Kultti (forthcoming), who allow for endogenous capacity choice and study the resulting firm size distribution.

Our model can also be framed as a choice of producers to trade with buyers via a middleman (firm) or do to trade with buyers directly. The papers in the literature on intermediation that are most closely related are Watanabe (2010, 2013). Unlike in those papers, the choice that producers (workers) face in our model is exclusive. Also, the meetings with the middlemen are subject to a friction. Wright and Wong (2014) offer a general model of middlemen with search and bargaining problems. We employ posting, allow for bypassing of the middlemen, and discuss labor policy implications.

Papers on efficient unemployment insurance for risk-averse individuals either do not take self-employment (e.g. Acemoglu and Shimer (1999)) or market frictions into account (e.g. Parker (1999)). Our paper shows that the interaction of risk-averse self-employed and goods market frictions is crucial for understanding efficient unemployment insurance.

The rest of the paper is organized as follows. In Section 2 we outline the structure of the model. Then, in section 3 we characterize the market equilibrium. In section 4 we present and characterize the conditions under which a unique mixed strategy equilibrium exists for risk-neutral preferences, and prove that it maximizes net production sold. In section 5 we show that the decentralized allocation is not efficient for risk-averse preferences, but that introducing a type-of-employment dependent tax and unemployment insurance policy can restore efficiency. Section 6 presents the steady state of a dynamic version of the model in which jobs in expectation last for multiple periods, and shows how the composition of employment depends on the key parameters of the model. The last section concludes.

2 Model environment

We consider a one-shot game of an economy populated by firms, buyers, and individuals that exchange indivisible labor for a divisible endowment in the labor market, and an indivisible consumption good for a divisible endowment in the goods market. Individuals face a career choice between self-employment and entering the labor market. There are coordination frictions in both the markets.

Population and technology. The measure of individuals is normalized to one. They value consumption according to a utility function $u(c)$, suffer no disutility of labor, but cannot consume their own production. There is a measure B of buyers in the goods market that have a valuation v from buying one unit of the consumption good, and a smaller valuation for an owned divisible endowment, which individuals can consume. We normalize buyers' utility from not buying to zero. The buyers can consume only one unit of the good. Finally, there is an endogenously determined mass V of vacancies opened by profit-maximizing firms upon paying a cost $k > 0$.

The career choice of individuals results in an endogenous measure SE of self-employed and $W = 1 - SE$ of workers. A match of a single worker and a vacancy results in an active firm that produces A units of the consumption good. Alternatively, an individual can be self-employed and produce a units of the good without a firm. Both self-employed and active firms are sellers in the goods market. Without loss of generality we normalize $v = 1$.

Goods Market. We assume that the units of the indivisible good are sold separately, one unit per selling outlet.[1] Self-employed and firms open as many outlets as units they produce, and post prices with commitment. Buyers observe prices, can only visit one outlet, but cannot coordinate which one to visit. For that reason, the goods market is subject to urn-ball frictions. As a result, some sellers face more customers than they can serve and others are not able to sell, while some buyers fail to buy the good. If there is a mass of buyers B_{SE} at the outlets open by the self-employed, then the average queue length at each outlet is $x_{SE} = \frac{B_{SE}}{aSE}$. The average queue length at an active firm x_F is defined analogously. The corresponding service probabilities for a buyer are denoted by $\eta(x_{SE})$ and $\eta(x_F)$, at self-employed and active firms respectively. The selling probabilities $\lambda(x_{SE}) = x_{SE}\eta(x_{SE})$ and $\lambda(x_F) = x_F\eta(x_F)$ are the complementary probabilities of having no buyers visiting at all. Using the large market assumption to characterize these probabilities, $\lambda(x) = 1 - \exp^{-x}$.

Labor Market. Upon paying an entry cost to open a vacancy, a firm posts a wage and commits to it. Workers observe wages but can apply to one vacancy only, while a vacancy can be filled by only one worker. We restrict our attention to symmetric strategies and assume that workers are unable to coordinate which vacancy to apply for. We denote the average queue length by $x_W = W/V$ with

[1] This is solely for analytical clarity. Alternatively, all units produced by one self-employed or firm are sold at one location, which yields a selling advantage to larger inventories. The corresponding queue lengths and service probabilities are given in Watanabe (2010).

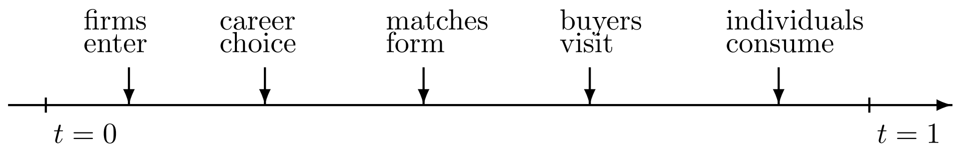

Figure 2: Timing of events

V for the measure of open vacancies. As the result of the coordination frictions, some firms fail to fill their vacancy and do not become active, while some workers become unemployed. The probability of filling a vacancy is denoted by $q(x_W)$, and by the large market assumption $q(x_W) = 1 - \mathrm{e}^{-x_W}$. The job finding probability is simply $\mu(x_W) = \frac{q(x_W)}{x_W}$. Finally, we assume that the firms can insure in a competitive market against the risk of not being able to pay the wage, so that the worker is guaranteed a wage once matched. The shares of firms are traded by financial investors (not modeled explicitly) who can buy a market portfolio of those shares so that the firms maximize expected profits.

Timing. The timing of the game is displayed in Figure 2. First, a measure of firms enter the labor market by opening vacancies. In the career choice stage the unit mass of individuals parts into self-employed and workers. In the third stage, the frictional labor market matches vacancies and workers, resulting in a measure $F = q(x_W)V$ of active firms and a measure $U = (1 - \mu(x_W))W$ of unemployed workers. In the fourth stage, all active firms and self-employed produce and become sellers in the goods market, and buyers direct their search to them such that the following adding-up constraint is satisfied:

$$ax_{SE}SE + Ax_F F = B, \tag{1}$$

which is equivalent to saying that no buyers stay at home not trying to visit any seller. Now we are in position to define the market equilibrium of this economy in the following section.

3 Decentralized Equilibrium

We decompose the one-shot game into two stages: the career choice and the labor market as the first stage, and the goods market as the second stage. We solve the game backwards, starting from the goods market. We focus on equilibria that feature both self-employment and payroll employment. The existence conditions for such a mixed strategy equilibrium of the career choice game are presented in the next section.

Goods market. As is standard in competitive search models, separate submarkets open and buyers choose between visiting each of the submarkets such that in equilibrium they are indifferent between them and obtain value V^B. Given the specification of buyers preferences, this value reads

$$V^B = \eta(x_F)(1 - p_F) = \eta(x_{SE})(1 - p_{SE}). \tag{2}$$

Given that the wage is sunk, active firms maximize expected revenue: $A\lambda(x_F)\,p_F$. Self-employed maximize expected utility, i.e. they maximize

$$V^{SE} = \sum_{j=1}^{a} \binom{a}{j} \lambda(x_{SE})^j (1-\lambda(x_{SE}))^{a-j} u(jp_{SE}).$$

In the remainder of this section and in other sections that allow for risk-averse preferences we will normalize $a = 1$, for analytical clarity. The expected value of self-employed sellers is then

$$V^{SE} = \lambda(x_{SE})\,u(p_{SE}). \tag{3}$$

The goods market outcomes and payoffs are depicted in Figure 3.

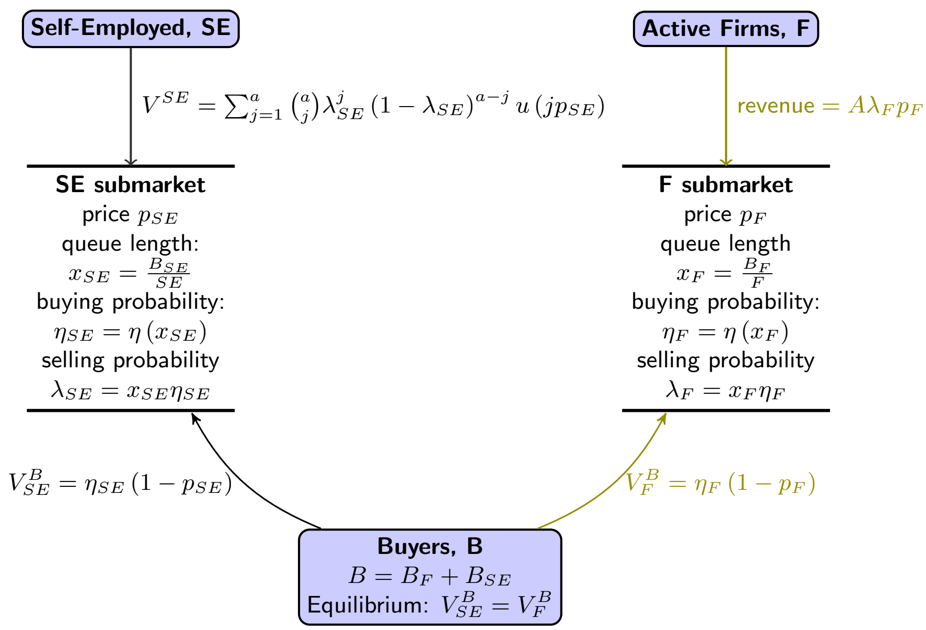

Figure 3: The goods market.

Sellers post prices to maximize their expected payoffs subject to the market utility of the buyers, V^B. The optimal prices and an associated goods market sub-equilibrium characterization are presented below.

Lemma 1 (Optimal price posting) *Assume $SE > 0$, $F > 0$, $B_F > 0$, $B_{SE} > 0$ fixed. Let $\phi(x) = -\frac{x\partial\eta(x)}{\eta(x)\partial x}$ be the elasticity of the buying probability with respect to the queue length x. Given queue lengths $x_{SE} = B_{SE}/SE$, $x_F = B_F/F$ the optimal price posting conditions are:*

$$\frac{\phi(x_{SE})(1-p_{SE})}{1-\phi(x_{SE})} = \frac{u(p_{SE})}{u'(p_{SE})}, \tag{4}$$

$$p_F = \phi(x_F). \tag{5}$$

Proof. See Appendix A. ■

Definition 1 (Goods market sub-equilibrium) *Let $SE > 0$, $F > 0$ be fixed. A goods market sub-equilibrium is a tuple $\{x_{SE}, x_F, p_{SE}, p_F\}$ such that given x_{SE}, x_F the sellers optimally post prices according to* (4) *and* (5)*, and buyers' indifference condition* (2) *and adding-up restriction* (1) *hold.*

Labor market. Now we consider a non-zero mass of workers $W > 0$ and analyze labor market outcomes. We do that in two steps. First, we fix the measure of vacancies $V > 0$; then we allow for free entry in posting vacancies by prospective firms. Given that the entry cost k is sunk, potential firms post a wage in the labor market to maximize expected profits, taking into account equilibrium outcomes in the goods market. They compete with other potential firms for workers, under the constraint that they must at least offer the market utility level of workers searching for jobs:

$$V^W = \mu(x_W)\, u(w)\,. \tag{6}$$

Lemma 2 (Optimal wage posting) *Assume $W > 0$ and $V > 0$, fixed. The optimal wage w that maximizes firms profits subject to workers' market utility* (6) *solves the following equation:*

$$\frac{\phi(x_W)\left[A\lambda(x_F)\, p_F - w\right]}{1 - \phi(x_W)} = \frac{u(w)}{u'(w)}, \tag{7}$$

where p_F and x_F come from the goods market sub-equilibrium $\{x_{SE}, x_F, p_{SE}, p_F\}$ induced by $SE = 1 - W$ and $F = q(x_W)\, V$ and where $\phi(x_W) = \frac{x_W \partial q(x_W)}{q(x_W) \partial x_W}$ is the elasticity of the job filling probability with respect to the queue length.

Proof. See Appendix A. ■

The free-entry condition drives the value of posting a vacancy net of the entry cost k down to zero. Firms' entry takes into account the resulting goods-market sub-equilibrium where $SE = 1 - W$. Formally, we can define the labor market sub-equilibrium as follows.

Definition 2 (Labor market sub-equilibrium) *Assume $W, SE > 0$. The labor market sub-equilibrium is a pair $\{x_W, w\}$ such that the firms optimally post wages according to* (7) *and the following free-entry condition holds:*

$$q(x_W)\left[A\lambda(x_F)p_F - w\right] - k = 0, \tag{8}$$

with p_F and x_F from the goods market sub-equilibrium $\{x_{SE}, x_F, p_{SE}, p_F\}$ induced by SE and $F = q(x_W)\, V$ with $V = \frac{W}{x_W}$.

The labor market equilibrium is represented in Figure 4. Now we are in the position to define a mixed strategy equilibrium for our career choice game.

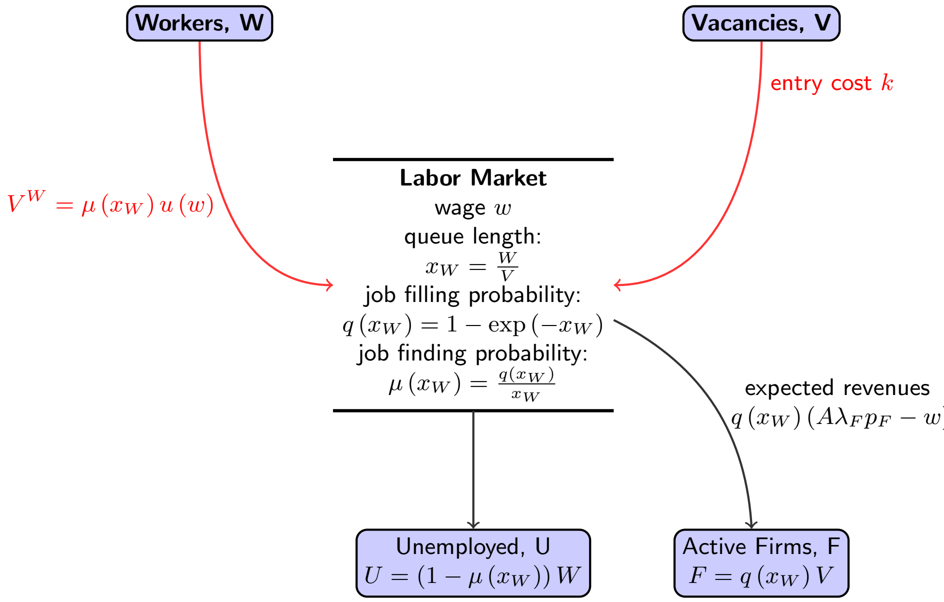

Figure 4: The labor market

Definition 3 (Mixed strategy career choice equilibrium) *A mixed strategy career choice equilibrium is a tuple* $\{SE^* > 0, W^* > 0, x_W^*, w^*, x_{SE}^*, x_F^*, p_F^*, p_{SE}^*\}$ *such that:*

1. *all individuals become either self-employed or a worker:* $SE^* + W^* = 1$*;*

2. *given* SE^*, W^**,* $\{x_W^*, w^*\}$ *is a labor market sub-equilibrium and* $\{x_{SE}^*, x_F^*, p_{SE}^*, p_F^*\}$ *is a corresponding goods market sub-equilibrium*

3. *individuals are indifferent between self-employment and entering the labor market, i.e.* $V^{SE^*} = V^{W^*}$ *as defined in* (3) *and* (6)*, respectively;*

Observe that the indifference condition requires that whenever the job finding probability μ exceeds the selling probability λ the income from self-employment is higher, conditional on selling, than the wage (and the opposite holds when $\lambda > \mu$).

In the next section we state the conditions for the existence of a mixed strategy equilibrium and to prove its efficiency. Afterwards, we investigate policies that make use of type dependent insurance and taxes to maximize output net of recruitment costs.

4 Existence of Equilibrium and Its Properties

The equilibrium can be shown to exist, to be unique, and to involve mixing of careers if the exogenous parameters $\{a, A, k\}$ are appropriately chosen. Outside of a certain set of $\{a, A, k\}$ the equilibrium still exists and is unique but features no mixing of careers. The proof is an application of the implicit function theorem.

As a first step, using buyers and workers adding up restrictions we arrive at the following *mixing condition*:

$$W = \frac{1 - a x_{SE}}{A x_F \mu\left(x_W\right) - a x_{SE}}, \quad 0 < W < 1. \tag{9}$$

Then, we need to solve for the queue lengths $\{x_{SE}, x_F, x_W\}$ and corresponding prices of goods and labor $\{p_{SE}, p_F, w\}$ using the remaining six equilibrium conditions. A necessary condition for this set of equation to have a solution is that the ratio $\frac{u(c)}{u'(c)}$ be increasing in c which holds for any utility function with $u'\left(c\right) > 0$ and $u''\left(c\right) < 0$. For analytical convenience we made the proof operational under CRRA preferences. However, as the previous remark implies, this is without loss of generality.

Theorem 1 (Existence of equilibrium) *Let $A, a \geq 1$ fixed and $u\left(c\right) = \frac{c^{1-\gamma}}{1-\gamma}$. There exist numbers $\underline{k}\left(A, a, \gamma\right)$, $\overline{k}\left(A, a, \gamma\right)$ such that the mixed equilibrium described in Definition 3 exists and is unique if and only if the following inequalities hold:*

$$A > a \tag{10}$$

$$\underline{k}\left(A, a, \gamma\right) < k < \overline{k}\left(A, a, \gamma\right). \tag{11}$$

Furthermore, if $k > \overline{k}\left(A, a, \gamma\right)$ then $SE^* = 1$ and if $k < \underline{k}\left(A, a, \gamma\right)$ then $SE^* = 0$.

A direct result of the working of the proof is the set of comparative statics encapsulated in Proposition 1. Intuitively, the net gain of setting up a vacancy can be neither too small, nor too big for agents to play a mixed strategy in the career choice game.

Proposition 1 (Comparative statics) *Consider a, A and k such that the mixing conditions (10) - (11) hold and let $\{SE^*, W^*, x_W^*, w^*, x_{SE}^*, x_F^*, p_F^*, p_{SE}^*\}$ be the corresponding mixed strategy career choice game equilibrium. Then, the following inequalities hold:*

$$\frac{\partial SE^*}{\partial k} > 0,$$

$$\frac{\partial \overline{k}}{\partial A} > 0, \quad \frac{\partial \underline{k}}{\partial A} > 0,$$

$$\frac{\partial \overline{k}}{\partial a} < 0, \quad \frac{\partial \underline{k}}{\partial a} < 0.$$

Proof. See Appendix A. ■

Discussion. There are two reasons that may make mixing by individuals suboptimal. First, there may be no firms willing to enter, so that there is no chance of finding a job on a payroll. This may happen, for example, when the vacancy posting cost k is prohibitively large. Second, the comparative advantages of firms may be too large to sustain self-employment as a valid alternative to seeking a payroll job.

Hence, the key driving force of the composition of employment in the model is the relative size of the productivities in the two types of employment, adjusted for entry costs, which can be loosely described by comparing $\frac{A}{k}$ to a. One can think of this ratio as a statistic for substitutability between the "self-employment technology" and "payroll employment technology" or the scale of returns to innovation on top of producing on one's own that comes up with setting up a firm. For example, a large-scale production industry like an automotive industry is a sector with a very high $\frac{A}{k}$ and low a. In contrast, one can expect the differnce between $\frac{A}{k}$ and a to be low in service industries like hairdressing or taxi-driving. A prediction of the model is therefore that when the share of low capital intensity services in the economy increases, the share of self-employment goes up as well. Another interpretation of the model are cross-country differentials in economic development and the composition of employment. In underdeveloped countries the technologies that are used in firms offer very little gains, or no gains at all, from organizing workers and capital into a firm. As our model predicts, those countries exhibit high self-employment rates.

Efficiency. We move onto investigating the efficiency of the decentralized equilibrium. We use production sold net of entry costs as our measure of efficiency and allow the social planner to choose the measure of vacancies to be opened and the measure of households to enter self-employment (and thus the measure to enter the labor market). The social planner faces the same coordination frictions within every (sub)market as present in the decentralized equilibrium, but can decide the measure of buyers to go shopping at the self-employed (and thus the measure of buyers that visits firms). Finally, note that choosing the latter, given SE and V, amounts to choosing x_{SE}. The problem of the social planner is then to maximize:

$$V^{SP}\left(V, x_{SE}, SE\right) = A\lambda\left(x_F\right) q\left(x_W\right) V + a\lambda\left(x_{SE}\right) SE - Vk,$$

where $x_W = \frac{1-SE}{V}$ and $x_F = \frac{1-ax_{SE}SE}{Aq(x_W)V}$ from the unit mass of individuals and the adding-up constraint in (1), respectively. Similar to Acemoglu and Shimer (1999), for this measure of efficiency the following result can be shown.

Theorem 2 (Efficiency of equilibrium) *If and only if individuals are risk neutral the decentralized allocation is constrained efficient.*

Proof. See Appendix A. ■

Consequently, the equilibrium allocation that is implicitly given by (2), (21), and (23), with $p_F = \phi\left(x_F\right)$ and $p_{SE} = \phi\left(x_{SE}\right)$, maximizes production sold net of entry costs.

From Theorem 2 it follows that the outcome of the market interactions under risk aversion does not maximize production sold net of entry costs. The drivers of this result are the price and wage posting decisions. When agents are risk-averse, the price p_{SE} that the self-employed charge is lower than the efficient p^*_{SE} for a given queue length x^*_{SE}. The self-employed self-insure by decreasing their price to improve the odds of selling their production. By doing so they generate an inefficient distribution of queues which decreases the total production sold. Furthermore, firms offer market insurance as well. They increase the job finding rate of workers by increasing entry at the expense of lower wages. This distorts the allocation by an inefficient increase in entry costs. On top of that, the underpricing by the self-employed forces the firms to lower their prices as well, which exerts another downward pressure on wages. Consequently, wages and both prices are lower than in the efficient allocation.

Risk aversion tilts the career choice decision towards the safer alternative, so that the composition of employment is distorted as well. The two other sources of inefficiency also affect the career choice decision, however, so that the self-employment rate can be either lower or higher than in the planner equilibrium. Thus, the self-employment rate may *increase* when we make *all* agents *identically* risk averse, a prediction of the model that goes against the conventional wisdom that postulates less risk averse individuals to self-select into self-employment.

As demonstrated in Figure 5, the composition of employment in the market equilibrium can even coincide with the planner equilibrium. Generically, however, the decentralized allocation features too little or too much self-employment, depending on entry cost k. When k is large, firms are reluctant to enter and the scope for labor market insurance is narrow, so that there is too much self-employment. This is in strong contrast to the conventional wisdom that risk aversion decreases self-employment. In fact, when firm entry costs are high and there can be large unemployment, the risk-averse agents prefer to self-insure. For lower values of k the firm entry margin dominates and self-employment is below the efficient level. Needless to say, a market equilibrium that features the right composition of employment is still inefficient, since the price and wage posting decisions continue to be distorted by inefficiently long queues at the self-employed.

The next section studies optimal insurance policies under risk-averse preferences, when the market is no longer efficient.

5 Risk Averse Workers: Efficient Insurance Policy

Efficient insurance policy. Having discussed the inefficiency of a market equilibrium under risk-averse preferences, we now move towards an analysis of an efficient insurance policy. We consider type-of-employment-dependent policies that satisfy the following definition:

Definition 4 (Balanced budget policies) *A balanced budget policy is a tuple of taxes and unemployment benefits $\mathcal{P} = \{\tau_{SE}, \tau_W, b_{SE}, b_W\}$ that satisfy the following condition:*

$$b_E\left(1-\mu\left(x_W\right)\right)W + b_{SE}\left(1-\lambda\left(x_{SE}\right)\right)SE = \tau_W W + \tau_{SE} SE. \tag{12}$$

For analytical tractability, we illustrate the features of the policy under CARA preferences with a risk aversion parameter θ:

$$u\left(c\right) = \frac{1-e^{-\theta c}}{\theta}.$$

Observe that the introduction of the policy instruments affects the price posting by self-employed, wage posting by firms, and values of workers and self-employed, respectively. These equations now

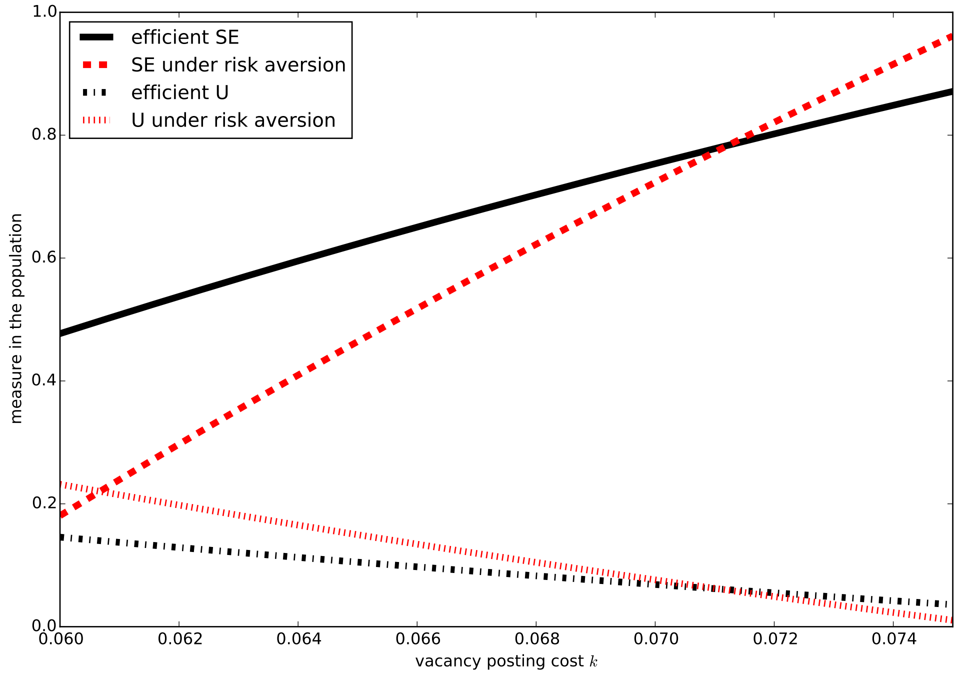

Figure 5: Self-employment and unemployment in the decentralized and planner equilibrium as a function of the vacancy posting cost k.

read:

$$\begin{aligned}
\frac{\phi\left(x_W\right)\left[A\lambda\left(x_F\right)p_F - w\right]}{1-\phi\left(x_W\right)} &= \frac{u\left(w-\tau_W\right)-u\left(b_W-\tau_W\right)}{u'\left(w-\tau_W\right)}, \\
\frac{\phi\left(x_{SE}\right)\left(1-p_{SE}\right)}{1-\phi\left(x_{SE}\right)} &= \frac{u\left(p_{SE}-\tau_{SE}\right)-u\left(b_{SE}-\tau_{SE}\right)}{u'\left(p_{SE}-\tau_{SE}\right)}, \\
V^W\left(\mathcal{P}\right) &= \mu\left(x_W\right)u\left(w-\tau_W\right)+\left(1-\mu\left(x_W\right)\right)u\left(b_W-\tau_W\right), \\
V^{SE}\left(\mathcal{P}\right) &= \lambda\left(x_{SE}\right)u\left(p_{SE}-\tau_{SE}\right)+\left(1-\lambda\left(x_{SE}\right)u\left(b_{SE}-\tau_{SE}\right)\right).
\end{aligned}$$

A natural question unfolds: is it possible to decentralize the planner equilibrium using a balanced budget policy of type-of-employment-dependent taxes and unemployment benefits? The answer, as provided in the following theorem, is positive. More interestingly, there is a clear pattern on how the unemployment benefits and otherwise lump-sum taxes should be conditioned on the type of employment.

Theorem 3 (Efficient insurance policy) *Let agents' preferences be described by a CARA utility function with a risk aversion parameter θ. There exists a balanced budget policy $\mathcal{P}^*$ that for every*

θ decentralizes a planner equilibrium such that:

$$b_W = w^* - \frac{1}{\theta}\log\left(1+\theta w^*\right),$$
$$b_{SE} = p^* - \frac{1}{\theta}\log\left(1+\theta p^*\right),$$
$$V^W\left(\mathcal{P}^*\right) = V^{SE}\left(\mathcal{P}^*\right).$$

Moreover, the taxes are characterized by the following inequality:

$$\tau_w \leq \tau_s \iff \log\left(1+\left(1-\mu\left(x_W\right)\right)\theta w\right) - \theta w \geq \log\left(1+\left(1-\lambda\left(x_{SE},1\right)\right)\theta p_s\right) - \theta p_s.$$

Proof. See Appendix A. ■

Observe that whenever the price that prevails in the decentralized equilibrium under risk neutrality is larger than the wage, the unemployment insurance for the self-employed should be more generous. From the career choice indifference condition that implements the efficient allocation we know that this happens if and only if the selling probability is lower than the job finding probability. Thus, whenever the income from self-employment is riskier, the benefits targeting the self-employed should be higher. This has nothing to do, however, with risk sharing considerations and is solely driven by efficiency.

One can show that the policy instruments separately target the three margins of inefficiency. The unemployment insurance for the self-employed corrects their pricing decision. The unemployment benefits for workers corrects the wage posting decision. Finally, the mix of taxes ensures the correct composition of employment and balances the budget.

6 A dynamic version of the model

In this section we describe the steady state of a dynamic version of the model, and perform some comparative statics exercises. The dynamic model captures the idea that employment at a firm is a long-term relationship. In particular, jobs last in expectation for multiple periods and are destructed exogenously with a constant probability δ. Time is discrete, individuals and buyers live forever, and they discount future periods with a factor β. Self-employment's and buyers' outcomes are assumed independent across periods. Therefore, the dynamic version of the model has no consequences for modeling self-employment and buyers, as well as the firms' pricing decision. The value of being a buyer is thus given by:

$$(1-\beta)V^B = \eta\left(x_F\right)\left(1-p_F\right) = \eta\left(x_{SE}\right)\left(1-p_{SE}\right),$$

while the value of the self-employed is given by

$$(1-\beta)V^{SE} = \lambda\left(x_{SE}\right)u\left(p_{SE}\right),$$

normalizing $a = 1$ once more.

To minimize the differences with the static model, the timing of the model is such that (1) existing jobs are destroyed, (2) vacancies enter, (3) individuals make their career choice, (4) matches form, (5) buyers visit, and (6) individuals consume. As a result, the value of entering the labor market is equal to

$$V^{W} = \mu (x_W) u (w) + (1 - \mu (x_W)) u (b) + \beta \left[(1 - \mu (x_W) (1 - \delta)) V^{W} + \mu (x_W) (1 - \delta) V^{E} \right], \quad (13)$$

with the value of employment V^E being:

$$V^{E} = u (w) + \beta \left[\delta V^{W} + (1 - \delta) V^{E} \right].$$

Solving for V^E and substituting the result in (13), it can be seen that the value of entering the labor market is still a weighted average of the expected time spent in employment and in unemployment. As a result, the career choice is not so much different in the dynamic version of the model, and individuals are indifferent between self-employment and entering the labor market if and only if

$$\lambda (x_{SE}) u (p_{SE}) = \frac{\mu (x_W) u (w) + (1 - \mu (x_W)) (1 - \beta (1 - \delta)) u (b)}{\mu (x_W) + (1 - \mu (x_W)) (1 - \beta (1 - \delta))}.$$

The assumption that jobs in expectation last for multiple periods also affects firm entry. The value of opening a vacancy is now

$$V^{V} = -k + q (x_W) V^{J}, \quad (14)$$

where V^J is the value of a filled vacancy (the value of a job to the firm):

$$V^{J} = A\lambda (x_F) p_F - w + \beta (1 - \delta) V^{J}.$$

Solving for V^J, substituting the result in (14), and closing the model by free entry implies:

$$q (x_W) \frac{A\lambda (x_F) p_F - w}{1 - \beta (1 - \delta)} = k \quad (15)$$

As before, firms maximize expected profits by posting a wage, taking into account its effect on x_W, constrained by the requirement to offer at least V^W to workers. As shown in Appendix B, this results in the following wage condition:

$$\frac{\phi (x_W)}{1 - \phi (x_W)} \left[A\lambda (x_F) p_F - w \right] = \frac{1 - \beta (1 - \delta)}{1 - \beta (1 - \delta) (1 - \mu (x_W))} \frac{u (w) - u (b)}{u' (w)}.$$

Finally, we consider the stocks and flows of the dynamic model. Let F_t now denote the measure of active firms at time t, and V_t the measure of vacancies opened in period t. The flows are such that

$$\begin{aligned}
F_t &= q\left(x_{W,t}\right)V_t + (1-\delta)\,F_{t-1},\\
U_t &= \left(1-\mu\left(x_{W,t}\right)\right)\left(1-SE_t-(1-\delta)E_{t-1}\right),\\
E_t &= \mu\left(x_{W,t}\right)\left(1-SE_t-(1-\delta)E_{t-1}\right)+(1-\delta)\,E_{t-1},
\end{aligned}$$

with the measure of active jobs $E_t = F_t$, the measure of workers in period t equal to $1 - SE_t - (1-\delta)E_{t-1}$, the queue length $x_{W,t} = \frac{1-SE_t-(1-\delta)E_{t-1}}{V_t}$, and the measure of labor market matches $\mu\left(x_{W,t}\right)\left(1-SE_t-(1-\delta)E_{t-1}\right) = q\left(x_{W,t}\right)V_t$. In steady state the measure of self-employed is constant, and by definition $1-SE-(1-\delta)E = U+\delta E$, so that the steady state satisfies:

$$q\left(x_W\right)V = \delta F = \delta E = \mu\left(x_W\right)\left(U+\delta E\right).$$

Efficiency. In Appendix B, we show that the decentralized steady state allocation of the dynamic model is efficient if individuals are risk-neutral, just as the static model. Here we define welfare as the present discounted number of goods sold net of recruiting costs. The social planner then solves the following problem:

$$\max_{\left\{x_{SE,t},V_t,E_t,SE_t\right\}_{t=1}^{\infty}} \sum_{t=1}^{\infty}\beta^t\left[x_{F,t}\eta\left(x_{F,t}\right)AE_t + x_{SE,t}\eta\left(x_{SE,t}\right)SE_t - V_t k\right],$$

subject to

$$E_t = q\left(x_{W,t}\right)V_t + (1-\delta)E_{t-1},$$

and an initial condition E_0, where $x_{W,t} = \frac{1-SE_t-(1-\delta)E_{t-1}}{V_t}$ from the unit mass of households, the career choice, and job survival, and where $x_{F,t} = \frac{1-x_{SE,t}SE_t}{AE_t}$ from the adding-up restriction in the goods market. In every period we again allow the social planner to choose the measure of vacancies to be opened and the measure of households to enter self-employment (and thus the measure to enter the labor market). The social planner still faces the same coordination frictions within every (sub)market as present in the decentralized equilibrium, but can still decide the measure of buyers to go shopping at the self-employed (and thus the measure of buyers that visits firms). Finally, note that choosing the latter, given SE_t and V_t, still amounts to choosing $x_{SE,t}$, because the only state variable E_t is also determined by SE_t and V_t (and E_{t-1}).

Comparative statics. Summing up, the steady state of a dynamic version of the model is not so much different from the static model. However, it captures the idea that jobs last for multiple periods and introduces two additional parameters: patience, and the expected duration of the wage contract.

In Figure 6 we show the response of the equilibrium self-employment rate to changes in the discount factor β and the job destruction probability δ. Not surprisingly, a higher discount factor and lower job separation probability make the long-term nature of a wage contract more valuable to individuals, which decreases the self-employment rate.

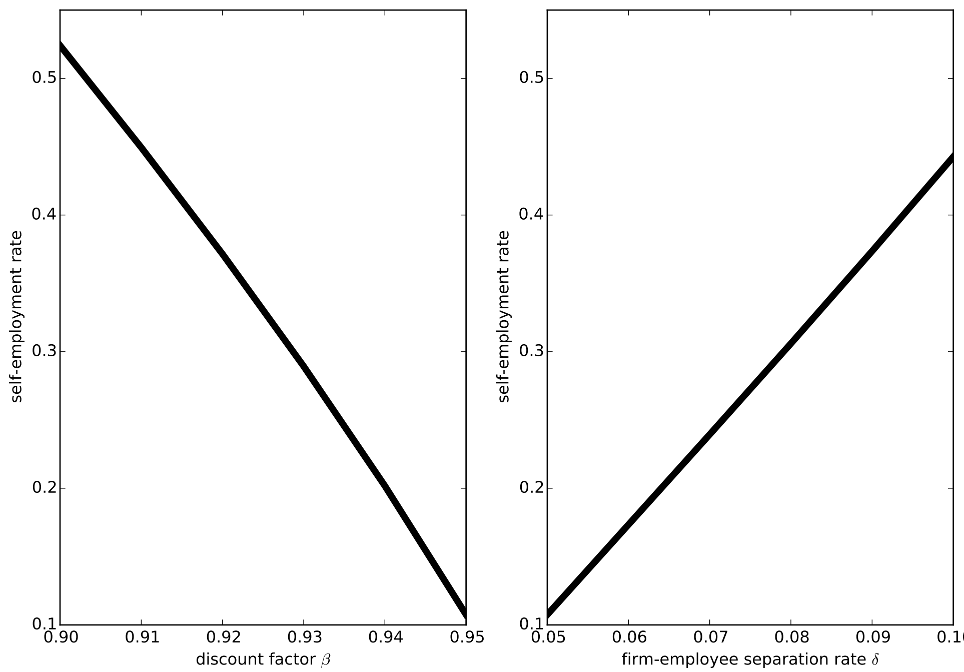

Figure 6: Equilibrium self-employment rate as a function of β and δ.

7 Conclusions

We have built a new theory of self-employment that emphasizes the trade-off between the frictions in the goods and in the labor market. Our theory, unlike a vast body of earlier research, does not rely on individual heterogeneity. It also offers microfoundations for the differences in the riskiness of payroll and self-employment incomes. In our model the self-employed forego the search friction in the labor market and the sharing of the match surplus with the firm. They are exposed, however, to the search friction in the goods market.

We also show that the decentralized equilibrium is inefficient if individuals are risk averse. In this case explicitly modeling the trade in the goods market is crucial, as risk-averse self-employed steal business from firms by under-pricing. The under-pricing is a form of self-insurance, which motive is absent in the firms' pricing decisions. Interestingly, we show that in this environment unemployment insurance for self-employed can improve efficiency, because it decreases the incentives to self-insure via pricing, increasing firm entry and improving prospects in the labor market.

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A Proofs for the static model

Proof of Lemma 1 (Optimal price posting).

The self-employed post prices to maximize their expected utility as given in (3), offering buyers at least their value V^B as given in (2). Using the latter, the price that the self-employed post can be written as a function of V^B and x_{SE}:

$$p_{SE} = 1 - \frac{V^B}{\eta\left(x_{SE}\right)}.$$

Substituting out p_{SE}, the self-employed problem can then be written as a choice op the optimal queue length:

$$\max_{x_{SE}} x_{SE}\eta\left(x_{SE}\right) u\left(1 - \frac{V^B}{\eta\left(x_{SE}\right)}\right).$$

The first-order condition yields:

$$\eta\left(x_{SE}\right) u\left(p_{SE}\right) + x_{SE}\eta'\left(x_{SE}\right) u\left(p_{SE}\right) + x_{SE}\eta\left(x_{SE}\right) u'\left(p_{SE}\right) \frac{\partial p_{SE}}{\partial \eta\left(x_{SE}\right)}\eta'\left(x_{SE}\right) = 0.$$

Dividing by $\eta\left(x_{SE}\right)$,

$$\left(1 - \phi\left(x_{SE}\right)\right) u\left(p_{SE}\right) - \eta\left(x_{SE}\right)\phi\left(x_{SE}\right) u'\left(p_{SE}\right) \frac{\partial p_{SE}}{\partial \eta\left(x_{SE}\right)} = 0$$

Using that $\frac{\partial p_{SE}}{\partial \eta(x_{SE})} = \frac{V^B}{\eta^2(x_{SE})}$, and substituting out V^B, we get:

$$\left(1 - \phi\left(x_{SE}\right)\right) u\left(p_{SE}\right) = \phi\left(x_{SE}\right) u'\left(p_{SE}\right)\left(1 - p_{SE}\right), \tag{16}$$

which is equal to (4). The price-posting problem of firms is the same, except that firms maximize expected revenue instead of utility. Replacing $u\left(p_{SE}\right)$ by p_F and $u'\left(p_{SE}\right)$ by 1 results in (5).

Proof of Lemma 2 (Optimal wage posting).

After paying an entry cost k, the firm optimally chooses the queue length to maximize profits

$$\Pi = q\left(x_w\right)\left(A\lambda\left(x_F\right) p_F - w\right),$$

subject to the market utility of workers V^W as given in (6). Via this outside option, the wage to be paid also depends on x_W, so that the first-order condition is

$$q'\left(x_W\right)\left(A\lambda\left(x_F\right)p_F-w\right)-q\left(x_w\right)\frac{dw}{dx_w}=0.$$

The derivative of the wage is obtained from totally differentiating V^W, which is fixed:

$$0=\mu'\left(x_W\right)u\left(w\right)+\mu\left(x_W\right)u'\left(w\right)\frac{dw}{dx_W},$$

so that the optimality condition reads

$$q'\left(x_W\right)\left(A\lambda\left(x_F\right)p_F-w\right)=-q\left(x_w\right)\frac{\mu'\left(x_W\right)}{\mu\left(x_w\right)}\frac{u\left(w\right)}{u'\left(w\right)}.$$

Finally, note that $-\frac{x_W\mu'(x_W)}{\mu(x_w)}=1-\phi\left(x_W\right)$, and (7) results.

Proof of Theorem 1 and Proposition 1 (Mixed equilibrium existence and its properties).

For simplicity fix $B=1$. The equilibrium is pinned by the system of the following 8 equations:

$$\begin{aligned}
ax_{SE}SE+Ax_FF&=1,\\
\frac{\phi\left(x_{SE}\right)\left(1-p_{SE}\right)}{1-\phi\left(x_{SE}\right)}&=\frac{u\left(p_{SE}\right)}{u'\left(p_{SE}\right)},\\
p_F&=\phi\left(x_F\right),\\
\frac{\phi\left(x_W\right)\left[A\lambda\left(x_F\right)p_F-w\right]}{1-\phi\left(x_W\right)}&=\frac{u\left(w\right)}{u'\left(w\right)},\\
\eta\left(x_F\right)\left(1-p_F\right)&=\eta\left(x_{SE}\right)\left(1-p_{SE}\right).\\
q\left(x_W\right)\left[A\lambda(x_F)p_F-w\right]&=k,\\
\mu\left(x_W\right)u\left(w\right)&=\lambda\left(x_{SE}\right)u\left(p_{SE}\right),\\
SE+W&=1.
\end{aligned}$$

In equilibrium agents play a mixed strategy iff all the equations hold and $1>SE,W>0$. Combining the adding-up restrictions in the goods and in the labor market we end up with the following *mixing condition*:

$$0<W=\frac{1-ax_{SE}}{Ax_F\mu\left(x_W\right)-ax_{SE}}<1, \tag{17}$$

It follows that $\lim_{x_{SE}\mapsto\frac{1}{a}^+}W=0^+$ and $\lim_{Ax_F\mu(x_W)\mapsto1^+}W=1^-$. as long as the denominator is not approaching zero and is positive. Then, we can substitute out prices and the wage so that together with the mixing condition we have the following three equations with three unknowns

$\{x_{SE}, x_F, x_W\}$:

$$\frac{\phi(x_W)\left[\frac{k}{q(x_W)}\right]}{1-\phi(x_W)} = \frac{u\left(A\lambda(x_F)\phi(x_F) - \frac{k}{q(x_W)}\right)}{u'\left(A\lambda(x_F)\phi(x_F) - \frac{k}{q(x_W)}\right)} \tag{18}$$

$$\mu(x_W)\, u\left(A\lambda(x_F)\phi(x_F) - \frac{k}{q(x_W)}\right) = \lambda(x_{SE})\, u\left(1 - \frac{\eta(x_F)(1-\phi(x_F))}{\eta(x_{SE})}\right) \tag{19}$$

$$\frac{\phi(x_{SE})\left(\frac{\eta(x_F)(1-\phi(x_F))}{\eta(x_{SE})}\right)}{1-\phi(x_{SE})} = \frac{u\left(1 - \frac{\eta(x_F)(1-\phi(x_F))}{\eta(x_{SE})}\right)}{u'\left(1 - \frac{\eta(x_F)(1-\phi(x_F))}{\eta(x_{SE})}\right)}. \tag{20}$$

The existence and uniqueness follow from invokign the Implicit Function Theorem on this system of equations. We start with the risk-neutral case as the mechanics of the proof become clearer.

Risk neutral preferences. If the self-employed are risk neutral, they maximize expected revenue $a\lambda(x_{SE})\, p_{SE}$. Because sellers sell one good per outlet, the normalization of $a = 1$ does not affect the derivation, and risk neutral self-employed post prices according to $p_{SE} = \phi(x_{SE})$, as derived above. Substituting this price in the buyers' indifference condition (2), it follows that firm and self-employed sellers can expect the same queue length and set the same prices.

If workers are risk neutral, the wage posting decision of firms as given in (7) implies $w = \phi(x_W)\, A\lambda(x_F)\, p_F$. Substituting this wage in (8) results in the following free entry condition:

$$q(x_W)\left[1-\phi(x_W)\right] A\lambda(x_F)p_F = k. \tag{21}$$

Besides, as $\mu(x_W)\,\phi(x_W) = q'(x_W)$, the value of being a worker can in this case be written as

$$V^W = q'(x_W)\, A\lambda(x_F)\, p_F. \tag{22}$$

As a result, indifference in the career choice game simply requires

$$V^{SE} = a\lambda(x_{SE})\, p_{SE} = q'(x_W)\, A\lambda(x_F)\, p_F = V^W. \tag{23}$$

Because self-employed and firms post the same prices, indifference in career choice implies

$$a/A = q'(x_W) = e^{-x_W} \to x_W = \ln\left(\frac{A}{a}\right) \tag{24}$$

Thus, if the mixed equilibrium exists, the queue length of prospective workers is independent of the vacancy posting cost k and only depends on a and A.

Equation (20) boils down to:

$$e^{-x_{SE}} = e^{-x_F} \to x_F = x_{SE}.$$

Then, as shown in the main body of the paper, equation (19) implies $x_W = \ln\left(\frac{A}{a}\right)$. The mixing condition can be restated as:

$$0 < \frac{1 - a x_{SE}}{\left(\frac{A-a}{\log\left(\frac{A}{a}\right)} - a\right) x_{SE}} < 1.$$

This result places bounds on x_{SE} for $1 > W > 0$, namely: $\frac{1}{a} > x_{SE} > \frac{\log\left(\frac{A}{a}\right)}{A-a}$ which can only hold if the following inequality is true:

$$\frac{A}{a} > 1 + \log\left(\frac{A}{a}\right). \tag{25}$$

Observe that this condition, the first inequality constraint on the exogenous parameters to have a mixed equilibrium, also guarantees that the share of workers W is non-negative. Also note that condition (25) is always satisfied if $A > a$. The only condition equilibrium that is left, after simple algebra and evaluation at $x_W = \log\left(\frac{A}{a}\right)$ reads:

$$h\left(x_{SE}, a, A, k\right) \equiv \frac{A - a - a\log\left(\frac{A}{a}\right)}{A}\left(1 - x_{SE}e^{-x_{SE}} - e^{-x_{SE}}\right) - k = 0. \tag{26}$$

Differentiating equation (26) we find the following relationships:

$$\begin{aligned}
\frac{\partial h}{\partial x_{SE}} &= \frac{A - a - a\log\left(\frac{A}{a}\right)}{A} x_{SE}e^{-x_{SE}} > 0, \\
\frac{\partial h}{\partial k} &= -1 < 0, \\
\frac{\partial h}{\partial A} &= \frac{a}{A^2}\log\left(\frac{A}{a}\right)\left(1 - x_{SE}e^{-x_{SE}} - e^{-x_{SE}}\right) > 0, \\
\frac{\partial h}{\partial a} &= -\frac{1}{A}\log\left(\frac{A}{a}\right)\left(1 - x_{SE}e^{-x_{SE}} - e^{-x_{SE}}\right) < 0.
\end{aligned}$$

Thus, from implicit function theorem we get, in particular, that $\frac{\partial x_{SE}}{\partial k} > 0$. Then, the bounds for the mixed equilibrium to exist follow from evaluating (26) at $x_{SE} \mapsto \frac{1}{a}^{-}$ to get $\overline{k}\left(A, a\right)$ and $x_{SE} \mapsto \frac{\log\left(\frac{A}{a}\right)}{A-a}^{+}$ to get $\underline{k}\left(A, a\right)$. The comparative statics of the self-employment rate follow from total differentiation of the mixing condition. Let $\vartheta \in \{a, A, k\}$:

$$\frac{\partial SE^*}{\partial \vartheta} = -\frac{\partial W}{\partial \vartheta}.$$

To find the derivative of W with respect to k we make use of chain rule: $\frac{\partial W}{\partial k} = \frac{\partial W}{\partial x_{SE}}\frac{\partial x_{SE}}{\partial k}$. From the Implicit Function Theorem applied to $h\left(a, A, k\right)$ we have that $\frac{\partial x_{SE}}{\partial k} > 0$. The derivative of W wrt to x_{SE} reads:

$$\frac{-\left(\frac{A-a}{\log\left(\frac{A}{a}\right)} - a\right)}{\left[\left(\frac{A-a}{\log\left(\frac{A}{a}\right)} - a\right) x_{SE}\right]^2} < 0,$$

which implies that

$$\frac{\partial SE^{*}}{\partial k} = -\underbrace{\frac{\partial W}{\partial x_{SE}}}_{<0} \overbrace{\frac{\partial x_{SE}}{\partial k}}^{>0} > 0. \tag{27}$$

The derivatives of bounds on k, $\underline{k}(A,a)$ and $\overline{k}(A,a)$ with respect to A and a also follow from the implicit function theorem:

$$\frac{\partial k}{\partial A} = -\frac{\frac{\partial h}{\partial k}}{\frac{\partial h}{\partial A}} > 0,$$
$$\frac{\partial k}{\partial A} = -\frac{\frac{\partial h}{\partial k}}{\frac{\partial h}{\partial a}} < 0.$$

Preferences with risk aversion. We assume CRRA preferences, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ so that $\frac{u(c)}{u'(c)} = \frac{c}{1-\gamma}$. To understand the effect of risk aversion on the equilibrium we will vary $\gamma \in [0,1]$. Our first result is that evaluating the price posting by the self-employed leads to the following relationship between queue lengths in the mixed equilibrium:

$$x_F = x_{SE} + \log\left(1 - \gamma\phi(x_{SE})\right). \tag{28}$$

Observe that this implies $x_F = x_{SE}$ if and only if $\gamma = 0$. Otherwise we have $\frac{x_F}{x_{SE}} < 1$ whenever the mixed equilibrium exists and this ratio is decreasing in risk aversion parameter γ. Then, we have two equations with two unknowns x_W and x_{SE} and one parameter k:

$$\frac{(1-\gamma)\phi(x_W)\left[\frac{k}{q(x_W)}\right]}{1-\phi(x_W)} = A\lambda(x_F)\phi(x_F) - \frac{k}{q(x_W)} \tag{29}$$

$$\mu(x_W)\left(A\lambda(x_F)\phi(x_F) - \frac{k}{q(x_W)}\right)^{1-\gamma} = \lambda(x_{SE})\left(1 - \frac{\eta(x_F)(1-\phi(x_F))}{\eta(x_{SE})}\right)^{1-\gamma} \tag{30}$$

This we can further simplify:

$$\frac{k}{q(x_W)} = A\lambda(x_F)\phi(x_F)\left[\frac{1-\phi(x_W)}{1-\gamma\phi(x_W)}\right],$$

so that the worker's career choice indifference condition reads:

$$\mu(x_W)\left(A\lambda(x_F)\phi(x_F)\frac{(1-\gamma)\phi(x_W)}{1-\gamma\phi(x_W)}\right)^{1-\gamma} = \lambda(x_{SE})\left(1 - \frac{\eta(x_F)(1-\phi(x_F))}{\eta(x_{SE})}\right)^{1-\gamma}.$$

Now we set this equation to zero and define it as $z(x_W, x_{SE}) = 0$. Collecting terms, taking logs and differentiating results in:

$$\frac{\partial z}{\partial x_W} = \frac{\mu'(x_W)}{\mu(x_W)} + (1-\gamma)\frac{\phi'(x_W)}{\phi(x_W)(1-\gamma\phi(x_W))} < 0 \quad \text{whenever } x_W \neq 0.$$

The part of $z(x_W, x_{SE})$ that is relevant for computing $\frac{\partial z}{\partial x_{SE}}$ reads:

$$\tilde{z}(x_W, x_{SE}) = \log\left(\frac{\lambda(x_F)}{\lambda(x_{SE})}\right) - \gamma\log(\lambda(x_F)) + (1-\gamma)\log\left(\frac{p_F}{p_{SE}}\right).$$

Under risk neutrality, $\gamma = 0$ and we have that the derivative of z with respect to x_{SE} is zero which also implies that x_W does not respond to x_{SE}. After some tedious algebra one can show that

$$\frac{\partial z}{\partial x_{SE}} < 0,$$

which also implies that there exists a function $x_W(x_{SE})$, decreasing in its argument. From here we can already establish the existence of equilibrium bounds on k, as the right hand side of identity:

$$k = q(x_W)A\lambda(x_F)\phi(x_F)\left[\frac{1-\phi(x_W)}{1-\gamma\phi(x_W)}\right],$$

is strictly increasing in x_{SE} so that the logic of the existence proof for the risk neutral preferences carries over.

Proof of Theorem 2 (Efficiency).

Using the adding-up constraint in (1), the objective can be written as:

$$V^{SP}(V, x_{SE}, SE) = a x_{SE} SE(\eta(x_{SE}) - \eta(x_F)) + \eta(x_F) - Vk. \tag{31}$$

For future reference, note that the partial derivatives of x_F with respect to the choice variables of the social planner are given by:

$$\begin{aligned} \frac{\partial x_F}{\partial V} &= \frac{x_F}{V}(\phi(x_W) - 1), \\ \frac{\partial x_F}{\partial x_{SE}} &= -\frac{aSE}{Aq(x_W)V}, \\ \frac{\partial x_F}{\partial SE} &= \frac{x_F A q'(x_W) - a x_{SE}}{Aq(x_W)V}. \end{aligned}$$

Let $\Delta\eta \equiv \eta(x_{SE}) - \eta(x_F)$. Taking the first order condition with respect to the measure of firms:

$$\frac{\partial V^{SP}}{\partial V} = \eta'(x_F)\frac{\partial x_F}{\partial V}(1 - ax_{SE}SE) - k = 0,$$
$$= \eta'(x_F)(x_F)^2 Aq(x_W)(\phi(x_W) - 1) - k = 0,$$
$$= (1 - \phi(x_W)) Aq(x_W)\lambda(x_F)\phi(x_F) - k = 0,$$

which is exactly the free entry condition in the decentralized equilibrium if workers are risk neutral as given in (21), since firms always post $p_F = \phi(x_F)$.

Taking the first order condition with respect to the queue length at the self-employed:

$$\frac{\partial V^{SP}}{\partial x_{SE}} = aSE\Delta\eta + ax_{SE}SE\eta'(x_{SE}) + (1 - ax_{SE}SE)\eta'(x_F)\frac{\partial x_F}{\partial x_{SE}} = 0,$$
$$= \eta(x_{SE})(1 - \phi(x_{SE})) - \eta(x_F)(1 - \phi(x_F)) = 0,$$

which (only) for risk neutral self-employed is exactly the buyer indifference condition in the goods market as in (2), since they post $p_{SE} = \phi(x_{SE})$ (and firms post $p_F = \phi(x_F)$).

Finally, the first order condition with respect to the measure of self-employed:

$$\frac{\partial V^{SP}}{\partial SE} = ax_{SE}\Delta\eta + (1 - ax_{SE}SE)\eta'(x_F)\frac{\partial x_F}{\partial SE} = 0,$$
$$= ax_{SE}\Delta\eta + x_F\eta'(x_F)\left(x_F Aq'(x_W) - ax_{SE}\right) = 0,$$
$$= \Delta\eta + \eta(x_F)\phi(x_F)\left(1 - \frac{x_F Aq'(x_W)}{x_{SEa}}\right) = 0,$$
$$= \eta(x_{SE})\left(1 - \frac{x_F\eta(x_F)Aq'(x_W)\phi(x_F)}{x_{SE}\eta(x_{SE})a}\right) - \eta(x_F)(1 - \phi(x_F)) = 0.$$

This equation is equal to the buyer indifference condition in the goods market if:

$$\phi(x_{SE}) = \frac{A\lambda(x_F)q'(x_W)\phi(x_F)}{a\lambda(x_{SE})},$$

which is exactly the condition that makes risk neutral households indifferent between entering the goods market as self-employed on the one hand and entering the labor market on the other. Indeed, it makes V^W as given in (22) equal to (3) when self-employed post $p_{SE} = \phi(x_{SE})$. Hence, risk-neutrality is sufficient for constrained efficiency.

Now, let us assume that in the decentralized allocation we have the measure of firms, the composition of employment, and the queues in the goods market that coincide with the planner solution. In other words, we assume that the decentralized allocation $\left\{\tilde{V}, \tilde{SE}, \tilde{x}_{SE}\right\}$ exactly matches its planner counterpart $\{V^*, SE^*, x^*_{SE}\}$. Then, let us assume that individuals are risk averse. Given that $x^*_{SE} = \tilde{x}_{SE}$, we can directly compare $\tilde{p}_{SE}$ and the $p^*_{SE} = \phi(x_{SE,1})$ that decentralizes the planner's solution. From the optimal price posting condition we get that under risk aversion $p^*_{SE} \neq \tilde{p}_{SE}$, which implies that the buyers' indifference condition is violated: the buying probabilities equal their counterparts from the planner's solution, but the buyers have an incentive to choose visits at the self-employed more often. Thus, one of the planner's solution conditions is violated. This demonstrates the necessity of the risk-neutrality assumption.

Consequently, the decentralized allocation is maximizing net production sold if and only if individuals are risk neutral.

Proof of the balanced budget policy properties

Formally, we consider policies that have to satisfy the balanced-budget identity:

$$b_W \left(1-\mu\left(x_W\right)\right) W + b_{SE} \left(1-\lambda\left(x_{SE},1\right)\right) SE = \tau_W W + SE\tau_{SE}. \tag{32}$$

The introduction of taxes and insurance changes the wage posting by firms and the price posting by the self-employed. These equations now read:

$$\frac{\phi\left(x_W\right)\left[\lambda\left(x_F,A\right)p_F - w\right]}{1-\phi\left(x_W\right)} = \frac{u\left(w-\tau_W\right)-u\left(b_W-\tau_W\right)}{u'\left(w-\tau_W\right)},$$

$$\frac{\phi\left(x_{SE},1\right)\left(1-p_{SE}\right)}{1-\phi\left(x_{SE},1\right)} = \frac{u\left(p_{SE}-\tau_{SE}\right)-u\left(b_{SE}-\tau_{SE}\right)}{u'\left(p_{SE}-\tau_{SE}\right)}.$$

Observe, that the taxes themselves are not relevant in the pricing/wage posting decision, as they affect agents' wealth in all the states (employed/unemployed/selling/not selling) but as CARA features no wealth effect τ_i drop out. They do matter, however, in making relative comparison of the career choices available. Let's start with pricing by the self-employed:

$$\frac{\phi\left(x_{SE},1\right)\left(1-p_{SE}\right)}{1-\phi\left(x_{SE},1\right)} = \frac{1}{\theta}\frac{e^{-\theta\left(b_{SE}-\tau_{SE}\right)}-e^{-\theta\left(p_{SE}-\tau_{SE}\right)}}{e^{-\theta\left(p_{SE}-\tau_{SE}\right)}} = \frac{1}{\theta}\left(e^{-\theta\left(b_{SE}-p_{SE}\right)}-1\right).$$

Now, suppose we wish to find b_{SE} that implements $p_{SE}=\phi\left(x^*_{SE},1\right)$ with the queue length as in the planner allocation. Then, it has to satisfy the following condition:

$$b_{SE} = \phi\left(x^*_{SE},1\right) - \frac{1}{\theta}\log\left(1+\theta\phi\left(x^*_{SE},1\right)\right).$$

The wage posting works in an analogous way, namely:

$$\frac{\phi\left(x_W\right)\left[\lambda\left(x_F,A\right)p_F - w\right]}{1-\phi\left(x_W\right)} = \frac{1}{\theta}\left(e^{-\theta\left(b_W-w\right)}-1\right).$$

Let's do the same for b_W. The efficient wage satisfies $w=\phi\left(x_W\right)\lambda\left(x_F,A\right)p_F$ so that:

$$\phi\left(x_W\right)\lambda\left(x_F,A\right)p_F = \frac{1}{\theta}\left(e^{-\theta\left(b_W-w\right)}-1\right)$$

$$b_W = \phi\left(x_W\right)\lambda\left(x^*_F,A\right)p^*_F - \frac{1}{\theta}\log\left(1+\theta\phi\left(x_W\right)\lambda\left(x^*_F,A\right)p^*_F\right)$$

Observe, that these conditions have an easy interpretation, namely:

$$b_W = w * -\frac{1}{\theta}\log\left(1+\theta w^*\right)$$
$$b_{SE} = p^* - \frac{1}{\theta}\log\left(1+\theta p^*\right)$$

The worker indifference condition reads:

$$\begin{aligned} V^W &= V^{SE} \text{ with:} \\ V^W &= \mu\left(x_W\right) u\left(w-\tau_W\right) + \left(1-\mu\left(x_W\right)\right) u\left(b_W-\tau_W\right) \\ V^{SE} &= \lambda\left(x_{SE},1\right) u\left(p_{SE}-\tau_{SE}\right) + \left(1-\lambda\left(x_{SE},1\right)\right) u\left(b_{SE}-\tau_{SE}\right) \end{aligned}$$

so that:

$$\mu\left(x_W\right) u\left(w-\tau_w\right) + \left(1-\mu\left(x_W\right)\right) u\left(b_w-\tau_w\right) = \lambda\left(x_{SE},1\right) u\left(p_s-\tau_s\right) + \left(1-\lambda\left(x_{SE},1\right)\right) u\left(b_s-\tau_s\right)$$

as we have $u\left(c\right) = \frac{1-e^{-\theta c}}{\theta}$ we arrive at the following:

$$\mu\left(x_W\right) e^{-\theta\left(w-\tau_w\right)} + \left(1-\mu\left(x_W\right)\right) e^{-\theta\left(b_w-\tau_w\right)} = \lambda\left(x_{SE},1\right) e^{-\theta\left(p_s-\tau_s\right)} + \left(1-\lambda\left(x_{SE},1\right)\right) e^{-\theta\left(b_s-\tau_s\right)}$$

so that:

$$e^{\theta\tau_w}\left(\mu\left(x_W\right) e^{-\theta w} + \left(1-\mu\left(x_W\right)\right) e^{-\theta b_w}\right) = e^{\theta\tau_s}\left(\lambda\left(x_{SE},1\right) e^{-\theta p_s} + \left(1-\lambda\left(x_{SE},1\right)\right) e^{-\theta b_s}\right)$$

Using the analytical expressions for b_W and b_{SE} we can rewrite the career choice equation as:

$$\begin{gathered} e^{\theta\tau_w} e^{-\theta w}\left(\mu\left(x_W\right) + \left(1-\mu\left(x_W\right)\right)\left(1+\theta w\right)\right) = e^{\theta\tau_{se}} e^{-\theta p_s}\left(\lambda\left(x_{SE},1\right) + \left(1-\lambda\left(x_{SE},1\right)\right)\left(1+\theta p_s\right)\right), \\ \theta\tau_w - \theta w + \log\left(1+\left(1-\mu\left(x_W\right)\right)\theta w\right) = \theta\tau_{se} - \theta p_s + \log\left(1+\left(1-\lambda\left(x_{SE},1\right)\right)\theta p_s\right). \end{gathered}$$

The taxes can therefore be ranked such that

$$\tau_w < \tau_s \iff \log\left(1+\left(1-\mu\left(x_W\right)\right)\theta w\right) - \theta w > \log\left(1+\left(1-\lambda\left(x_{SE},1\right)\right)\theta p_s\right) - \theta p_s.$$

If θ is small, this ranking can be approximated by

$$\tau_w < \tau_s \iff \mu\left(x_W\right)\theta w < \lambda\left(x_{SE},1\right)\theta p_s,$$

so that self-employment should be taxed more heavily than labor market participation whenever risk-neutral individuals would prefer self-employment over entering the labor market in an environment without taxation, given the unemployment benefits. Whenever we find that $b_{SE} > b_W$ (which happens always when $\mu^* > \lambda^*$) we may expect to have $\tau_{SE} > \tau_W$.

B Derivations of the dynamic model

B.1 Optimal wage posting under general preferences

To derive the wage condition under general preferences, let firms maximize the left-hand side of the free-entry condition:

$$\begin{aligned}
\frac{\phi\left(x_W\right)}{1-\phi\left(x_W\right)}\left[A\lambda\left(x_F\right)p_F-w\right] &= \frac{u\left(w\right)-\left(1-\beta\left(1-\delta\right)\right)u\left(b\right)-\beta\left(1-\delta\right)\left(1-\beta\right)V^W}{u'\left(w\right)}, \\
&= \left(1-\frac{\beta\left(1-\delta\right)\mu\left(x_W\right)}{\mu\left(x_W\right)+\left(1-\mu\left(x_W\right)\right)\left(1-\beta\left(1-\delta\right)\right)}\right)\frac{u\left(w\right)-u\left(b\right)}{u'\left(w\right)}, \\
&= \frac{1-\beta\left(1-\delta\right)}{1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\right)}\frac{u\left(w\right)-u\left(b\right)}{u'\left(w\right)}.
\end{aligned}$$

B.2 Risk-neutral individuals

If workers are risk-neutral and $b_{SE}=0$, then the optimal wage posting simplifies to

$$\begin{aligned}
\frac{\phi\left(x_W\right)}{1-\phi\left(x_W\right)}\left[A\lambda\left(x_F\right)p_F-w\right] &= \frac{1-\beta\left(1-\delta\right)}{1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\right)}w, \\
\left[\phi\left(x_W\right)A\lambda\left(x_F\right)p_F-\phi\left(x_W\right)w\right]\left[1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\right)\right] &= \left(1-\phi\left(x_W\right)\right)\left[1-\beta\left(1-\delta\right)\right]w, \\
\phi\left(x_W\right)A\lambda\left(x_F\right)p_F\left[1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\right)\right] &= \left[1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\phi\left(x_W\right)\right)\right]w,
\end{aligned}$$

$$w=\frac{\phi\left(x_W\right)A\lambda\left(x_F\right)p_F\left[1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\right)\right]}{1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\phi\left(x_W\right)\right)}. \tag{33}$$

Under these conditions, the value of entering the labor market is given by

$$(1-\beta)V^W=\frac{\mu\left(x_W\right)w}{1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\right)},$$

so that individuals are indifferent between careers if and only if

$$\left[1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\right)\right]\lambda\left(x_{SE}\right)p_{SE}=\mu\left(x_W\right)w. \tag{34}$$

Substituting the wage of (33) in this individuals' indifference condition, yields:

$$\begin{aligned}
\left[1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\right)\right]\lambda\left(x_{SE}\right)p_{SE} &= \mu\left(x_W\right)\frac{\phi\left(x_W\right)A\lambda\left(x_F\right)p_F\left[1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\right)\right]}{1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\phi\left(x_W\right)\right)}, \\
\lambda\left(x_{SE}\right)p_{SE} &= \frac{\mu\left(x_W\right)\phi\left(x_W\right)A\lambda\left(x_F\right)p_F}{1-\beta\left(1-\delta\right)\left(1-\mu\left(x_W\right)\phi\left(x_W\right)\right)}. \qquad (35)
\end{aligned}$$

Finally, substituting the wage of (33) in the free entry condition of (15), yields:

$$
\begin{aligned}
q(x_W)\left[A\lambda(x_F)p_F - \frac{\phi(x_W)A\lambda(x_F)p_F\left[1-\beta(1-\delta)(1-\mu(x_W))\right]}{1-\beta(1-\delta)(1-\mu(x_W)\phi(x_W))}\right] &= k\left[1-\beta(1-\delta)\right],\\
q(x_W)A\lambda(x_F)p_F\frac{\left[1-\beta(1-\delta)\right](1-\phi(x_W))}{1-\beta(1-\delta)(1-\mu(x_W)\phi(x_W))} &= k\left[1-\beta(1-\delta)\right],\\
\frac{q(x_W)A\lambda(x_F)p_F(1-\phi(x_W))}{1-\beta(1-\delta)(1-\mu(x_W)\phi(x_W))} &= k. \qquad (36)
\end{aligned}
$$

B.3 Efficiency

Using $x_{F,t} = \frac{1-x_{SE,t}SE_t}{AE_t}$, and defining $\Delta\eta_t \equiv \eta(x_{SE,t}) - \eta(x_{F,t})$, simplifies the objective of the social planner to:

$$
\max_{\{x_{SE,t},V_t,E_t,SE_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t\left[x_{SE,t}SE_t\Delta\eta_t + \eta(x_{F,t}) - V_t k\right]. \qquad (37)
$$

Choosing $x_{SE,t}$ is only an intra-temporal problem, but both SE_t and V_t determine future values of employment E. For that reason, we set up a Lagrangian:

$$
\mathcal{L} = \sum_{t=1}^{\infty}\left\{\beta^t\left[x_{SE,t}SE_t\Delta\eta_t + \eta(x_{F,t}) - V_t k\right] + \nu_t\left[q(x_{W,t})V_t + (1-\delta)E_{t-1} - E_t\right]\right\},
$$

where ν_t is the Lagrange multiplier on the law of motion for E_t.

The first-order condition with respect to $x_{SE,t}$ is

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial x_{SE,t}} = \beta^t\left[SE_t\Delta\eta_t + x_{SE,t}SE_t\left(\eta'(x_{SE,t}) + \eta'(x_{F,t})\frac{SE_t}{AE_t}\right) - \eta'(x_{F,t})\frac{SE_t}{AE_t}\right] &= 0,\\
= SE_t\Delta\eta_t + SE_t x_{SE,t}\eta'(x_{SE,t}) - SE_t x_{F,t}\eta'(x_{F,t}) &= 0,\\
= \eta(x_{SE,t})(1-\phi(x_{SE,t})) - \eta(x_{F,t})(1-\phi(x_{F,t})) &= 0, \qquad (38)
\end{aligned}
$$

which is the same intra-temporal condition for the goods market as in the static model. If the self-employed are risk-neutral, then $p_{SE,t} = \phi(x_{SE,t})$. Together with the price-setting of firms, this condition then coincides with the buyers' indifference condition of the decentralized allocation as given above. Consequently, the decentralized allocation in the goods market is efficient if individuals are risk-neutral.

The first-order condition with respect to V_t is

$$
\frac{\partial\mathcal{L}}{\partial V_t} = -\beta^t k + \nu_t\left[q(x_{W,t}) - q'(x_{W,t})x_{W,t}\right] = 0, \qquad (39)
$$

$$
\frac{\beta^t k}{q(x_{W,t})(1-\phi(x_{W,t}))} = \nu_t. \qquad (40)
$$

The first-order condition with respect to E_t is

$$\frac{\partial \mathcal{L}}{\partial E_t} = \beta^t \eta'(x_{F,t}) \frac{x_{F,t}}{E_t}\left[x_{SE,t} SE_t - 1\right] - \nu_t + \nu_{t+1}\left[(1-\delta) - q'(x_{W,t})(1-\delta)\right] = 0,$$
$$= -\beta^t \eta'(x_{F,t}) A x_{F,t}^2 - \nu_t + \nu_{t+1}(1-\delta)(1-\phi(x_{W,t})\mu(x_{W,t})) = 0,$$
$$= \beta^t A \lambda(x_{F,t})\phi(x_{F,t}) - \nu_t + \nu_{t+1}(1-\delta)(1-\phi(x_{W,t})\mu(x_{W,t})) = 0.$$

Substituting (40), yields

$$\beta^t A\lambda(x_{F,t})\phi(x_{F,t}) = \frac{\beta^t k}{q(x_{W,t})(1-\phi(x_{W,t}))} - \frac{\beta^{t+1}k(1-\delta)(1-\phi(x_{W,t})\mu(x_{W,t}))}{q(x_{W,t+1})(1-\phi(x_{W,t+1}))},$$
$$k = \frac{q(x_{W,t})(1-\phi(x_{W,t}))A\lambda(x_{F,t})\phi(x_{F,t})}{1 - \frac{q(x_{W,t})(1-\phi(x_{W,t}))}{q(x_{W,t+1})(1-\phi(x_{W,t+1}))}\beta(1-\delta)(1-\phi(x_{W,t})\mu(x_{W,t}))},$$

which is a first-order difference equation for optimal entry. In steady state the ratio of today's and tomorrow's matching rates and elasticities is equal to one, and this equation simplifies to

$$k = \frac{q(x_W)A\lambda(x_F)\phi(x_F)(1-\phi(x_W))}{1-\beta(1-\delta)(1-\mu(x_W)\phi(x_W))}, \tag{41}$$

which equals the free entry condition in (36) given that $p_F = \phi(x_F)$. Consequently, free entry in the decentralized allocation is optimal if wages are set for the case that workers are risk-neutral.

The first-order condition with respect to SE_t is

$$\frac{\partial \mathcal{L}}{\partial SE_t} = \beta^t\left[x_{SE,t}\Delta\eta_t + \frac{x_{SE,t}^2 SE_t \eta'(x_{F,t})}{AE_t} - \frac{\eta'(x_{F,t})x_{SE,t}}{AE_t}\right] - \nu_t q'(x_{W,t}) = 0,$$
$$= \beta^t\left[x_{SE,t}\Delta\eta_t - x_{SE,t}x_{F,t}\eta'(x_{F,t})\right] - \nu_t\mu(x_{W,t})\phi(x_{W,t}) = 0,$$
$$= \beta^t x_{SE,t}\left[\eta(x_{SE,t}) - \eta(x_{F,t})(1-\phi(x_{F,t}))\right] - \nu_t\mu(x_{W,t})\phi(x_{W,t}) = 0.$$

Substituting (40) and rearranging, results in

$$\beta^t x_{SE,t}\left[\eta(x_{SE,t}) - \eta(x_{F,t})(1-\phi(x_{F,t}))\right] = \frac{\beta^t k\mu(x_{W,t})\phi(x_{W,t})}{q(x_{W,t})(1-\phi(x_{W,t}))},$$
$$\eta(x_{SE,t}) - \eta(x_{F,t})(1-\phi(x_{F,t})) = \frac{\eta(x_{SE,t})k\mu(x_{W,t})\phi(x_{W,t})}{\lambda(x_{SE,t})q(x_{W,t})(1-\phi(x_{W,t}))},$$
$$\eta(x_{SE,t})\left(1 - \frac{k\mu(x_{W,t})\phi(x_{W,t})}{\lambda(x_{SE,t})q(x_{W,t})(1-\phi(x_{W,t}))}\right) = \eta(x_{F,t})(1-\phi(x_{F,t})),$$

which coincides with the goods market condition in (38) if and only if

$$\phi(x_{SE,t}) = \frac{k\mu(x_{W,t})\phi(x_{W,t})}{\lambda(x_{SE,t})q(x_{W,t})(1-\phi(x_{W,t}))}.$$

Using the steady state optimal firm entry decision in (41) to substitute for k,

$$\begin{aligned}
\phi\left(x_{SE,t}\right) &= \frac{\mu\left(x_{W,t}\right)\phi\left(x_{W,t}\right)q\left(x_{W,t}\right)A\lambda\left(x_{F,t}\right)\phi\left(x_{F,t}\right)\left(1-\phi\left(x_{W,t}\right)\right)}{\lambda\left(x_{SE,t}\right)q\left(x_{W,t}\right)\left(1-\phi\left(x_{W,t}\right)\right)\left[1-\beta\left(1-\delta\right)\left(1-\mu\left(x_{W,t}\right)\phi\left(x_{W,t}\right)\right)\right]}, \\
\lambda\left(x_{SE,t}\right)\phi\left(x_{SE,t}\right) &= \frac{\mu\left(x_{W,t}\right)\phi\left(x_{W,t}\right)A\lambda\left(x_{F,t}\right)\phi\left(x_{F,t}\right)}{1-\beta\left(1-\delta\right)\left(1-\mu\left(x_{W,t}\right)\phi\left(x_{W,t}\right)\right)}.
\end{aligned}$$

Given that firms set $p_F = \phi\left(x_{F,t}\right)$ and that risk-neutral self-employed set $p_{SE} = \phi\left(x_{SE,t}\right)$, this is exactly the individuals' indifference condition in the career choice game if they are risk-neutral, as can be seen in (35). Consequently, also the career choice of risk-neutral individuals maximizes total production sold. We conclude that $\{x_{SE,t}, V_t, E_t, SE_t\}_{t=1}^{\infty}$ are chosen efficiently by the market.