

On the Puzzle of Diversification in Social Networks with Occupational Mismatch

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Abstract

This paper incorporates social networks into a frictional labour market framework. There are two worker types and two occupations. Both occupations are subject to correlated business cycle fluctuations in labour demand. The equilibrium in this model is characterized by occupational mismatch which is associated with a wage penalty. This paper shows that there exists a unique value of network homophily maximizing the present value of income. Therefore, there is a gain for risk-neutral workers if their network is diversified between the two occupations. The reason for diversification is that the present value of income is a non-linear function of the network composition. Thus, it is not the desire to reduce the volatility of income as in standard portfolio theory which is driving the decision of workers. Nevertheless, the optimal diversification level is higher with stronger negative correlation in labour demand between the two occupations, with a lower unemployment benefit and with a higher probability of recession in the primary occupation. On the other hand, the optimal diversification level is reduced if there is on-the-job search in the state of mismatch.

JEL classification: J23, J31, J64

Keywords: Occupational mismatch, social networks, homophily, diversification

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1 Introduction

This paper investigates the link between social networks and welfare in the context of a frictional labour market with occupational mismatch. Empirical studies show that 30-60% of new hires find jobs with a help of social contacts¹. At the same time, there is evidence that up to 47% of workers in some occupations are mismatched (Robst (2007)). Thus it is natural to ask whether the network channel of job search contributes to the high levels of occupational mismatch. From a theoretical perspective, Bentolila et al. (2010) and Horvath (2014b) show that social networks with weak homophily may generate more mismatch compared to the formal channel of search. Weak homophily here means that workers have relatively many social contacts in occupations other than their own. However, more mismatch is not equivalent to lower welfare, especially in the presence of occupation-specific fluctuations in labour demand. On the contrary, it may be optimal for workers to diversify their networks across occupations in order to reduce the risk of being unemployed even if this strategy is associated with more mismatch. This study fills the gap in the analysis of network implications for social welfare and analyzes the optimal level of network diversification in a setting with business-cycle fluctuations.

The ingredients of the model are as follows. There are two worker types and two occupations. The type of worker is given by the initial training in one of the two occupations. Every worker can be unemployed, employed in the primary occupation or mismatched, which is associated with a wage penalty. Fluctuations in labour demand are described by a time-homogeneous transition matrix and are correlated between the two occupations. In the benchmark case there are binary vacancy fluctuations in each occupation, e.g. a period of expansion with many vacancies and a period of recession with fewer vacancies. Every worker has a fixed total number of social contacts, which is the network size. The level of network homophily is characterized by the proportion of contacts to other workers of the same type. Thus a higher level of homophily implies a less diversified network and vice versa.

In this setting the primary contribution of this paper is a full characterization of the optimal network diversification level which maximizes the expected present value of income for a given worker type. This diversification effect is new in the literature as workers in the model are risk-neutral and their decisions are not driven by risk-aversion. Indeed, in the standard Markowitz portfolio theory a risk-neutral investor will avoid diversification and invest into the stock with the highest expected return. The main reason for this puzzling result is that expected income is a non-linear function of network homophily. Thus the optimal level of network diversification obtains at the point where the marginal gain from diversification is equal to the marginal cost. The marginal gain from diversification is due to a higher probability of finding a mismatch job especially in times of low labour demand in the primary occupation. The marginal cost is due to a lower probability of finding a job in the primary occupation. More intuitively, since it is always possible that the mismatch occupation is booming while the primary occupation of the worker is experiencing a recession, it is optimal for workers to have a fraction of their contacts in the mismatch occupation even though it is associated with a significant wage penalty.

This diversification result is robust to parameter changes in the comparative statics analysis.

¹The incidence of referrals is 34 – 36% in France (Margolis and Simonnet (2003) and Delattre and Sabatier (2007)), 47% in Italy and Portugal (Pistaferrì (1999) and Addison and Portugal (2002)), 50 – 56% in the United States (Granovetter (1995) and Bentolila, Michelacci and Suarez (2010)).

For example, I find that the optimal homophily level is higher and diversification is weaker if labour demands in the two occupations are positively correlated. This is because with positive correlation it is less likely that the mismatch occupation is expanding while the primary occupation is in a recession. Thus the gain from diversification is reduced. On the contrary, the optimal homophily level is lower and diversification is stronger with a lower unemployment benefit. Consider the situation when the primary occupation of the worker is experiencing a recession and the person becomes unemployed. If unemployment insurance is relatively low, it is optimal for the worker to have more contacts in the mismatch occupation in order to leave the state of unemployment as soon as possible. In addition, diversification becomes less (more) important with a higher (lower) frequency of expansions in the primary occupation.

Finally, in the last section of the paper the model is extended to allow for on-the-job search in the state of mismatch. On the one hand, searching on-the-job is a valuable option for workers which should raise the expected present value of income. But on the other hand, searching mismatched workers reduce the job-finding chances of unemployed workers, which raises unemployment and has a negative effect on the expected present value of income. I find that this negative effect is dominating if social networks are relatively well diversified, which is intuitive since workers are never mismatched with a fully homophilous network. Thus the optimal level of homophily is higher and diversification is weaker with on-the-job search. Underlying this result is the fact that primary contacts become more important if the worker wants to leave a mismatch job and come back to the primary occupation. Thus the cost of diversification is higher and the gain is lower with on-the-job search.

This paper is closely related to the literature on social networks in the labour market. The first idea to introduce a separate homophily parameter into an economic model is due to Montgomery (1991). This author and later Simon and Warner (1992) emphasize the point that friends and acquaintances are likely to have similar skills and ability (homophily by skills). Thus referrals from high ability employees reveal positive information to the firm about the quality of the match. This idea is empirically confirmed by Hensvik and Skans (2013) who show that in Sweden entrants are more likely to be linked to high ability incumbent employees than to low ability incumbents (defined from test scores or wages). Stupnytska and Zaharieva (2015) extend this idea by separating family and professional contacts in their model. In the equilibrium there is a self-selection of low ability workers into family referrals and high ability workers into professional referrals, which generates a U-shape hiring pattern. Overall, transmitting information about applicants' characteristics to the employer is a first influence channel of social networks, which is particularly important in a setting with heterogeneous workers.

Ioannides and Soetevent (2006) and Fontaine (2008) describe a second influence channel which is based on the transmission of information about vacancies between connected workers. In the former study better connected workers experience lower unemployment rates and receive higher wages. Fontaine (2008) considers a frictional labour market and shows that differences in networks can generate wage dispersion among equally productive workers. Other studies incorporating networks into the search and matching framework include Kugler (2003), Cahuc and Fontaine (2009), Zaharieva (2013) and Galenianos (2014). Kugler (2003) explains that referees may exert peer pressure on newly hired workers, which makes it more profitable for firms to pay efficiency wages. Zaharieva (2013) investigates network externalities and shows that

bargained wages are inefficiently high because workers do not internalize the positive externality on their network connections. Galenianos (2014) predicts and confirms empirically a positive correlation between referral hiring and matching efficiency across industries. In all three studies referrals are modeled as an additional channel of search, however, none of these models includes the underlying structure of the network into the matching function. So Cahuc and Fontaine (2009) is a first study incorporating an explicit structure of the network into the matching function. Their network approach is also used in the present study but there is no mismatch and diversification in their model.

To the best of my knowledge, there are only two studies combining social networks and on-the-job search. Both Horvath (2014a) and Zaharieva (2015) consider a setting with heterogeneous firms, hence employed workers accept job offers from more productive employers and forward other offers from less productive employers to their network connections. This setup implies that referral offers are associated with wage penalties. This feature is also present in the current study as mismatch jobs pay low wages and there is an incentive for workers to continue searching on-the-job in the hope of better payment in the primary occupation. Despite this similarity neither Horvath (2014a) nor Zaharieva (2015) consider network diversification in the presence of business cycle fluctuations which is the primary issue analyzed in this paper.

As mentioned in the beginning of this section there are only two studies combining networks and occupational mismatch in a unified labour market framework. They are Bentolila et al. (2010) and Horvath (2014b). Bentolila et al. (2010) report that social networks and referrals can generate a mismatch between the occupational choice of the worker and his/her acquired qualification. An implicit assumption for this result is that workers are never mismatched if they find a job in a formal way. Horvath (2014b) extends this approach with a homophily parameter, which is the extent of workers with similar educational background to be connected in the social network. He finds that occupational mismatch is falling with a higher level of homophily in the social network but leaves the issue of network diversification for future research. Finally, Blazquez and Jansen (2008) also consider mismatch in a frictional labour market and show that the equilibrium outcome with random search and ex-post bargaining is never efficient, but there are no networks or referrals in their model.

Most other studies on occupational mismatch are empirical and distinguish between vertical and horizontal mismatch. The former approach investigates whether workers are over- or under-qualified for the job. In contrast, the horizontal mismatch approach doesn't consider the level of education but rather takes into account the type of education the worker obtained and the type of skills required in the job. Horizontal mismatch appears in situations when the worker doesn't have the "right" type of education to perform the job successfully, thus this approach is about the degree of correspondence between the field of study and the occupation choice. This latter idea is also used in the present work. A number of early empirical papers on horizontal mismatch include Allen and van der Velden (2001), Wolbers (2003) and Robst (2007). For example, Wolbers (2003) considers data on school graduates in a number of Western European Economies and finds that school-leavers from humanities, arts and agriculture are more likely to be mismatched than those from engineering, manufacturing, business and law. He also considers the business cycle perspective and reports that in times of high unemployment school-leavers more often have to accept a job that does not fit their field of education. Another interesting finding of that

paper is that for school-leavers with a job mismatch, the odds of looking for another job is 1.4 times larger than for the properly matched school-leavers. Robst (2007) finds similar results for college graduates in the United States, where 27 – 47% of workers in arts, social sciences, psychology, languages and biology are mismatched. He also reports that horizontal mismatch is associated with a wage loss of about 10%.

More recent empirical studies on occupational mismatch include Allen and de Weert (2007), Nordin et al. (2008), and Beduwe and Giret (2011). The first study finds that the proportion of workers employed with an appropriate level of schooling but in a different field is 6% in Spain, 10% in Germany, 11% in the Netherlands and 18% in the UK. Moreover, this study confirms that mismatched German and British respondents are significantly more likely to look for other work. Nordin et al. (2008) finds that in Sweden 23% of men are strongly mismatched and 16% are weakly mismatched in their job. In addition, they find a very large penalty associated with occupational mismatch in Sweden equal to 32%. People with dentist, police and law education are least often mismatched, whereas those with a biology, psychology or artistic education are again more often mismatched. Finally, Beduwe and Giret (2011) investigate job characteristics of workers who accomplished vocational training and find that 30% of them are vertically matched but horizontally mismatched in France. They also find wage penalties of 2-3% for mismatched workers. Overall, empirical evidence suggests that horizontal mismatch is a frequent phenomenon. Moreover, mismatched workers are more likely to be involved in on-the-job search. There is also a significant wage penalty associated with occupational mismatch which varies across countries.

The plan of the paper is as follows. Section 2 explains notation and the general economic environment. Section 3 presents the model with two occupations and correlated business-cycle fluctuations. Section 4 illustrates the results by means of a large numerical example. Section 5 extends the model to account for on-the-job search. Section 6 concludes the paper.

2 The Framework

Consider the model with two groups of infinitely lived risk neutral workers and two occupations. Workers of type A obtained training in occupation A , which is their primary occupation, but they can also work in occupation B , which is a mismatch occupation for them. In a similar way, occupation B is a primary occupation for type B workers, whereas there is mismatch if type B workers are employed in occupation A . Every worker can be either employed or unemployed. Let u_j denote the measure of unemployed workers of type j and e_j – the measure of employed type j workers, $j = A, B$. Each group of workers is a continuum of measure 1, thus $u_j + e_j = 1$. Every employed worker may lose the job at rate δ . Unemployed workers of both types receive the flow unemployment benefit z . Every occupation pays a flow wage w to workers trained in this occupation and a lower wage $w_0 \in [z, w]$ to mismatched workers, such as type A workers in occupation B and type B workers in occupation A . Workers discount future flows at rate r .

Every worker has n social contacts; γn of the same type and $(1 - \gamma)n$ contacts of the different type. Variable $\gamma \in [0.5, 1]$ can be interpreted as a level of homophily in the society. Montgomery (1991) refers to it as an "inbreeding bias" by type. If $\gamma = 1$ the society is homophilous as only workers of the same type are connected in networks. In contrast, if $\gamma = 0.5$ the two groups

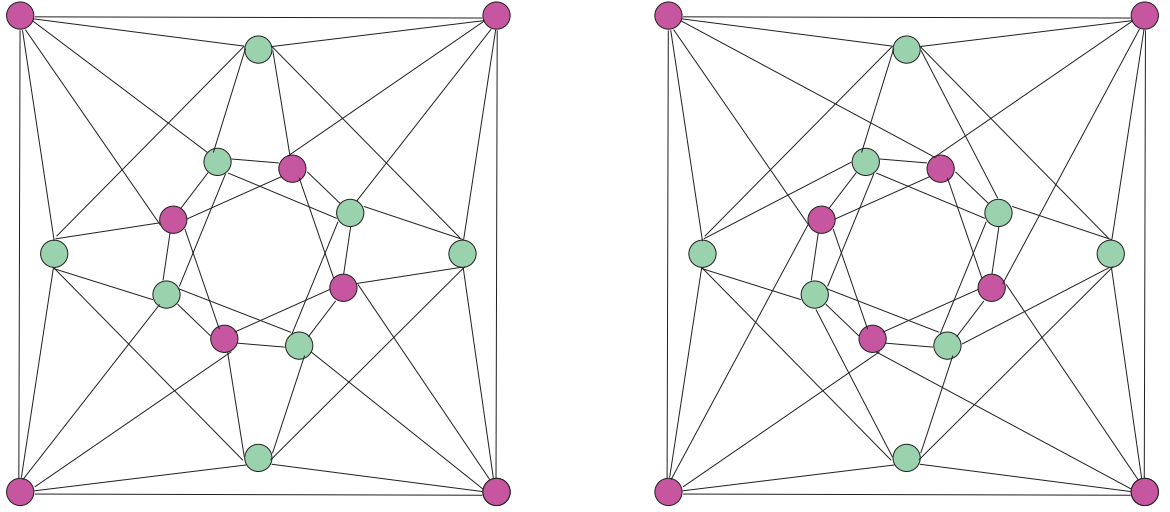


Figure 1: Left panel: $n = 6$, $\gamma = 3/6$. Right panel: $n = 6$, $\gamma = 4/6$.

are strongly mixed and there is no "inbreeding bias". In general, homophily refers to the fact that people are more prone to maintain relationships with people who are similar to themselves. There can be homophily measured by age, race, gender, religion or profession and it is generally a robust observation in social networks (see McPherson et al. (2001) for an overview of research on homophily). The focus of this paper is on the latter type of homophily by profession or occupation. Jackson (2008) distinguishes between homophily due to opportunity and due to choice. In this respect, homophily by occupation is likely to arise due to the fact that workers with the same profession studied or worked together in the beginning of their career. Thus it is rather a limited opportunity of meeting workers from different professions which can generate homophily rather than an explicit choice.

Figures 1 and 2 provide an intuitive illustration of the model with $N = 8$ workers of type A (pink nodes) and 8 workers of type B (green nodes). On the left panel of figure 1 there is a regular network of degree $n = 6$, which means that all workers have an identical number of links equal to 6. The homophily parameter of this network γ is $3/6$ since every worker has 3 links with other workers of the same type and another 3 links with workers of the opposite type². Comparing both panels on figure 1 reveals that homophily is higher on the right panel: $\gamma = 4/6$. It is then further increased on figure 2 reaching the levels $\gamma = 5/6$ and $\gamma = 6/6$. This latter case corresponds to a fully homophilous network since there are no links between the groups and the network falls apart into two independent components. In this setting, the main research question addressed in this study is which of these network compositions corresponding to different levels of γ will maximize the present value of expected income for workers? Is it a network with $\gamma = 0.5$, $\gamma = 1$ or an intermediate value of γ ?

In order to keep the model tractable I assume that there is only one channel of job search by means of referrals. Thus hiring takes place if employed workers who get information about open vacancies recommend their contacts for the job. To model this process I follow the approach of Cahuc and Fontaine (2009). Firms with open vacancies in occupation A contact type A workers

²For every integer $\gamma n \in \{1..N-1\}$ such a network can be constructed in two steps. In the first step one should construct two disconnected regular networks each of a given degree γn which is always possible if N is divisible by 2. In the second step one should construct a regular bipartite network of degree $(1-\gamma)n$ by matching workers between the two groups.

employed in their occupation at rate s per unit time. This is an exogenous search intensity of employers. Every type A employee has γn type A contacts, where every contact is employed with probability e_A . Therefore, the probability that there is at least one unemployed contact of type A who is willing to take the job is equal to $1 - e_A^{\gamma n}$. Thus, the number of matches between workers and vacancies of type A is equal to $sv_A(1 - e_A^{\gamma n})$, where v_A is a number of vacancies in occupation A .

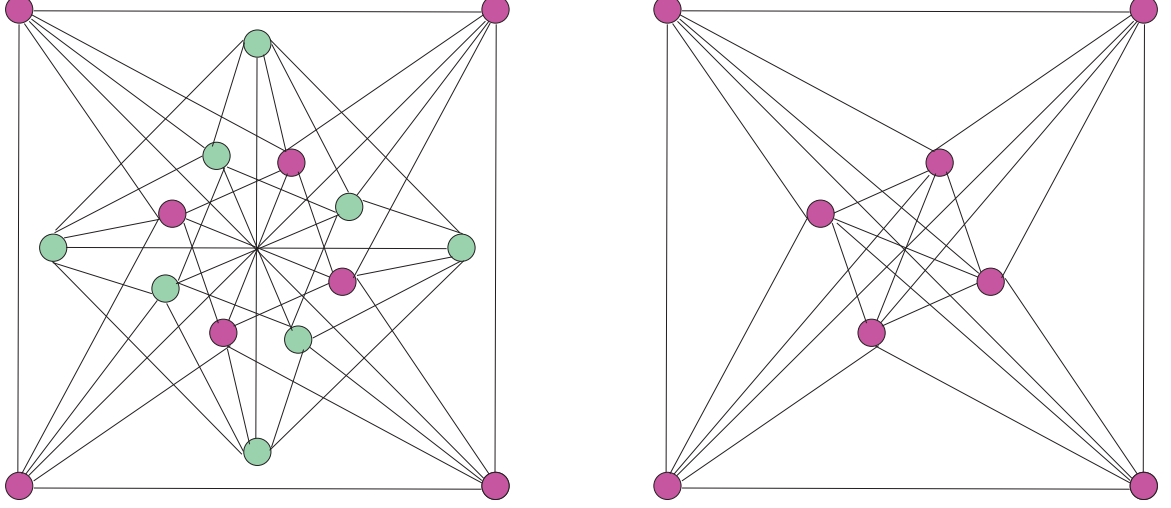


Figure 2: Left panel: $n = 6$, $\gamma = 5/6$. Right panel: $n = 6$, $\gamma = 6/6$.

However, if all type A contacts of the chosen employee are also employed, which happens with probability $e_A^{\gamma n}$, this employee may recommend a contact of type B . The probability that there is at least one type B contact out of $(1 - \gamma)n$ is equal to $1 - e_B^{(1-\gamma)n}$. This is because type B workers are employed with probability e_B . Hence, the total number of matches between type B workers and type A vacancies is equal to $sv_A e_A^{\gamma n} (1 - e_B^{(1-\gamma)n})$. With this information I can find the job arrival rate to workers A getting jobs in occupation A , let it be denoted by λ_{AA} , and the job arrival rate to workers B in occupation A , let it be denoted by λ_{BA} . Each of these rates is equal to the ratio between the number of corresponding matches per unit time and the number of searching unemployed workers of the corresponding type:

$$\lambda_{AA} = \frac{sv_A}{u_A} (1 - e_A^{\gamma n}) \quad \lambda_{BA} = \frac{sv_A}{u_B} \cdot e_A^{\gamma n} \cdot (1 - e_B^{(1-\gamma)n})$$

In a similar way, one can find the job arrival rate to workers B getting jobs in occupation B , denoted by λ_{BB} , and the job arrival rate to workers A in occupation B , denoted by λ_{AB} :

$$\lambda_{BB} = \frac{sv_B}{u_B} \cdot (1 - e_B^{\gamma n}) \quad \lambda_{AB} = \frac{sv_B}{u_A} \cdot e_B^{\gamma n} \cdot (1 - e_A^{(1-\gamma)n})$$

where v_B is a number of vacancies in occupation B . Furthermore, at rate δ every job can be destroyed for exogenous reasons. Labour market transitions are illustrated on figure 3, however, job destruction flows are not included on this figure for the ease of illustration.

Note that the four job-finding rates above are presented for a given macroeconomic state i , which is characterized by the exogenous vector of vacancies $\{v_A^i, v_B^i\}$, $i = 1, \dots, m$. Thus formally all endogenous variables above should have an upper index i , that is $\{\lambda_{AA}^i, \lambda_{AB}^i, \lambda_{BB}^i,$

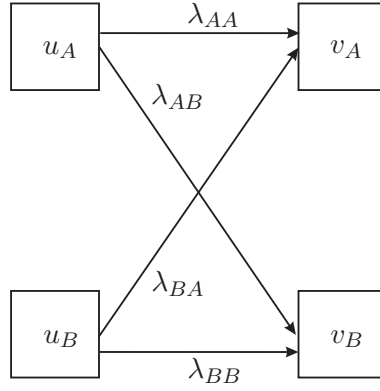


Figure 3: Labour market transitions

$\lambda_{BA}^i, e_A^i, e_B^i, u_A^i, u_B^i\}$. However, the upper index was suppressed for the ease of notation. At Poisson rate ϕ the macroeconomic state of the economy may change according to the time-homogeneous discrete Markov chain with a transition matrix Π :

$$\Pi = \begin{pmatrix} \pi_1 & \cdots & \pi_m \\ \vdots & \cdots & \vdots \\ \pi_1 & \cdots & \pi_m \end{pmatrix} \quad \text{where} \quad \sum_{i=1}^m \pi_i = 1$$

Intuitively, this means that $\phi\pi_i$ is a constant arrival rate of state i with a vector of vacancies $\{v_A^i, v_B^i\}$, $i = 1, \dots, m$. Variable ϕ measures the frequency of business cycle fluctuations in labour demand. For example, consider state i and a short period of time Δt . With probability $e^{-\phi\Delta t}$ no shock will arrive during this period of time. For a small Δt this probability can be approximated as $1 - \phi\Delta t$. So the probability that state i will remain unchanged by the end of the period is given by $1 - \phi\Delta t + \phi\Delta t\pi_i = 1 - (1 - \pi_i)\phi\Delta t$. The first term in this expression is the probability that no shock will arrive, and the second term is the probability that state i will persist conditional on the shock. In addition, with probability $\phi\Delta t\pi_j$ the new state after the shock will be state j , $j \neq i$. Hence a higher value of ϕ implies more frequent fluctuations, while a lower value of ϕ is associated with higher persistence of economic states.

3 The Model

3.1 Unemployment rates

This section is dedicated to the analysis of unemployment rates u_A and u_B , where the upper index i is again suppressed. Specifically, I follow the approach by Hall and Milgrom (2008). Although in principle both unemployment rates are separate state variables, they move so much faster than i that one can use the equilibrium values as close approximations of the actual values of unemployment rates. Thus in every state i , the inflow of workers into unemployment should be equal to the outflow of workers. On the one hand, at rate δ every worker loses the job, thus, the inflow of type A workers is equal to $\delta(1 - u_A)$. On the other hand, every unemployed type A worker finds some job at rate $\lambda_{AA} + \lambda_{AB}$, either in the primary occupation or in the mismatched occupation. So the differential equation for the unemployment rate of type A workers becomes:

$\dot{u}_A = \delta(1 - u_A) - u_A(\lambda_{AA} + \lambda_{AB})$. And the differential equation for the unemployment rate of type B workers is: $\dot{u}_B = \delta(1 - u_B) - u_B(\lambda_{BB} + \lambda_{BA})$. In the equilibrium it holds that $\dot{u}_A = 0$ and $\dot{u}_B = 0$, so that:

$$u_A = \frac{\delta}{\delta + \lambda_{AA} + \lambda_{AB}} \quad u_B = \frac{\delta}{\delta + \lambda_{BA} + \lambda_{BB}} \quad (3.1)$$

Inserting values for λ_{AA} , λ_{AB} , λ_{BB} and λ_{BA} into (3.1) produces a system of two equations in two employment variables e_A and e_B . This is summarized in the following lemma:

Lemma 1: *For every state i , $i = 1, \dots, m$, the equilibrium employment rates e_A and e_B are uniquely determined from the following system of equations:*

$$sv_A(1 - e_A^{\gamma n}) + sv_B \cdot e_B^{\gamma n} \cdot (1 - e_A^{(1-\gamma)n}) = \delta e_A \quad \Rightarrow \quad e_A(e_B) \quad (3.2)$$

$$sv_A \cdot e_A^{\gamma n} \cdot (1 - e_B^{(1-\gamma)n}) + sv_B \cdot (1 - e_B^{\gamma n}) = \delta e_B \quad \Rightarrow \quad e_B(e_A) \quad (3.3)$$

A higher number of vacancies v_A leads to higher employment in both occupations e_A and e_B and higher job-finding rates λ_{AA} , λ_{AB} , λ_{BB} and λ_{BA} . The same is true for a higher v_B .

Proof: Appendix I.

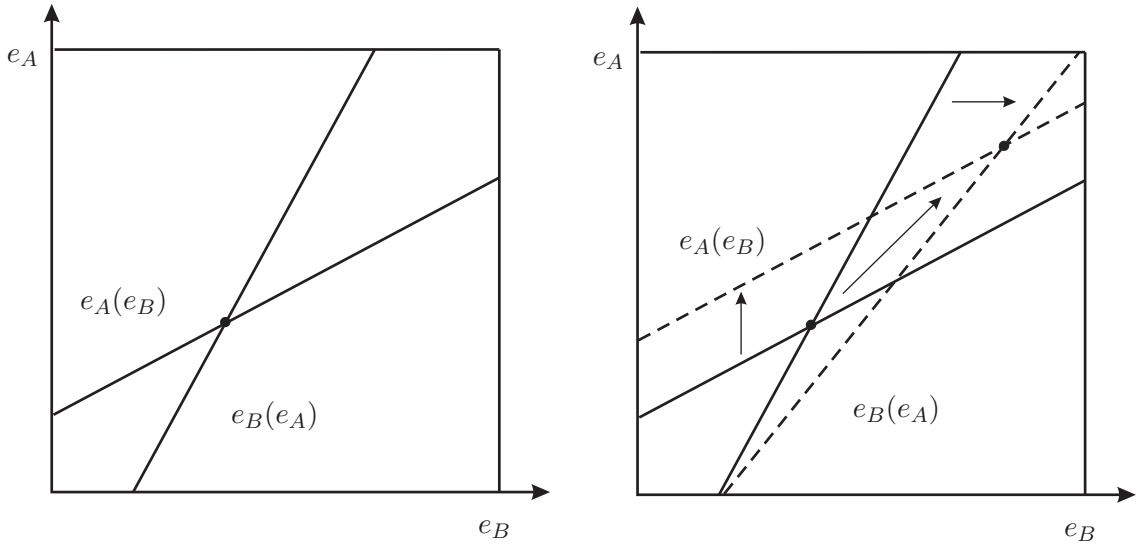


Figure 4: Left panel: equilibrium. Right panel: positive shock in v_A

Consider equation (3.2). The right-hand side of this equation is a linear function increasing from 0 to δ when e_A is increasing from 0 to 1. On the contrary, the left-hand side of this equation is decreasing down to zero when $e_A = 1$. So there exists a unique intersection between these curves. A larger value e_B raises the left-hand side of equation (3.2), thereby increasing e_A . Thus, one can write e_A as an increasing function of e_B , where $e_A(0) > 0$ and $e_A(1) < 1$. This relation highlights spillovers between the two occupations. If a larger fraction of type B workers is employed (i.e. e_B is rising), then more type B workers will be recommending their type A social contacts for jobs in occupation B . So the equilibrium employment rate of type A workers is higher. The same holds true for a higher employment rate e_A , which has a positive effect on the employment rate of type B workers. This follows from equation (3.3), where $e_B(0) > 0$ and $e_B(1) < 1$. Hence the equilibrium values of e_A and e_B can be obtained at the intersection

between the two positively sloping curves $e_A(e_B)$ and $e_B(e_A)$ which is illustrated on the left panel of figure 4.

Next consider a positive change in vacancies v_A which is illustrated on the right panel of figure 4. First, there is a direct positive effect on the employment of type A workers as finding jobs in occupation A becomes easier. Graphically this corresponds to the upward shift in $e_A(e_B)$. Second, a higher employment of type A workers brings more type B workers into jobs, so there is a downward rotation in the curve $e_B(e_A)$. Combining these two shifts together one can see that due to networks a higher number of vacancies in one occupation is propagating employment in the other occupation. However, this network multiplier is not necessarily a desirable feature for the labour market as it will unambiguously amplify the rise of unemployment in the economy-wide recession. As a final remark in this subsection, notice that the corner case $\gamma = 1$, corresponding to full homophily, implies that $e_A(e_B)$ is a horizontal curve independent of e_B and $e_B(e_A)$ is a vertical curve independent of e_A . In this case, unemployment shifts in response to vacancy fluctuations are not amplified as the two types of workers are not connected.

3.2 Numerical example

This subsection illustrates the theoretical result from lemma 1 by means of a numerical example. Consider the case of binary labour demand in each of the two occupations. This is the model with $m = 4$ states (see table 1), where every occupation can be either expanding or shrinking. In addition, this example allows me to estimate the fractions of mismatched type A and type B workers, let them be denoted by e_{AB} and e_{BA} respectively.

| | Expansion in B, $v_B = 0.05$ | Recession in B, $v_B = 0.04$ |
|--------------------------------|--|--|
| Expansion in A $v_A = 0.05$ | State 1 with prob. π_1 $e_A = e_B = 0.934$ $e_{AB} = e_{BA} = 0.191$ $\lambda_{AA} = \lambda_{BB} = 1.130$ $\lambda_{AB} = \lambda_{BA} = 0.290$ | State 2 with prob. π_2 $e_A = 0.907, e_B = 0.875$ $e_{AB} = 0.048, e_{BA} = 0.131$ $\lambda_{AA} = 0.921, \lambda_{BB} = 0.597$ $\lambda_{AB} = 0.050, \lambda_{BA} = 0.105$ |
| Recession in A $v_A = 0.04$ | State 3 with prob. π_3 $e_A = 0.875, e_B = 0.907$ $e_{AB} = 0.131, e_{BA} = 0.048$ $\lambda_{AA} = 0.597, \lambda_{BB} = 0.921$ $\lambda_{AB} = 0.105, \lambda_{BA} = 0.050$ | State 4 with prob. π_4 $e_A = e_B = 0.800$ $e_{AB} = e_{BA} = 0.009$ $\lambda_{AA} = \lambda_{BB} = 0.395$ $\lambda_{AB} = \lambda_{BA} = 0.005$ |

Table 1: The model with $m = 4$ states, $\delta = 0.1$, $s = 2$, $n = 40$, $\gamma = 0.5$

Consider an economy in state 4 which is an economy-wide recession ($v_A = 0.04$, $v_B = 0.04$). At rate $\phi\pi_2$ occupation A starts recovering which is reflected by the sharp rise of vacancies v_A from 0.04 to 0.05. In accordance with lemma 1 this is associated with a rise of employment of type A workers e_A from 0.800 to 0.907. In addition, there is a positive spillover effect on type B workers whose job-finding rate in the mismatch occupation λ_{BA} is increasing from 0.005 to 0.105 and their employment rate e_B is increasing from 0.800 to 0.875. At rate $\phi\pi_1$ occupation B starts recovering and the economy is moving from state 2 to state 1. This is associated with a moderate rise of employment e_B from 0.875 to 0.934 followed by a small increase of employment

e_A from 0.907 to 0.934 due to the network spillover.

Finally, note that 19.1% of workers are mismatched if both occupations are expanding. This is due to the fact that vacancy stocks are relatively high in both occupations and workers prefer to be mismatched rather than unemployed. In contrast, the mismatch level is only 0.9% if there is an economy-wide recession in state 4.

3.3 Present value equations

Next consider present value equations for workers of type A . Let U_A^i denote the present value of unemployed type A workers in state i . The objective of this section is to express variable U_A^i as a function of model parameters. Moen (1997) proved that maximizing the present value of unemployment is equivalent to the maximization of welfare for a given risk-neutral worker type. This is because the present value of unemployment is a forward-looking variable, which takes into account the probability of finding a job and the expected wage. Thus it is U_A^i which will be maximized with respect to γ in the next sections of the paper. In addition, let variables W_{AA}^i and W_{AB}^i denote the present values of type A employees in occupations A and B respectively. The present value U_A^i can then be written as:

$$\begin{aligned} rU_A^i &= z + \lambda_{AA}^i(W_{AA}^i - U_A^i) + \lambda_{AB}^i(W_{AB}^i - U_A^i) + \\ &+ \phi[\pi_1(U_A^1 - U_A^i) + \dots + \pi_{i-1}(U_A^{i-1} - U_A^i) + \pi_{i+1}(U_A^{i+1} - U_A^i) + \dots + \pi_m(U_A^m - U_A^i)] \end{aligned}$$

Unemployed workers receive the flow unemployment benefit z and may find a job in their primary occupation which happens at rate λ_{AA}^i or in the mismatch occupation which happens at rate λ_{AB}^i . Alternatively, the macroeconomic state may change at rate ϕ , where the new state is determined according to the probability distribution $\{\pi_1, \dots, \pi_m\}$. The present values of employment W_{AA}^i and W_{AB}^i can be written as:

$$\begin{aligned} rW_{AA}^i &= w - \delta(W_{AA}^i - U_A^i) + \phi[\pi_1(W_{AA}^1 - W_{AA}^i) + \dots + \pi_{i-1}(W_{AA}^{i-1} - W_{AA}^i) + \\ &+ \pi_{i+1}(W_{AA}^{i+1} - W_{AA}^i) + \dots + \pi_m(W_{AA}^m - W_{AA}^i)] \\ rW_{AB}^i &= w_0 - \delta(W_{AB}^i - U_A^i) + \phi[\pi_1(W_{AB}^1 - W_{AB}^i) + \dots + \pi_{i-1}(W_{AB}^{i-1} - W_{AB}^i) + \\ &+ \pi_{i+1}(W_{AB}^{i+1} - W_{AB}^i) + \dots + \pi_m(W_{AB}^m - W_{AB}^i)] \end{aligned}$$

These equations reflect the fact that workers get a high wage w if employed in their primary occupation and a low wage w_0 if mismatched. The three equations above form a system of $3m$ equations for type A workers. In addition, there are $3m$ similar equations for type B workers, so the total number of equations is $6m$. In order to solve this system I am going to use matrix notation. Let U_A denote a column-vector containing values U_A^i for every state i . When unemployed, all workers receive the unemployment benefit z , so let ζ denote a column-vector of z -values, which has dimension $m \times 1$. Moreover, type A workers can find a job in occupation A , which corresponds to the matrix of job-finding rates Λ_{AA} of dimension $m \times m$, and a mismatch job in occupation B , which corresponds to the matrix of job-finding rates Λ_{AB} . This notation allows me to rewrite the present value equations in matrix form:

$$rU_A = \zeta + \Lambda_{AA}(W_{AA} - U_A) + \Lambda_{AB}(W_{AB} - U_A) + \phi(\Pi - I)U_A$$

$$\begin{aligned}
rW_{AA} &= \omega - \delta(W_{AA} - U_A) + \phi(\Pi - I)W_{AA} \\
rW_{AB} &= \omega_0 - \delta(W_{AB} - U_A) + \phi(\Pi - I)W_{AB}
\end{aligned}$$

$$U_A = \begin{pmatrix} U_A^1 \\ \vdots \\ U_A^m \end{pmatrix} \quad W_{Aj} = \begin{pmatrix} W_{Aj}^1 \\ \vdots \\ W_{Aj}^m \end{pmatrix} \quad \Lambda_{Aj} = \begin{pmatrix} \lambda_{Aj}^1 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & \lambda_{Aj}^m \end{pmatrix} \quad j = A, B$$

where r, δ, ϕ are scalars, I is the identity matrix and ω, ω_0 are the $(m \times 1)$ vectors of wages w and w_0 respectively. Note that at rate ϕ the macroeconomic state of the economy may change according to the transition matrix Π . With this matrix notation, present values of employed type A workers, i.e. W_{AA} and W_{AB} , can be expressed as:

$$W_{AA} = M(\omega + \delta U_A) \quad W_{AB} = M(\omega_0 + \delta U_A) \quad \text{where} \quad M = [(r + \delta + \phi)I - \phi\Pi]^{-1}$$

M is the auxiliary matrix. Inserting these expressions into the equation for U_A and repeating the same procedure for workers of type B , I obtain the following results:

$$\begin{aligned}
U_A &= [(r + \phi)I - \phi\Pi + (\Lambda_{AA} + \Lambda_{AB})(I - \delta M)]^{-1}(z + \Lambda_{AA}Mw + \Lambda_{AB}Mw_0) \\
U_B &= [(r + \phi)I - \phi\Pi + (\Lambda_{BA} + \Lambda_{BB})(I - \delta M)]^{-1}(z + \Lambda_{BB}Mw + \Lambda_{BA}Mw_0)
\end{aligned}$$

Note that the job-finding matrices Λ_{Aj} and Λ_{Bj} , $j = A, B$ depend on the homophily parameter γ , thus the present value of searching workers also depends on γ which allows me to investigate the question whether full homophily $\gamma = 1$ will maximize the present value of income, or it will be maximized for some interior value of γ , implying a diversified (heterophilous) network. Next, define $G_A = [(r + \phi)I - \phi\Pi + (\Lambda_{AA} + \Lambda_{AB})(I - \delta M)]^{-1}$ and $G_B = [(r + \phi)I - \phi\Pi + (\Lambda_{BA} + \Lambda_{BB})(I - \delta M)]^{-1}$ as additional auxiliary matrices. Interestingly, these matrices strongly depend on different transition rates but they don't depend on the monetary variables z, w and w_0 . This allows me to recover the individual elements of U_A and U_B switching back from the matrix notation to scalars. These results are summarized in proposition 1:

Proposition 1: Let g_{il}^A denote the elements of matrix G_A and m_{lk} denote the elements of matrix M . The present value of income for unemployed type A workers U_A^i is given by a linear combination of the unemployment benefit and wages:

$$U_A^i = z \sum_{l=1}^m g_{il}^A + w \sum_{l=1}^m g_{il}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + w_0 \sum_{l=1}^m g_{il}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk}$$

Proof: Appendix II.

This proposition shows that the present value of income U_A^i is linearly increasing in each of the three monetary variables z, w and w_0 , but it is a non-linear function of the homophily parameter γ through the job-finding rates λ_{Aj}^i .

One last question which should be clarified in this section, is that of reservation wages. The equilibrium solution derived in proposition 1 is based on the assumption that workers accept mismatch jobs in every state of the economy. Thus it should be checked that this condition is satisfied in the equilibrium. To address this issue let w_0^R be the reservation wage of type

A workers in the best macroeconomic state, i.e. when vacancy rates are at their maximum in both occupations. Without loss of generality, let this be state 1. Intuitively, if $w_0 > w_0^R$, then unemployed workers would accept mismatch jobs in all states of the economy, even in state 1, which is defined as an economy-wide expansion. The reservation wage w_0^R is defined as a wage level which makes type "A" workers indifferent between accepting and rejecting the mismatch job: $U_A^1 = W_{AB}^1$. In Lemma 2 this reservation wage is derived as a function of model parameters:

Lemma 2: *Let state 1 be an economy-wide expansion. The reservation wage of type A workers in this state is given by:*

$$w_0^R = \frac{z \sum_{l=1}^m g_{1l}^A + w \sum_{l=1}^m g_{1l}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} - \delta z \sum_{i=1}^m m_{1i} \sum_{l=1}^m g_{il}^A - \delta w \sum_{i=1}^m m_{1i} \sum_{l=1}^m g_{il}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk}}{\sum_{i=1}^m m_{1i} + \delta \sum_{i=1}^m m_{1i} \sum_{l=1}^m g_{il}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk} - \sum_{l=1}^m g_{1l}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk}}$$

Proof: Appendix II.

The reservation wage of type B workers can be derived analogously. Note that the above solution is obtained for the equilibrium values of all transition rates λ_{Aj}^l in state 1. This corresponds to the idea that deviating from the equilibrium strategy and rejecting a mismatch job should not be profitable for a single worker. However, if one worker decides to deviate it will not change the labour market conditions such as the job-finding rates λ_{Aj}^l . This implies that the reservation wage should be estimated at the equilibrium transitions rates, which gives rise to the solution in lemma 2. The next subsection is dealing with a case of binary fluctuation in labour demand allowing for a more intuitive reduced representation of the model and its results.

3.4 Binary labour demand fluctuations

This section is dealing with an intuitive case of binary labour demand in each of the two occupations. This is the model with $m = 4$ states already used in the numerical example above. In this economy every occupation can be either expanding or hit by recession. Let p denote the probability that occupation A is in the state of expansion, thus $1 - p$ is the probability of recession in occupation A. Similarly, let q be the probability that occupation B is expanding. In addition, let ρ be the correlation coefficient between the two occupations. With this notation, the matrix of transition probabilities Π can be expressed as follows:

| | Expansion in B, q | Recession in B, $1 - q$ |
|-------------------------|--|--|
| Expansion in A, p | $\pi_1 = pq + \rho\sqrt{p(1-p)q(1-q)}$ | $\pi_2 = p(1-q) - \rho\sqrt{p(1-p)q(1-q)}$ |
| Recession in A, $1 - p$ | $\pi_3 = (1-p)q - \rho\sqrt{p(1-p)q(1-q)}$ | $\pi_4 = (1-p)(1-q) + \rho\sqrt{p(1-p)q(1-q)}$ |

Table 2: Parameterization of the transition matrix Π

The derivation of this matrix is presented in appendix III. Note that $\rho = 0$ corresponds to the case of independent fluctuations in labour demand between the occupations. However, a positive value of the correlation coefficient leads to more probability mass on the diagonal, thus

symmetric states with two expansions or two recessions become more likely. In contrast, when ρ is negative asymmetric states become more likely. In the following consider two special cases associated with table 2.

Special case 1: Symmetric occupations ($p = q = 0.5$):

If the two occupations are symmetric, i.e. $p = q = 0.5$, the four transition probabilities are:

$$\pi_1 = \pi_4 = 0.25(1 + \rho) \quad \pi_2 = \pi_3 = 0.25(1 - \rho)$$

This case is convenient for analyzing comparative statics results with respect to ρ .

Special case 2: Perfect negative correlation $\rho = -1$:

Note that this case automatically implies that $p = 1 - q$, thus the model is reduced from 4 to only 2 states ($m = 2$): state 2 – expansion in occupation A , recession in occupation B , and state 3 – recession in occupation A , expansion in occupation B . The matrix of transition probabilities Π is then simplified in the following way:

$$\Pi = \begin{pmatrix} 1 - q & q \\ 1 - q & q \end{pmatrix}$$

This case is convenient for analyzing comparative statics results with respect to q . Considering the model with perfectly negatively correlated fluctuations allows me to find an explicit equation for the income present value of unemployed workers as a scalar function of the homophily parameter γ . Further, diversification will be more important if the primary occupation of the worker has low labour demand, which is state 3 for type A workers and state 2 for type B workers. Thus proposition 2 presents solution for the present values U_A^3 and U_B^2 . The other two values U_A^2 and U_B^3 can be found in appendix IV.

Proposition 2: *In the case of binary fluctuations in labour demand and perfect negative correlation between the occupations ($\rho = -1$), the present value of unemployment is a weighted average between wages w , w_0 and the unemployment benefit z :*

$$\begin{aligned} rU_A^3(\gamma) &= z(1 - f_{AA}^3(\gamma) - f_{AB}^3(\gamma)) + wf_{AA}^3(\gamma) + w_0f_{AB}^3(\gamma) \\ rU_B^2(\gamma) &= z(1 - f_{BB}^2(\gamma) - f_{BA}^2(\gamma)) + wf_{BB}^2(\gamma) + w_0f_{BA}^2(\gamma) \end{aligned}$$

where the weights $f_{AA}^3, f_{AB}^3, f_{BA}^2, f_{BB}^2 \in [0..1]$ are given by:

$$\begin{aligned} f_{Aj}^3 &= \frac{c\lambda_{Aj}^2 + d\lambda_{Aj}^3}{(r + \delta)(c + d) + c\lambda_{AA}^2 + d\lambda_{AA}^3 + c\lambda_{AB}^2 + d\lambda_{AB}^3} \quad j = A, B \\ f_{Bj}^2 &= \frac{a'\lambda_{Bj}^2 + b'\lambda_{Bj}^3}{(r + \delta)(a' + b') + a'\lambda_{BB}^2 + b'\lambda_{BB}^3 + a'\lambda_{BA}^2 + b'\lambda_{BA}^3} \quad j = A, B \end{aligned}$$

and variables c, d, a', b' are provided in the appendix.

Proof: Appendix IV.

If the two sectors are symmetric (i.e. $p = q = 0.5$), then it holds that $\lambda_{AA}^3 = \lambda_{BB}^2$, $\lambda_{AA}^2 = \lambda_{BB}^3$, $\lambda_{BA}^2 = \lambda_{AB}^3$ and $\lambda_{AB}^2 = \lambda_{BA}^3$. This means $a' = d$, $b' = c$, so that $f_{AA}^3 = f_{BB}^2$ and $f_{AB}^3 = f_{BA}^2$. This however means that type A workers obtain the same present value of income in state 3 (when sector A has low labour demand) as type B workers in state 2 (when sector

B has low labour demand): $U_A^3(\gamma) = U_B^2(\gamma)$. Otherwise, for any $q \neq 0.5$ the two present values are different. Next, consider the situation from the perspective of type A workers. Based on proposition 2 one can find the optimal level of diversification which would maximize their present value $U_A^3(\gamma)$. Let it be denoted by γ^* . It can be found from the following first order condition:

$$\underbrace{-\frac{\partial f_{AA}^3(\gamma^*)}{\partial \gamma}(w - z)}_{\text{Marginal cost of diversification}} = \underbrace{\frac{\partial f_{AB}^3(\gamma^*)}{\partial \gamma}(w_0 - z)}_{\text{Marginal gain of diversification}}$$

where $\partial f_{AA}^3/\partial \gamma > 0$ and $\partial f_{AB}^3/\partial \gamma < 0$. Consider diversifying the network by reducing γ . The left-hand side of this condition is the marginal cost of diversification, which is reflected in worse employment chances in the primary occupation for type A workers if gamma is lower. The right-hand side of this equation is the marginal gain, as lower γ implies a higher number of type B contacts and better employment chances in occupation B . Is it always the case that the marginal cost is larger than the marginal gain? Let the number of contacts n be sufficiently large and $\gamma = 1$, then reducing the number of contacts of the same type down to $n - 1$ is likely to have a small impact on the probability of finding a job in the primary occupation λ_{AA}^i and a large impact on the probability of finding a job in the mismatch occupation λ_{AB}^i . This means, in the case of full homophily, one can expect a large gain and a small cost of network diversification. Continuing this way, one arrives at the point where the marginal gain is exactly equal to the marginal cost, which is precisely the optimal level of network diversification γ^* .

Finally, note that the marginal gain of diversification strongly depends on the difference $w_0 - z$. When accepting mismatched jobs in occupation B type A workers increase their flow income from z to w_0 but give up the option of finding a job in their primary occupation. This means that diversification of social contacts is never optimal if $w_0 = z$. In this situation there are no gains from diversification. Thus diversification is more likely with a higher value of w_0 and a lower value of z .

4 Comparative statics analysis

Figure 5 illustrates the shape of the unemployment present value U_A^3 for different values of the correlation coefficient ρ . The two expansion probabilities are fixed at $p = q = 0.5$, moreover, if the two occupations have identical parameters it is true that $U_A^3 = U_B^2$. This means that both worker types obtain a maximum present value at the same diversification level γ^* . The case of asymmetric occupations will be investigated later in this section. The left panel of this figure shows the main result of this paper. The present value of income is maximized for some interior homophily level. For example, if the two occupations are perfectly negatively correlated. i.e. $\rho = -1$, workers will achieve a maximum present value when $\gamma^* = 0.625$, which is the proportion of contacts in their primary occupation (A) (see the pink solid curve). This corresponds to 25 contacts out of $n = 40$. At the same time, it is optimal for them to have $(1 - \gamma)n = 15$ contacts in the mismatch occupation in order to exit unemployment as soon as possible if there is low labour demand in the primary occupation. Note that the optimal γ^* is increasing with higher values of the correlation coefficient. For example, when the two occupations are perfectly positively correlated, i.e. $\rho = 1$, the optimal level of homophily is equal to 0.675, which means 27 contacts

in the primary occupation and 13 contacts in the mismatch occupation (see the green solid curve).

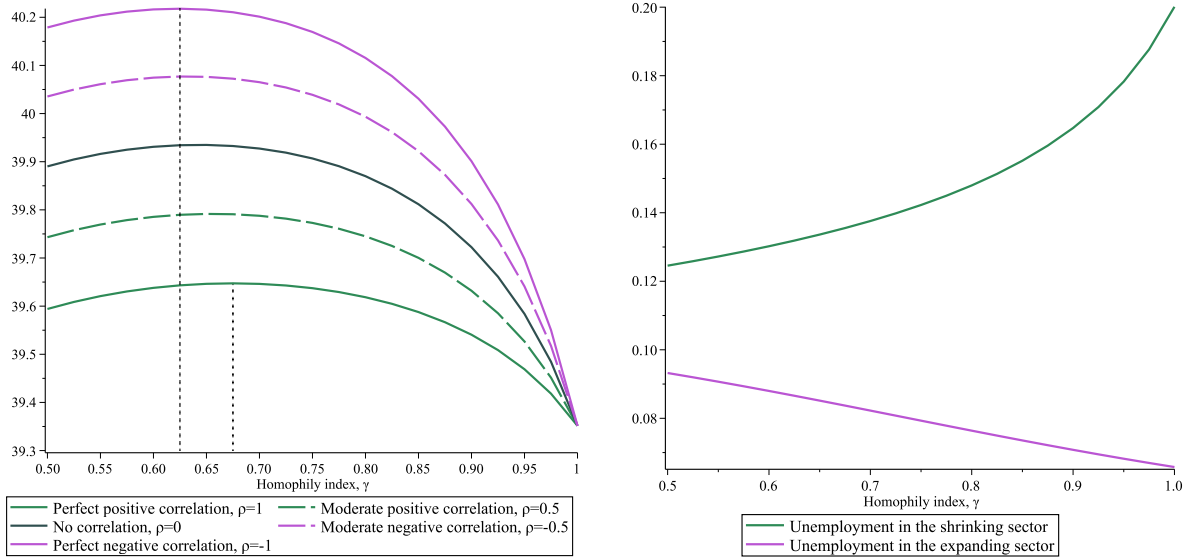


Figure 5: Left panel: PV of unemployment U_A^3 . Right panel: Unemployment rates u_A^3 , u_B^3 . Common parameters: $\delta = 0.1$, $s = 2$, $n = 40$, $z = 0.2$, $w_0 = 0.85$, $w = 0.9$, $p = q = 0.5$.

The right panel of figure 5 illustrates unemployment rates u_A^3 and u_B^3 for different levels of homophily. If both occupations are expanding (i.e. $v_A^1 = v_B^1 = 0.05$), then the two unemployment rates are the same and equal to 0.066 for any value of γ . The same is true when both occupations are in a recession (i.e. $v_A^4 = v_B^4 = 0.04$), so that the two unemployment rates are equal to 0.2 for any value of γ . In contrast, the level of homophily has a strong impact on the unemployment rates if the two occupations are in the opposite states. As follows from figure 5, increasing homophily from $\gamma = 0.5$ to 1 is associated with a drop in unemployment from 0.093 to 0.066 for workers trained in the expanding occupation (see the pink solid curve on the right panel). Moreover, the same increase in the level of homophily is associated with a dramatic rise of unemployment from 0.125 to 0.2 for workers trained in the recessing occupation (see the green solid curve on the right panel). This is intuitive, since having all contacts in the recessing occupation implies a very low job-finding rate, so many workers remain unemployed.

To understand the role of wages and unemployment benefits for network diversification, consider the case of perfect negative correlation ($\rho = -1$). As follows from the theoretical part of the paper, diversification is more desirable if the unemployment benefit z is relatively low whereas the mismatch wage w_0 is relatively high. This is illustrated on the left panel of figure 6. This figure shows that $\gamma^* = 0.6$ is optimal if the unemployment benefit is reduced down to $z = 0.1$, whereas $\gamma^* = 0.65$ if the unemployment benefit is increased to $z = 0.3$.

The right panel of figure 6 shows comparative statics with respect to the probability of expansion in occupation B which is captured by variable q . Note that this probability is the same as the probability of recession in occupation A , since $q = 1 - p$ in the case of perfect negative correlation. The present value of unemployment is very sensitive to this parameter. For example, $\gamma^* = 0.5$ (corner solution) if occupation A is almost always in a recession, which happens for $q = 0.99$ and $p = 0.01$. This means that workers trained in a constantly recessing occupation

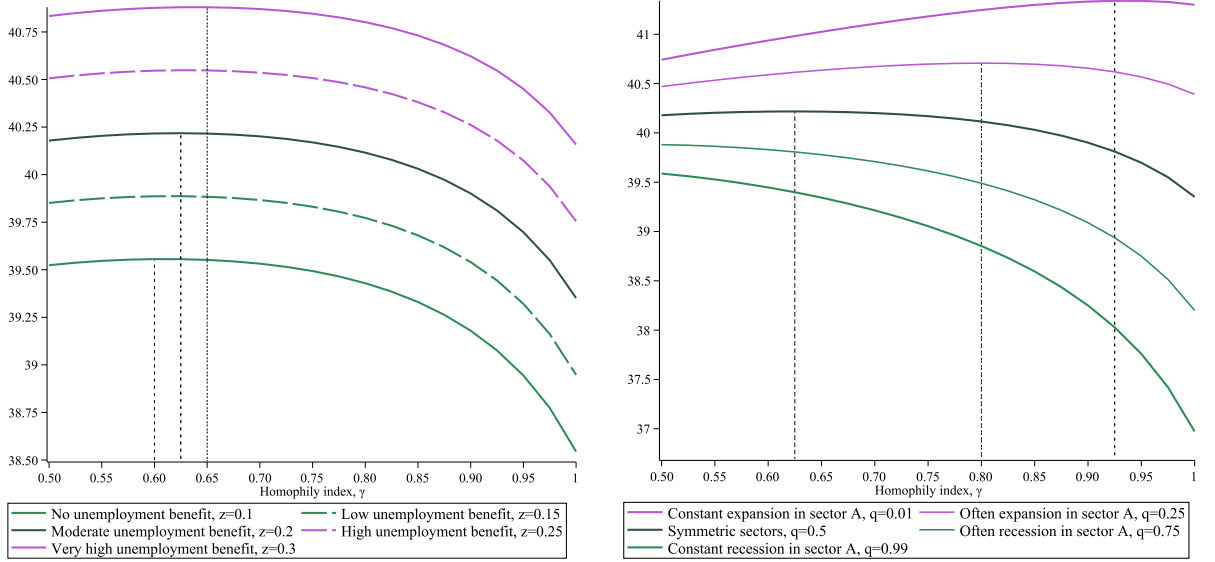


Figure 6: Both panels show the present value of unemployment U_A^3 . Common parameters: $\delta = 0.1$, $s = 2$, $n = 40$, $\rho = -1$, $w_0 = 0.85$, $w = 0.9$.

will gain most from full diversification. However, as q is decreasing and p is increasing, the optimal level of homophily is going up, reaching the level $\gamma^* = 0.925$ for $q = 0.01$ and $p = 0.99$. This corresponds to having 37 contacts in the primary occupation and only 3 contacts in the mismatch occupation. Indeed, there are no reasons to have many contacts in the mismatch occupation if your own occupation is constantly expanding.

In addition, the right panel of figure 6 shows that there doesn't exist a unique value of the diversification parameter γ maximizing the income present value of both worker types simultaneously if the two occupations are not symmetric. Recall that the optimal number of contacts was equal to 0.625 when $\rho = -1$ and $p = q = 0.5$. If the probability of expansion is relatively low in occupation A and relatively high in occupation B (case $p = 0.25$ and $q = 0.75$), then it is optimal for type A workers to have a fully diversified network (see the green curve on the right panel of figure 6). In contrast, for type B workers the maximum is reached if they have a fraction $\gamma^* = 0.8$ of their contacts in occupation B (see the pink curve on the same figure). So the optimal composition of the network differs between the two types of workers.

Figure 7 shows that this asymmetric network composition is not an artifact of the perfect negative correlation. For example, the left panel of figure 7 presents results for the case of independent occupations, i.e. $\rho = 0$. Here again the optimal number of contacts for type A workers is falling from 0.65 to 0.5 (see the shift from the black curve to the green curve) when the probability of expansion in occupation A is decreasing from $p = 0.5$ (and $q = 0.5$) to $p = 0.25$ (and $q = 0.75$). On the contrary, the optimal number of contacts for type B workers is increasing to $\gamma^* = 0.85$ in this case (see the shift from the black curve to the pink curve).

The right panel of figure 7 shows the reservation wage w_0^R for different levels of γ and ρ . First, note that for all parameter values on the figure it holds that $w_0 = 0.85 > w_0^R$, which means that all workers accept jobs in the mismatch occupation even in the case of an economy-wide expansion. This guarantees existence of the equilibrium described above. In addition, one can see that the reservation wage is increasing as the correlation coefficient becomes more negative.

This is in line with the previous findings as negative correlation between occupations is beneficial for workers and is associated with a higher present value of unemployment. Negative correlation implies a lower probability of an economy-wide recession, so that unemployed workers face better chances of finding a job in the nearest future which makes them choosier with respect to accepted wages. Considering again the case of perfect negative correlation, and different values of q , it is not surprising that lower values of q (which means higher levels of p) improve the present value of unemployment for type A workers. However, I find that the reservation wage is not very sensitive to this parameter rising up to 0.845 if $q = 0.01$ ($p = 0.99$). This means even in a good scenario with perfect negative correlation and frequent expansions in occupation A , type A workers continue accepting mismatch jobs with a flow wage $w_0 = 0.85$. The same is true for type B workers.

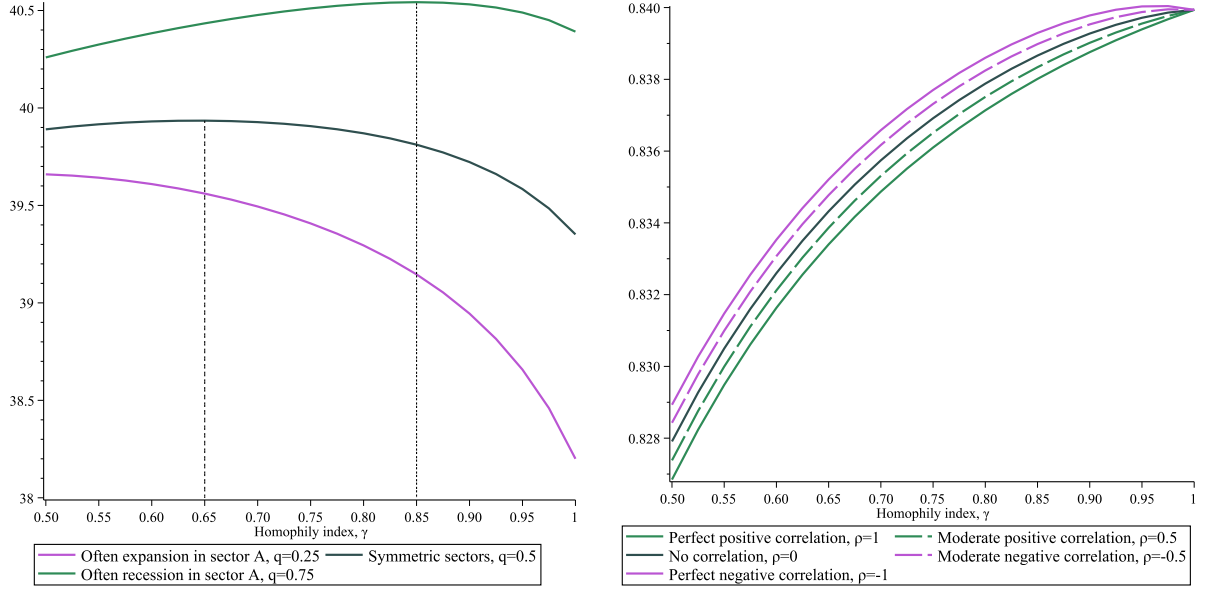


Figure 7: Left panel: the present value of unemployment U_A^3 for $\rho = 0$. Right panel: the reservation wage w_0^R for $q = p = 0.5$. Common parameters: $\delta = 0.1$, $s = 2$, $n = 40$, $z = 0.2$, $w_0 = 0.85$, $w = 0.9$.

Overall, figure 7 shows that the reservation wage w_0^R is relatively high reaching the level of 0.84 for some values of γ and ρ . This is because the model presented above does not account for the possibility of searching on-the-job. Nevertheless, empirical evidence (e.g. Wolbers (2003) and Beduwe and Giret (2011)) shows that mismatched workers are searching more intensively for alternative jobs compared to those employed in their primary occupation. These findings are taken into account in the next section where the model is extended to the case of on-the-job search by mismatched workers. At the same time, Burdett and Mortensen (1998) show that reservation wages are typically lower with on-the-job search. When accepting mismatch employment in a situation without on-the-job search workers give up a valuable option to get a job in the primary occupation. So they require a high compensation for the loss of this option. In a similar situation with on-the-job search workers do not fully lose the possibility of getting a job in the primary occupation, so their required compensation is reduced. For the next section this implies that the level of reservation wages is less important for equilibrium existence and will not be considered.

5 On-the-job search

5.1 Job arrival rates

This section extends the model to the case of on-the-job search. It is intuitive to think that mismatched workers may continue searching for jobs in their primary occupation. Let η_A denote the job arrival rate to workers A employed in the mismatch occupation B and associated with on-the-job search. In a similar way, η_B denotes the job arrival rate to workers B associated with on-the-job search (see figure 8). Recall that e_{AA} is the mass of type A workers employed in their primary occupation and e_{AB} – the mass of mismatched type A workers, so that $e_A = e_{AA} + e_{AB}$. I use similar notation for type B workers, so that $e_B = e_{BB} + e_{BA}$. As before firms with open vacancies in occupation A contact one of the incumbent employees of type A employed in their occupation at rate s per unit time. The incumbent employee who is chosen to recommend a friend out of his/her network of n contacts acts according to the following scheme:

- If there is at least one unemployed type A contact, forward the offer to this person. Randomize if there are several unemployed type A contacts;
- If there are no unemployed type A contacts, check if there is at least one mismatched type A contact and forward the offer to this person. Randomize if there are several mismatched type A contacts;
- If none of type A contacts is either unemployed or mismatched, check if there is at least one unemployed type B contact and forward the offer to this person. Randomize if there are several unemployed type B contacts;
- If none of type A contacts is either unemployed or mismatched and none of type B contacts is unemployed the offer is lost.

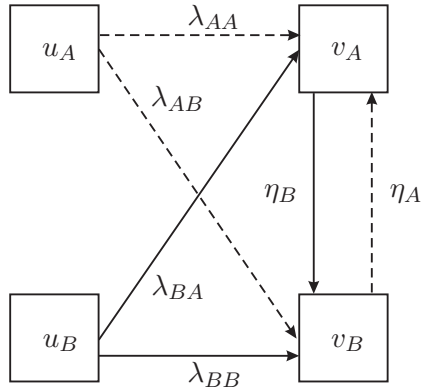


Figure 8: Labour market transitions with on-the-job search

This sequence of actions implies that workers always first try to recommend their contacts of the same type whether they are unemployed or mismatched and only afterwards forward the offer to the unemployed contacts of the opposite type. One rationale for this assumption is that workers with the same training are often working in teams and their knowledge is complementary to each other. Thus there may be private productivity gains from recommending workers of the

same type. Nevertheless, no specific technology assumptions are necessary for the purpose of this paper. Since unemployed workers of the same type are still prioritized, there is no change in the expressions for λ_{AA} and λ_{BB} compared to the model without on-the-job search. Next suppose the worker doesn't have any unemployed type A contacts which happens with probability $e_A^{\gamma n}$. Conditional on being employed, a given contact is working in occupation A with probability e_{AA}/e_A and is mismatched with a counterprobability e_{AB}/e_A . Thus with probability $(e_{AA}/e_A)^{\gamma n}$ all employed contacts are working in occupation A . This means that $1 - (e_{AA}/e_A)^{\gamma n}$ is the probability that there is at least one mismatched employee out of γn employed type A contacts. This reasoning allows me to calculate the job arrival rate η_A in the following way:

$$\eta_A = \frac{sv_A}{e_{AB}} \cdot e_A^{\gamma n} \cdot \left(1 - \left(\frac{e_{AA}}{e_A}\right)^{\gamma n}\right) = \frac{sv_A}{e_{AB}} \cdot (e_A^{\gamma n} - e_{AA}^{\gamma n})$$

Further note that with probability $e_{AA}^{\gamma n} = e_A^{\gamma n} \cdot (e_{AA}/e_A)^{\gamma n}$ neither of type A contacts of the incumbent employee is unemployed nor mismatched. However, with probability $1 - e_B^{(1-\gamma)n}$ there is at least one unemployed type B contact, so the job-finding rate λ_{BA} becomes:

$$\lambda_{BA} = \frac{sv_A}{u_B} \cdot e_{AA}^{\gamma n} \cdot (1 - e_B^{(1-\gamma)n})$$

In a similar way, one can derive the job-finding rate λ_{AB} and the job arrival rate to mismatched type B workers associated with on-the-job search η_B :

$$\lambda_{AB} = \frac{sv_B}{u_A} \cdot e_{BB}^{\gamma n} \cdot (1 - e_A^{(1-\gamma)n}) \quad \text{and} \quad \eta_B = \frac{sv_B}{e_{BA}} \cdot (e_B^{\gamma n} - e_{BB}^{\gamma n})$$

Differential equations for unemployment don't change with on-the-job search, thus $\dot{u}_j = \delta e_j - (\lambda_{jA} + \lambda_{jB})(1 - e_j)$, $j = A, B$. Next consider changes in variable e_{AA} . The inflow of workers into this category consists of unemployed type A workers $\lambda_{AA}u_A$ and mismatched workers coming from occupation B , that is $\eta_A e_{AB}$. Given that $e_{AB} = e_A - e_{AA}$, $u_A = 1 - e_A$ and repeating the analysis for type B workers one gets the following steady state conditions for e_{AA} and e_{BB} :

$$\begin{aligned} e_{AA}(\eta_A + \delta) &= \lambda_{AA}(1 - e_A) + \eta_A e_A \\ e_{BB}(\eta_B + \delta) &= \lambda_{BB}(1 - e_B) + \eta_B e_B \end{aligned}$$

Solving these two equations jointly with the steady state equations $\dot{u}_A = 0$ and $\dot{u}_B = 0$ with respect to variables $\{e_A, e_{AA}, e_B, e_{BB}\}$ gives rise to lemma 3:

Lemma 3: *For every state i , $i = 1, \dots, m$, the equilibrium rates e_A , e_{AA} , e_B and e_{BB} are uniquely determined from the following system of equations:*

$$sv_A(1 - e_A^{\gamma n}) + sv_B \cdot e_{BB}^{\gamma n} \cdot (1 - e_A^{(1-\gamma)n}) = \delta e_A \quad \text{where} \quad \delta e_{BB} = sv_B(1 - e_{BB}^{\gamma n}) \quad (5.4)$$

$$sv_A \cdot e_{AA}^{\gamma n} \cdot (1 - e_B^{(1-\gamma)n}) + sv_B \cdot (1 - e_B^{\gamma n}) = \delta e_B \quad \text{where} \quad \delta e_{AA} = sv_A(1 - e_{AA}^{\gamma n}) \quad (5.5)$$

A higher number of vacancies v_j leads to higher employment in both occupations e_A and e_B , $j = A, B$. However, a higher v_A raises the fraction of mismatched type B workers, while a higher v_B raises the fraction of mismatched type A workers. For the same state, employment in both occupations is lower in the model with on-the-job search.

These findings are illustrated on figure 9. Note that e_{BB} is the point where function $e_B(e_A)$ is crossing the horizontal axis, so that $e_{BB} = e_B(0)$. This means that $e_A = e_A(e_B(0))$. Intuitively, on-the-job search reduces employment of type A workers as their chances of finding jobs in occupation B are getting worse. In a similar way, e_{AA} is the point where function $e_A(e_B)$ is crossing the vertical axis, so that $e_{AA} = e_A(0)$. This means that $e_B = e_B(e_A(0))$, thus the equilibrium employment of type B workers is also lower with on-the-job search (see the left panel of figure 9).

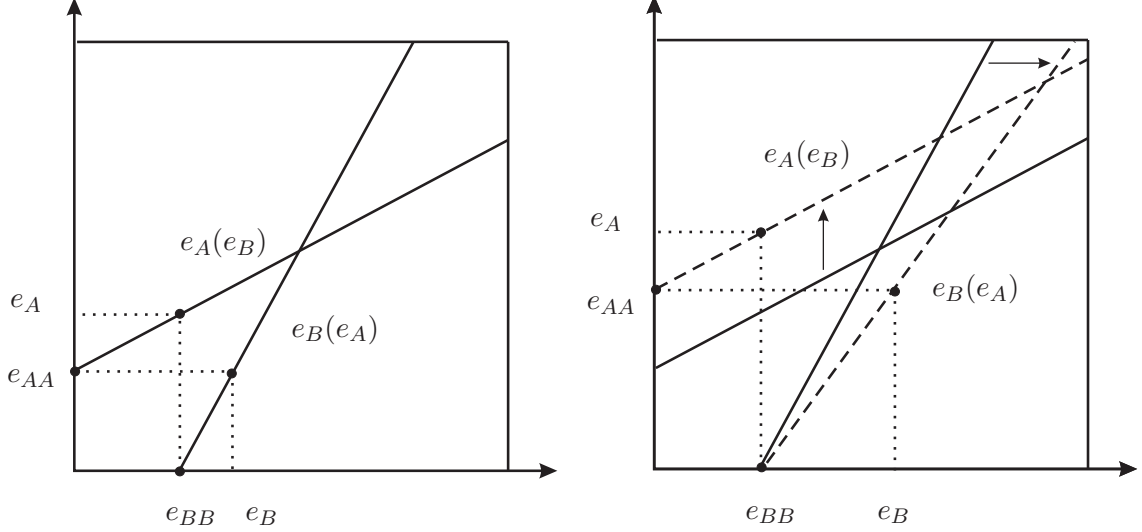


Figure 9: Left panel: equilibrium with OJS. Right panel: positive shock in vacancies v_A

Further, consider a higher number of vacancies v_A which is illustrated on the right panel of figure 9. This vacancy shock is associated with an upward shift of the curve $e_A(e_B)$ and a downward rotation of the curve $e_B(e_A)$, thus triggering a rise of employment rates e_A and e_B . However, there is no change in the number of type B workers employed in their primary occupation, that is e_{BB} remains unchanged with on-the-job search. If occupation A has low labour demand, then relatively many type B workers find jobs in their primary occupation directly from unemployment and relatively few workers go through the state of intermediate mismatch employment in occupation A . In contrast, when occupation A is expanding, then relatively many type B workers go through the state of intermediate employment in occupation A and relatively few of them find jobs directly from unemployment. However, the sum of the two inflows (from unemployment and mismatched jobs) remains unchanged whatever the situation in occupation A , leading to the unchanged rate e_{BB} . Hence, the number of mismatched type B workers $e_{BA} = e_B - e_{BB}$ is unambiguously higher when occupation A is expanding.

5.2 Numerical example with on-the-job search

This section continues the numerical example from section 4 with the same parameter values but extended to the case of on-the-job search. The equilibrium employment rates in each of the four macroeconomic states are presented in table 3. First, with on-the-job search unemployment is higher and employment is lower in every state. This is because unemployed workers of the opposite type are now less likely to hear about a job as priority is given to unemployed and

mismatched workers of the same type. Even more striking is that the fraction of mismatched workers is also lower. For example, in state 1 when both occupations are expanding, mismatched workers constitute only 2.2% of the total labour force. This is much lower than 19.1% in the model without on-the-job search. These stock variables can be low for two reasons. Either because the inflow of mismatched workers is relatively low or because the outflow is too high. Focusing on type A workers, table 3 reveals that the outflow rate of mismatched type A workers ($\eta_A = 0.302$) is almost three times larger than the inflow rate $\lambda_{AB} = 0.104$. Thus the stock of mismatched workers is low mostly due to the high outflow rate. Hence many workers accept jobs in the mismatch occupation with a fast transition to their primary occupation thereafter.

| | Expansion in B, $v_B = 0.05$ | Recession in B, $v_B = 0.04$ |
|--------------------------------|--|--|
| Expansion in A $v_A = 0.05$ | State 1 with prob. π_1 $e_A = e_B = 0.915$ $e_{AB} = e_{BA} = 0.022$ $\eta_A = \eta_B = 0.302$ $\lambda_{AA} = \lambda_{BB} = 0.982$ $\lambda_{AB} = \lambda_{BA} = 0.104$ | State 2 with prob. π_2 $e_A = 0.896, e_B = 0.861$ $e_{AB} = 0.002, e_{BA} = 0.069$ $\eta_A = 0.242, \eta_B = 0.047$ $\lambda_{AA} = 0.853, \lambda_{BB} = 0.546$ $\lambda_{AB} = 0.007, \lambda_{BA} = 0.072$ |
| Recession in A $v_A = 0.04$ | State 3 with prob. π_3 $e_A = 0.861, e_B = 0.896$ $e_{AB} = 0.069, e_{BA} = 0.002$ $\eta_A = 0.047, \eta_B = 0.242$ $\lambda_{AA} = 0.546, \lambda_{BB} = 0.853$ $\lambda_{AB} = 0.072, \lambda_{BA} = 0.007$ | State 4 with prob. π_4 $e_A = e_B = 0.799$ $e_{AB} = e_{BA} = 0.006$ $\eta_A = \eta_B = 0.021$ $\lambda_{AA} = \lambda_{BB} = 0.393$ $\lambda_{AB} = \lambda_{BA} = 0.004$ |

Table 3: The model with $m = 4$ states, $\delta = 0.1$, $s = 2$, $n = 40$, $\gamma = 0.5$

Further, as follows from table 3 mismatch can still be significant despite on-the-job search in the asymmetric states 2 and 3. For example, when occupation A is expanding, while occupation B is in a recession, 6.9% of type B workers are mismatched ($e_{BA} = 0.069$). This is intuitive as these workers experience difficulties finding jobs in their primary occupation B and accept jobs in the mismatch occupation at rate $\lambda_{BA} = 0.072$. However, the corresponding transition rate from A to B is relatively low, $\eta_B = 0.047$, and 6.9% of type B workers remain mismatched on average in state 2.

5.3 Present value equations

The only change in the initial present value equations due to on-the-job search concerns equations for the mismatched present values W_{AB}^i and W_{BA}^i . Consider workers of type A and macroeconomic state i . Then the present value equation for W_{AB}^i becomes:

$$\begin{aligned}
 rW_{AB}^i &= w_0 - \delta(W_{AB}^i - U_A^i) + \eta_A(W_{AA}^i - W_{AB}^i) + \phi[\pi_1(W_{AB}^1 - W_{AB}^i) + \dots \\
 &+ \pi_{i-1}(W_{AB}^{i-1} - W_{AB}^i) + \pi_{i+1}(W_{AB}^{i+1} - W_{AB}^i) + \dots + \pi_m(W_{AB}^m - W_{AB}^i)]
 \end{aligned}$$

The new term in this equation $\eta_A(W_{AA}^i - W_{AB}^i)$ corresponds to the fact that type A workers employed in occupation B may change the job at rate η_A . To write this equation in the matrix form let N_A be the $m \times m$ matrix containing the job arrival rates η_A^i on the main diagonal. In

addition, let N_B be the corresponding matrix for type B workers:

$$N_A = \begin{pmatrix} \eta_A^1 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & \eta_A^m \end{pmatrix} \quad N_B = \begin{pmatrix} \eta_B^1 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & \eta_B^m \end{pmatrix}$$

With this notation modified vectors of present values W_{AB} and W_{BA} can be written as:

$$\begin{aligned} rW_{AB} &= \omega_0 - \delta(W_{AB} - U_A) + N_A(W_{AA} - W_{AB}) + \phi(\Pi - I)W_{AB} \\ rW_{BA} &= \omega_0 - \delta(W_{BA} - U_B) + N_B(W_{BB} - W_{BA}) + \phi(\Pi - I)W_{BA} \end{aligned}$$

So modified present values of unemployed workers U_A and U_B become:

$$\begin{aligned} U_A &= [(r + \phi)I - \phi\Pi + \Lambda_{AA}(I - \delta M) + \Lambda_{AB}(I - \delta M_A(I + N_A M))]^{-1} \\ &\quad \times (z + \Lambda_{AB}M_A w_0 + (\Lambda_{AA} + \Lambda_{AB}M_A N_A)Mw) \\ U_B &= [(r + \phi)I - \phi\Pi + \Lambda_{BB}(I - \delta M) + \Lambda_{BA}(I - \delta M_B(I + N_B M))]^{-1} \\ &\quad \times (z + \Lambda_{BA}M_B w_0 + (\Lambda_{BB} + \Lambda_{BA}M_B N_B)Mw) \end{aligned}$$

where $M = [(r + \delta + \phi)I - \phi\Pi]^{-1}$, $M_A = [(r + \delta + \phi)I - \phi\Pi + N_A]^{-1}$ and $M_B = [(r + \delta + \phi)I - \phi\Pi + N_B]^{-1}$ are the three auxiliary matrices. The proof is presented in appendix V. With these results I can proceed to the numerical characterization of the labour market with on-the-job search in the next section.

5.4 Comparative statics with on-the-job search

This section is dedicated to the numerical analysis of the model with on-the-job search. Figure 10 (right panel) illustrates changes in the unemployment rate for both types of workers. In state 3 there is a recession in occupation A and the unemployment of type A workers is relatively high (solid green curve). At the same time having less contacts with type B workers (that is higher γ) raises their unemployment even further. The situation is different for type B workers as their occupation is expanding in state 3 and their unemployment is relatively low (solid pink curve). Moreover it is falling with a stronger homophily index γ . With on-the-job search unemployed workers of the opposite type are ranked behind mismatched workers of the same type, thus the probability of leaving unemployment is lower with on-the-job search and unemployment is higher. This change is represented by the solid curves on the right panel of figure 10.

Changes in the unemployment rate reveal that there are two counteracting effects of on-the-job search on the present value of income. On the one hand, on-the-job search is beneficial as the option to continue searching in the mismatched state always has a positive value. However, this effect is based on the assumption of unchanged job-finding rates which is not the case in the present model. In contrast, unemployment is increasing and the probability of finding a job for unemployed workers is falling with on-the-job search. This is a negative effect for the present value of income. The left panel of figure 10 shows that the negative effect is dominating especially for low values of γ since the fall in unemployment is particularly pronounced for strong diversification levels. Note that the present value U_A^3 on figure 10 is compared to its counterpart

without on-the-job search for two different values of the correlation coefficient $\rho = -1$ and $\rho = 1$. In both situations the optimal level of diversification is decreasing (i.e. higher γ^*) with on-the-job search. For example, γ^* goes up from 0.625 to 0.725 in the case of perfect negative correlation ($\rho = -1$), which means that workers should have 29 social contacts of the same type and only 11 contacts of the opposite type. Underlying this result is the fact that primary contacts become more important if the worker wants to leave a mismatch job and come back to the primary occupation. Thus the cost of diversification is higher and the gain is lower with on-the-job search. Similar changes can be reported for the intermediate values of the correlation coefficient which is illustrated on the left panel of figure 11.

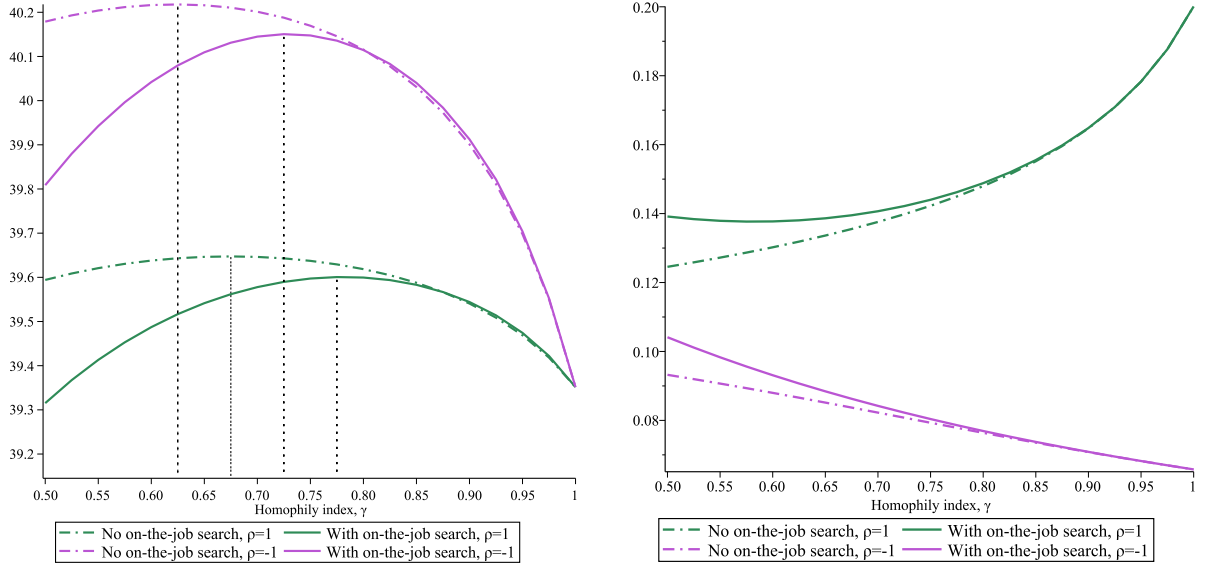


Figure 10: Left panel: PV of unemployment U_A^3 with and without on-the-job search for different values of ρ , $q = p = 0.5$. Right panel: PV of unemployment u_A^3 with and without on-the-job search for different values of $q = 1 - p$, $\rho = -1$. Common parameters: $\delta = 0.1$, $s = 2$, $n = 40$, $z = 0.2$, $w_0 = 0.85$, $w = 0.9$

The right panel of figure 11 illustrates consequences of on-the-job search in a labour market with asymmetric occupations and perfect negative correlation. Specifically, I consider a vector of recession probabilities $q = 1 - p \in \{0.01, 0.25, 0.5, 0.75, 0.99\}$. As already shown before, raising q is associated with a sharp decline of the unemployment present value for type A workers (solid green curves) while reducing q is associated with a sharp rise of U_A^3 (solid pink curves). One can see that full diversification is never optimal with on-the-job search. Even when $q = 0.99$, which means that occupation A suffers from a continuous recession, the optimal level of homophily is equal to 0.6 rather than 0.5 which was the optimal level of homophily in the model without on-the-job search. Intuitively, primary contacts become more valuable with on-the-job search as they are helpful not only during unemployment but also in the state of mismatched employment. However, the difference between the optimal level of homophily with and without on-the job search is diminishing with a higher γ . For example, γ^* is again equal to 0.925 when $q = 0.01$. In this situation, on-the-job search doesn't have any impact on unemployment and so there is hardly any change in the present value when γ is relatively high.

Next consider changes in the network size n . Based on lemma 3 one can show that the

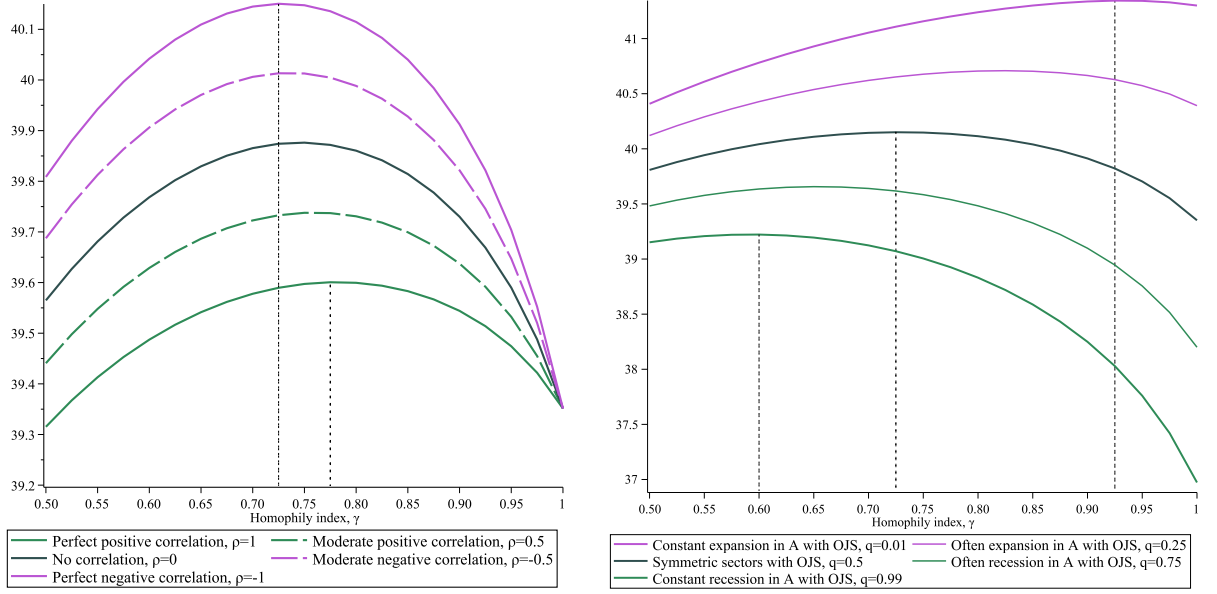


Figure 11: Left panel: PV of unemployment U_A^3 with on-the-job search for different values of ρ , $q = p = 0.5$. Right panel: PV of unemployment U_A^3 with on-the-job search for different values of $q = 1 - p$, $\rho = -1$. Common parameters: $s = 2$, $n = 40$, $z = 0.2$, $w_0 = 0.85$, $w = 0.9$

proportions of properly matched workers e_{AA} and e_{BB} are strictly increasing in n . However, the probability that all contacts of the same type are employed ($e_{AA}^{\gamma n}$ and $e_{BB}^{\gamma n}$) is decreasing. This means that the probability that all contacts of the same type are employed is relatively high in a small network but it is relatively small in a very large network. Writing it down from the perspective of type B workers I get:

$$\frac{\partial e_{BB}}{\partial n} = -\frac{\gamma s v_B e_{BB}^{\gamma n} \ln e_{BB}}{\delta + s v_B \gamma n e_{BB}^{\gamma n-1}} > 0 \quad \frac{\partial e_{BB}^{\gamma n}}{\partial n} = \frac{\delta \gamma n e_{BB}^{\gamma n} \ln e_{BB}}{\delta + s v_B \gamma n e_{BB}^{\gamma n-1}} < 0$$

Note that $e_{BB}^{\gamma n}$ is sensitive to the job destruction rate δ . If jobs are frequently destroyed, relatively few workers are properly matched, that is e_{BB} is low and responds strongly to higher n . In contrast, when jobs are stable, most workers are properly matched, that is e_{BB} is close to 1 and shows low responsiveness to the network size n . Further consider type A workers. The term $s v_B e_{BB}^{\gamma n}$ is a probability that type A worker will get a job offer forwarded by one of the employed type B workers in his/her network. Thus with larger networks type A workers are less likely to hear about a job in the mismatch occupation B . According to lemma 2 this has a negative indirect effect on their employment e_A :

$$\frac{\partial e_A}{\partial n} = \frac{\overbrace{-s \ln e_A [v_A \gamma e_A^{\gamma n} + v_B (1 - \gamma) e_{BB}^{\gamma n} e_A^{(1-\gamma)n}]}^{\text{Positive direct effect}} + \overbrace{s v_B (1 - e_A^{(1-\gamma)n}) \cdot \frac{\partial e_{BB}^{\gamma n}}{\partial n}}^{\text{Negative indirect effect}}}{\delta + s v_A \gamma n e_A^{\gamma n-1} + s v_B \gamma n e_{BB}^{\gamma n} (1 - \gamma) n e_A^{(1-\gamma)n-1}}$$

At the same time, if the network is larger, the probability of finding a job in the primary occupation is increasing. This is a positive direct effect of the network size. Note that the positive effect is likely to dominate if the primary occupation of type A workers is expanding, whereas the mismatch occupation is not (state 2). In this state mismatch employment is less

relevant in the job search process. In contrast, the indirect negative effect is likely to dominate if the mismatch occupation B is expanding, whereas the primary occupation is not (state 3). The situation in state 3 is illustrated on the left panel on figure 12.

If the job destruction δ is low, employment e_A^3 is increasing with a larger network size and the unemployment u_A^3 is falling (shift from dashed to solid curves). This is because having more contacts improves chances of getting job offers in the primary occupation. This effect dominates for the benchmark parameter value $\delta = 0.1$. On the contrary, if δ is large employed type B workers can choose among a larger group of type B contacts and are less likely to forward the job offer to type A contacts. Thus employment e_A^3 is falling with a larger network size and the unemployment rate u_A^3 is increasing.

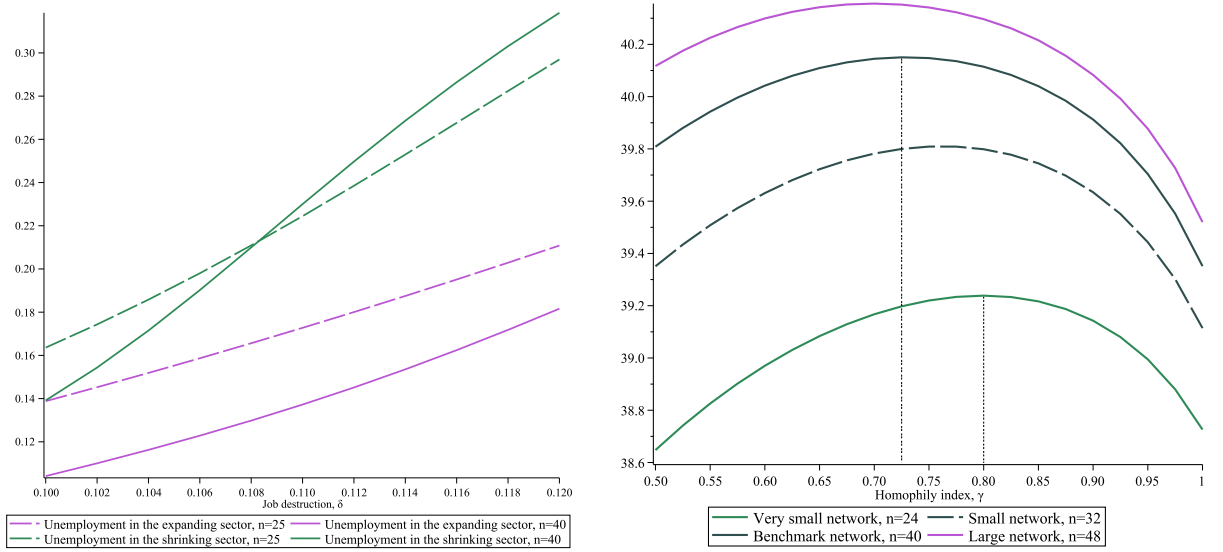


Figure 12: Left panel: Unemployment in state 3 (u_A^3 and u_B^3) with OJS for different values of δ and n , $\gamma = 0.5$. Right panel: PV of unemployment U_A^3 with on-the-job search for different values of n , $\delta = 0.1$. Common parameters: $s = 2$, $n = 40$, $z = 0.2$, $w_0 = 0.85$, $w = 0.9$, $q = p = 0.5$, $\rho = -1$

The right panel of figure 12 shows changes in the optimal diversification level γ^* . Note that the four present value curves U_A^3 are depicted for type A workers in state 3. One can see that the optimal homophily level γ^* falls from 0.8 down to 0.725 as the network size is increasing from $n = 24$ to $n = 40$. This implies a change from about 19 to 29 contacts in the primary occupation if the network is enlarged from 24 to 40 contacts. Hence in larger networks the present value of income will be maximized with stronger diversification, whereas in smaller networks a low level of diversification is optimal. This is intuitive since redistributing one more contact to the mismatch occupation is costly if the total number of contacts (network size) is small and every contact in the primary occupation is very valuable. However, it is less costly if the network is large and there are already many contacts in the primary occupation. Thus the cost of diversification is large in a small network but it is relatively small in a large network. This explains the fact that the optimal level of diversification is increasing (lower γ) with the network size. Based on this reasoning one can conclude that larger networks are more likely to be diversified than smaller networks.

6 Conclusions

This paper develops a search model with two risk-neutral worker types, two occupations and correlated business-cycle fluctuations in labour demand. This model is used to analyze the effect of social networks on occupational mismatch and workers' expected income. In the expectation of low labour demand in the primary occupation, workers' expected income will be maximized for some interior value of network homophily. Thus it is optimal for risk-neutral workers to diversify their social network and keep a fraction of their contacts in the mismatch occupation even though it is associated with a considerable wage penalty. The reason for diversification is that the present value of income is a non-linear function of the network composition. Thus, the optimal level of diversification is obtained at the point where the marginal gain from having one more contact in the mismatch occupation is equal to the marginal cost from having one contact less in the primary occupation. This is different from portfolio theory, where expected gain is a linear function of the investment share and the optimal portfolio composition is always a corner solution for the risk-neutral investor.

Comparative statics results show that the optimal diversification level is higher with a lower correlation in labour demand between the two occupations and with a lower unemployment benefit. On the contrary, a higher probability of expansion in the primary occupation and on-the-job search are both associated with weaker diversification. In addition, this paper identifies positive employment spillovers between the two occupations. A positive labour demand shock in one occupation raises employment in both occupations due to the fact that workers exchange vacancy information with their contacts of the opposite type. This is the network multiplier effect. Finally, I show that larger networks are more likely to be diversified than smaller networks, but unemployment is not necessarily decreasing in the network size.

7 Appendix

Appendix I: Proof of lemma 1

$$\frac{\partial \lambda_{AA}}{\partial e_A} = \frac{sv_A}{(1-e_A)^2} \left[-\gamma n e_A^{\gamma n-1} (1-e_A) + 1 - e_A^{\gamma n} \right] = \frac{sv_A}{(1-e_A)^2} (1 - \sigma(e_A)) \geq 0,$$

where $\sigma(e_A) = e_A^{\gamma n-1} (\gamma n (1-e_A) + e_A) \leq 1$

Note that auxilliary function $\sigma(e_A)$ is always smaller than 1 for $e_A < 1$. This is because $\sigma(0) = 0$, $\sigma(1) = 1$ and $\sigma'(e_A) = (\gamma n - 1) e_A^{\gamma n-2} \gamma n (1-e_A) > 0$. Therefore, all job-finding rates are increasing in v_A and v_B due to the direct effect and the indirect effect through higher employment rates e_A and e_B .

Appendix II: Proof of proposition 1:

$$\Lambda_{Aj} M = \begin{bmatrix} \lambda_{Aj}^1 m_{11} & \lambda_{Aj}^1 m_{12} & \cdots & \lambda_{Aj}^1 m_{1m} \\ \lambda_{Aj}^2 m_{21} & \lambda_{Aj}^2 m_{22} & \cdots & \lambda_{Aj}^2 m_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ \lambda_{Aj}^m m_{m1} & \lambda_{Aj}^m m_{m2} & \cdots & \lambda_{Aj}^m m_{mm} \end{bmatrix}$$

$$\Lambda_{AA}M\omega = \begin{bmatrix} w\lambda_{AA}^1 \sum_{k=1}^m m_{1k} \\ w\lambda_{AA}^2 \sum_{k=1}^m m_{2k} \\ \vdots \\ w\lambda_{AA}^m \sum_{k=1}^m m_{mk} \end{bmatrix} \quad \Lambda_{AB}M\omega_0 = \begin{bmatrix} w_0\lambda_{AB}^1 \sum_{k=1}^m m_{1k} \\ w_0\lambda_{AB}^2 \sum_{k=1}^m m_{2k} \\ \vdots \\ w_0\lambda_{AB}^m \sum_{k=1}^m m_{mk} \end{bmatrix}$$

$$\begin{aligned} U_A &= \begin{bmatrix} g_{11}^A & g_{12}^A & \cdots & g_{1m}^A \\ g_{21}^A & g_{22}^A & \cdots & g_{2m}^A \\ \vdots & \vdots & \cdots & \vdots \\ g_{m1}^A & g_{m2}^A & \cdots & g_{mm}^A \end{bmatrix} \times \begin{bmatrix} z + w\lambda_{AA}^1 \sum_{k=1}^m m_{1k} + w_0\lambda_{AB}^1 \sum_{k=1}^m m_{1k} \\ z + w\lambda_{AA}^2 \sum_{k=1}^m m_{2k} + w_0\lambda_{AB}^2 \sum_{k=1}^m m_{2k} \\ \vdots \\ z + w\lambda_{AA}^m \sum_{k=1}^m m_{mk} + w_0\lambda_{AB}^m \sum_{k=1}^m m_{mk} \end{bmatrix} \\ &= \begin{bmatrix} z \sum_{l=1}^m g_{1l}^A + w \sum_{l=1}^m g_{1l}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + w_0 \sum_{l=1}^m g_{1l}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk} \\ z \sum_{l=1}^m g_{2l}^A + w \sum_{l=1}^m g_{2l}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + w_0 \sum_{l=1}^m g_{2l}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk} \\ \vdots \\ z \sum_{l=1}^m g_{ml}^A + w \sum_{l=1}^m g_{ml}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + w_0 \sum_{l=1}^m g_{ml}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk} \end{bmatrix} \end{aligned}$$

Proof of lemma 2:

$$\begin{aligned} W_{AB} &= M \times \begin{bmatrix} w_0 + \delta(z \sum_{l=1}^m g_{1l}^A + w \sum_{l=1}^m g_{1l}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + w_0 \sum_{l=1}^m g_{1l}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk}) \\ w_0 + \delta(z \sum_{l=1}^m g_{2l}^A + w \sum_{l=1}^m g_{2l}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + w_0 \sum_{l=1}^m g_{2l}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk}) \\ \vdots \\ w_0 + \delta(z \sum_{l=1}^m g_{ml}^A + w \sum_{l=1}^m g_{ml}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + w_0 \sum_{l=1}^m g_{ml}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk}) \end{bmatrix} \\ &= \begin{bmatrix} w_0 \sum_{i=1}^m m_{1i} + \delta z \sum_{i=1}^m m_{1i} \sum_{l=1}^m g_{il}^A + \delta w \sum_{i=1}^m m_{1i} \sum_{l=1}^m g_{il}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + \delta w_0 \sum_{i=1}^m m_{1i} \sum_{l=1}^m g_{il}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk} \\ w_0 \sum_{i=1}^m m_{2i} + \delta z \sum_{i=1}^m m_{2i} \sum_{l=1}^m g_{il}^A + \delta w \sum_{i=1}^m m_{2i} \sum_{l=1}^m g_{il}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + \delta w_0 \sum_{i=1}^m m_{2i} \sum_{l=1}^m g_{il}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk} \\ \vdots \\ w_0 \sum_{i=1}^m m_{mi} + \delta z \sum_{i=1}^m m_{mi} \sum_{l=1}^m g_{il}^A + \delta w \sum_{i=1}^m m_{mi} \sum_{l=1}^m g_{il}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + \delta w_0 \sum_{i=1}^m m_{mi} \sum_{l=1}^m g_{il}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk} \end{bmatrix} \end{aligned}$$

So the reservation wage w_0^R comes from equation $U_A^1 = W_{AB}^1$:

$$\begin{aligned} & z \sum_{l=1}^m g_{1l}^A + w \sum_{l=1}^m g_{1l}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + w_0^R \sum_{l=1}^m g_{1l}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk} = w_0^R \sum_{i=1}^m m_{1i} + \\ & + \delta z \sum_{i=1}^m m_{1i} \sum_{l=1}^m g_{il}^A + \delta w \sum_{i=1}^m m_{1i} \sum_{l=1}^m g_{il}^A \lambda_{AA}^l \sum_{k=1}^m m_{lk} + \delta w_0^R \sum_{i=1}^m m_{1i} \sum_{l=1}^m g_{il}^A \lambda_{AB}^l \sum_{k=1}^m m_{lk} \end{aligned}$$

Appendix III: Derivation of the transition matrix II.

Let x be the indicator function taking value 1 if occupation A is expanding (with probability p) and 0 otherwise. Similarly, let y be the indicator function for occupation B , which is expanding with probability q . Then the means and variances of x and y are given by: $E[x] = p$, $E[y] = q$, $V[x] = p(1-p)$ and $V[y] = q(1-q)$.

Further, let $\pi_1 = P\{x = 1, y = 1\}$, $\pi_2 = P\{x = 1, y = 0\}$, $\pi_3 = P\{x = 0, y = 1\}$,

$\pi_4 = P\{x = 0, y = 0\}$. For example, π_1 is the probability that both occupations are expanding. Then it holds that $\pi_2 = p - \pi_1$ and $\pi_3 = q - \pi_1$ and $\pi_4 = 1 - p - q + \pi_1$. The covariance between the two variables x and y can then be expressed as:

$$\begin{aligned} \text{cov}(x, y) &= (1-p)(1-q)\pi_1 + (1-p)(0-q)(p-\pi_1) + (0-p)(1-q)(q-\pi_1) \\ &+ (0-p)(0-q)(1-q-p-\pi_1) = \pi_1 - pq \end{aligned}$$

So the correlation coefficient ρ can be expressed as:

$$\rho = \frac{\text{cov}(x, y)}{\sqrt{V[x]V[y]}} = \frac{\pi_1 - pq}{\sqrt{p(1-p)q(1-q)}} \quad \Rightarrow \quad \pi_1 = pq + \rho\sqrt{p(1-p)q(1-q)}$$

which gives rise to table 2.

Appendix IV: Proof of proposition 2:

$$(r + \delta + \phi)I - \phi\Pi = \begin{pmatrix} r + \delta + \phi q & -\phi q \\ -\phi(1-q) & r + \delta + \phi(1-q) \end{pmatrix}$$

$$M = \frac{1}{(r + \delta)(r + \delta + \phi)} \begin{pmatrix} r + \delta + \phi(1-q) & \phi q \\ \phi(1-q) & r + \delta + \phi q \end{pmatrix}$$

$$U_A = \frac{1}{ad - bc} \begin{pmatrix} a & b \\ c & d \end{pmatrix} (\zeta + \Lambda_{AA}M\omega + \Lambda_{AB}M\omega_0)$$

$$\begin{aligned} a &= r + \frac{r(\lambda_{AA}^3 + \lambda_{AB}^3)}{r + \delta} + c & b &= \phi q + \frac{(\lambda_{AA}^2 + \lambda_{AB}^2)}{r + \delta} \frac{\delta \phi q}{r + \delta + \phi} \\ c &= \phi(1-q) + \frac{(\lambda_{AA}^3 + \lambda_{AB}^3)}{r + \delta} \frac{\delta \phi(1-q)}{r + \delta + \phi} & d &= r + \frac{r(\lambda_{AA}^2 + \lambda_{AB}^2)}{r + \delta} + b \end{aligned}$$

$$\begin{aligned} ad - bc &= ad - \left(d - r - \frac{r(\lambda_{AA}^2 + \lambda_{AB}^2)}{r + \delta}\right) \left(a - r - \frac{r(\lambda_{AA}^3 + \lambda_{AB}^3)}{r + \delta}\right) \\ &= ad - ad + d\left(r + \frac{r(\lambda_{AA}^3 + \lambda_{AB}^3)}{r + \delta}\right) + \left(r + \frac{r(\lambda_{AA}^2 + \lambda_{AB}^2)}{r + \delta}\right) \left(a - r - \frac{r(\lambda_{AA}^3 + \lambda_{AB}^3)}{r + \delta}\right) \\ &= d\left(r + \frac{r(\lambda_{AA}^3 + \lambda_{AB}^3)}{r + \delta}\right) + c\left(r + \frac{r(\lambda_{AA}^2 + \lambda_{AB}^2)}{r + \delta}\right) \\ &= \frac{r}{r + \delta} [(c + d)(r + \delta) + c(\lambda_{AA}^2 + \lambda_{AB}^2) + d(\lambda_{AA}^3 + \lambda_{AB}^3)] \end{aligned}$$

$$U_A^3 = z \frac{c + d}{(ad - bc)} + \frac{w}{r + \delta} \frac{[c\lambda_{AA}^2 + d\lambda_{AA}^3]}{(ad - bc)} + \frac{w_0}{r + \delta} \frac{[c\lambda_{AB}^2 + d\lambda_{AB}^3]}{(ad - bc)}$$

$$rU_A^3 = z(1 - f_{AA}^3 - f_{AB}^3) + wf_{AA}^3 + w_0f_{AB}^3$$

$$f_{Aj}^3 = \frac{c\lambda_{Aj}^2 + d\lambda_{Aj}^3}{(r+\delta)(c+d) + c\lambda_{AA}^2 + d\lambda_{AA}^3 + c\lambda_{AB}^2 + d\lambda_{AB}^3} \quad j = A, B$$

Alternatively, $ad - bc$ can be rewritten as:

$$ad - bc = \frac{r}{r+\delta}[(a+b)(r+\delta) + a(\lambda_{AA}^2 + \lambda_{AB}^2) + b(\lambda_{AA}^3 + \lambda_{AB}^3)]$$

which allows me to show that:

$$rU_A^2 = z(1 - f_{AA}^2 - f_{AB}^2) + wf_{AA}^2 + w_0f_{AB}^2$$

$$f_{Aj}^2 = \frac{a\lambda_{Aj}^2 + b\lambda_{Aj}^3}{(r+\delta)(a+b) + a\lambda_{AA}^2 + b\lambda_{AA}^3 + a\lambda_{AB}^2 + b\lambda_{AB}^3} \quad j = A, B$$

Next consider type B workers:

$$U_B = \frac{1}{a'd' - b'c'} \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} (\zeta + \Lambda_{BB}M\omega + \Lambda_{BA}M\omega_0)$$

$$\begin{aligned} a' &= r + \frac{r(\lambda_{BB}^3 + \lambda_{BA}^3)}{r+\delta} + c' & b' &= \phi q + \frac{(\lambda_{BB}^2 + \lambda_{BA}^2)}{r+\delta} \frac{\delta\phi q}{r+\delta+\phi} \\ c' &= \phi(1-q) + \frac{(\lambda_{BB}^3 + \lambda_{BA}^3)}{r+\delta} \frac{\delta\phi(1-q)}{r+\delta+\phi} & d' &= r + \frac{r(\lambda_{BB}^2 + \lambda_{BA}^2)}{r+\delta} + b' \end{aligned}$$

$$\begin{aligned} a'd' - b'c' &= \left(r + \frac{r(\lambda_{BB}^3 + \lambda_{BA}^3)}{r+\delta} + c'\right) \left(r + \frac{r(\lambda_{BB}^2 + \lambda_{BA}^2)}{r+\delta} + b'\right) - b'c' \\ &= \left(r + \frac{r(\lambda_{BB}^3 + \lambda_{BA}^3)}{r+\delta} + c'\right) \left(r + \frac{r(\lambda_{BB}^2 + \lambda_{BA}^2)}{r+\delta}\right) + \left(r + \frac{r(\lambda_{BB}^3 + \lambda_{BA}^3)}{r+\delta}\right) b' + b'c' - b'c' \\ &= a' \left(r + \frac{r(\lambda_{BB}^2 + \lambda_{BA}^2)}{r+\delta}\right) + b' \left(r + \frac{r(\lambda_{BB}^3 + \lambda_{BA}^3)}{r+\delta}\right) \\ &= \frac{r}{r+\delta} [(a' + b')(r+\delta) + a'(\lambda_{BB}^2 + \lambda_{BA}^2) + b'(\lambda_{BB}^3 + \lambda_{BA}^3)] \end{aligned}$$

$$U_B^2 = z \frac{a' + b'}{(a'd' - b'c')} + \frac{w}{r+\delta} \frac{[a'\lambda_{BB}^2 + b'\lambda_{BB}^3]}{(a'd' - b'c')} + \frac{w_0}{r+\delta} \frac{[a'\lambda_{BA}^2 + b'\lambda_{BA}^3]}{(a'd' - b'c')}$$

$$rU_B^2 = z(1 - f_{BB}^2 - f_{BA}^2) + wf_{BB}^2 + w_0f_{BA}^2$$

$$f_{Bj}^2 = \frac{a'\lambda_{Bj}^2 + b'\lambda_{Bj}^3}{(r+\delta)(a' + b') + a'\lambda_{BB}^2 + b'\lambda_{BB}^3 + a'\lambda_{BA}^2 + b'\lambda_{BA}^3} \quad j = A, B$$

Alternatively, $a'd' - b'c'$ can be rewritten as:

$$a'd' - b'c' = \frac{r}{r+\delta} [(c' + d')(r+\delta) + c'(\lambda_{BB}^2 + \lambda_{BA}^2) + d'(\lambda_{BB}^3 + \lambda_{BA}^3)]$$

which allows me to show that:

$$rU_B^3 = z(1 - f_{BB}^3 - f_{BA}^3) + wf_{BB}^3 + w_0f_{BA}^3$$

$$f_{Bj}^3 = \frac{c'\lambda_{Bj}^2 + d'\lambda_{Bj}^3}{(r + \delta)(c' + d') + c'\lambda_{BB}^2 + d'\lambda_{BB}^3 + c'\lambda_{BA}^2 + d'\lambda_{BA}^3} \quad j = A, B$$

Appendix V Let $M = [(r + \delta + \phi)I - \phi\Pi]^{-1}$ and $M_A = [(r + \delta + \phi)I - \phi\Pi + N_A]^{-1}$ be the two auxilliary matrices, so that:

$$\begin{aligned} ((r + \delta)I - \phi(\Pi - I))W_{AA} &= w + \delta U_A \quad \Rightarrow \quad W_{AA} = M(w + \delta U_A) \\ ((r + \delta)I - \phi(\Pi - I) + N_A)W_{AB} &= \omega_0 + \delta U_A + N_A W_{AA} \end{aligned}$$

Thus the vector of present values W_{AB} can be expressed as:

$$\begin{aligned} W_{AB} &= M_A(\omega_0 + \delta U_A + N_A W_{AA}) = M_A(\omega_0 + \delta U_A + N_A M(w + \delta U_A)) \\ &= M_A(\omega_0 + N_A M w + \delta(I + N_A M)U_A) \end{aligned}$$

Thus I get the following equation for U_A :

$$\begin{aligned} rU_A &= \zeta + \Lambda_{AA}[M(w + \delta U_A) - U_A] \\ &+ \Lambda_{AB}[M_A(\omega_0 + N_A M w + \delta(I + N_A M)U_A) - U_A] + \phi(\Pi - I)U_A \\ &= [(r + \phi)I - \phi\Pi + \Lambda_{AA}(I - \delta M) + \Lambda_{AB}(I - \delta M_A(I + N_A M))]U_A \\ &= \zeta + \Lambda_{AB}M_A\omega_0 + (\Lambda_{AA} + \Lambda_{AB}M_A N_A)M w \end{aligned}$$

Inverting the matrix in the square bracket on the left-hand side produces the final equation for U_A . In a similar way one can obtain the equation for U_B .

8 References

- ADDISON J.T., P. PORTUGAL (2002): "Job Search Methods and Outcomes", *Oxford Economic Papers*, 54(3): 505-533.
- ALLEN J., E. DE WEERT (2007): "What do Educational Mismatches Tell us about Skill Mismatches? A Cross-Country Analysis", *European Journal of Education*, 42(1): 59-73.
- ALLEN J., R. VAN DER VELDEN (2001): "Educational Mismatches versus Skill Mismatches: Effects on Wages, Job Satisfaction, and On-the-Job Search", *Oxford Economic Papers* Special issue on Skills Measurement and Economic Analysis, 434-452.
- BEDUWE C., J.-F. GIRET (2011): "Mismatch of Vocational Graduates: What Penalty on the French Labour Market", *Journal of Vocational Behaviour*, 78: 68-79.
- BENTOLILA S., C. MICHELACCI, J. SUAREZ (2010): "Social Contacts and Occupational Choice", *Economica*, 77(305): 20-45.
- BLAZQUEZ M., M. JANSSEN, (2008): "Search, Mismatch and Unemployment", *European Economic Review*, 52: 498-526.

- BURDETT K., D.T. MORTENSEN (1998): "Wage Differentials, Employer Size and Unemployment", *International Economic Review*, 39(2): 257-273.
- CAHUC P., F. FONTAINE (2009): "On the Efficiency of Job Search with Social Networks", *Journal of Public Economic Theory*, 11(3): 411-439.
- DELATTRE E., M. SABATIER (2007): "Social Capital and Wages: an Econometric Evaluation of Social Networking's Effects", *Labour*, 21(2): 209-236.
- FONTAINE F. (2008): "Why Are Similar Workers Paid Differently? The Role of Social Networks", *Journal of Economic Dynamics and Control*, 32(12): 3960-3977.
- GALENIANOS M. (2014): "Hiring through Referrals", *Journal of Economic Theory*, 152: 304-323.
- GRANOVETTER M. (1995): "Getting a Job: A Study of Contacts and Careers", Chicago: University of Chicago Press.
- HALL R. E., P.R. MILGROM (2008): "The Limited Influence of Unemployment on the Wage Bargain", *The American Economic Review*, 98(4):1653-1674.
- HENSVIK L., O. N. SKANS (2013): "Social Networks, Employee Selection and Labor Market Outcomes", *Working Paper*, Institute for Evaluation of Labor Market and Education Policy, No. 2013:15.
- HORVATH G. (2014a) "On-the-Job Search and Finding a Good Job Through Social Contacts", *The B.E. Journal of Theoretical Economics*, 14(1): 93-125.
- HORVATH G. (2014b): "Occupational Mismatch and Social Networks", *Journal of Economic Behavior and Organization*, 106: 442-468.
- IOANNIDES Y.M., A.R. SOETEVENT (2006): "Wages and Employment in a Random Social Network with Arbitrary Degree Distribution", *The American Economic Review*, 96(2): 270-274.
- JACKSON M. O. (2008): "Social and Economic Networks", Princeton University Press.
- KUGLER A. D. (2003): "Employee Referrals and Efficiency Wages", *Labour Economics*, 10: 531-556.
- MARGOLIS D.M., V. SIMMONET (2003): "Educational Track, Networks and Labor Market Outcomes", *IZA Discussion paper*, No. 699.
- MCPHERSON M., L. SMITH-LOVIN, J.M. COOK (2001): "Birds of a Feather: Homophily in Social Networks", *Annual Review of Sociology*, 27: 415-444.
- MOEN E. (1997): "Competitive Search Equilibrium", *Journal of Political Economy*, 105(2): 385-411.
- MONTGOMERY J.D. (1991): "Social Networks and Labor-Market Outcomes: Toward an Economic Analysis", *The American Economic Review*, 81(5): 1408-1418.
- NORDIN M., I. PERSSON, ROTH D.-O. (2008): "Education-Occupation Mismatch: Is There an Income Penalty?", *IZA Discussion paper* No. 3806.
- PISTAFERRI L. (1999): "Informal Networks in the Italian Labor Market", *Giornale degli Economisti e Annali di Economia*, 58(3-4): 355-375.
- ROBST J. (2007): "Education, College Major, and Job Match: Gender Differences in Reasons for Mismatch", *Education Economics*, 15(2): 159-175.
- SIMON C.J., J.T. WARNER (1992): "Matchmaker, Matchmaker: The Effect of Old Boy Networks on Job Match Quality, Earnings, and Tenure", *Journal of Labor Economics*, 10(3): 306-330.
- STUPNYTSKA Y., A. ZAHARIEVA (2015): "Explaining U-shape of the Referral Hiring Pattern in a Search Model with Heterogeneous Workers", *Journal of Economic Behavior and Organizations*, 119: 211-233.
- WOLBERS M. S. J. (2003): "Job Mismatches and Their Labour Market Effects Among School-Leavers in Europe", *European Sociological Review*, 19(3): 249-266.
- ZAHARIEVA A. (2013): "Social Welfare and Wage Inequality in Search Equilibrium with Personal Contacts", *Labour Economics*, 23: 107-121.
- ZAHARIEVA A. (2015): "Social Contacts and Referrals in a Labour Market with On-the-Job Search", *Labour Economics* 32, 27-43.