Skill Accumulation in the Market and at Home

Jean Flemming*

June 16, 2016

Abstract

This paper introduces learning by doing in home production into a stochastic directed search model. Workers' labor supply choices affect skill accumulation in both the home and market sectors, making the distributions of workers across employment states endogenous and persistent state variables. The optimal search behavior by an unemployed worker implies that the job finding probability is much more sensitive than the reemployment wage to the duration of unemployment, two facts which have been documented empirically. The calibrated model is used to decompose the declining hazard out of unemployment, implying a nontrivial role for true duration dependence due to changes in skills. The model framework can easily accommodate business cycles, and the mechanism predicts that the optimal response of agents' decisions after an aggregate shock generates an asymmetric response of the unemployment rate during and after recessions, and more severe recessions resulting in stronger hysteresis in labor force participation.

1 Introduction

It is a widely documented fact that the probability that an unemployed worker finds a job is falling over the duration of unemployment. Additionally, recent empirical work has found that reemployment and reservation wages are only mildly sensitive to duration (Fernández-Blanco and Preugschat 2015, Krueger and Mueller 2016, Schmieder et al. 2013). This paper aims to reconcile these two facts by introducing learning by doing in home production during unemployment and in market production during employment.

*University of Rome, Tor Vergata and Einaudi Institute for Economics and Finance (EIEF). I thank Guido Menzio, Francesco Lippi, Facundo Piguillem, Mark Aguiar, Stefano Gagliarducci, Iourii Manovskii, Fabrizio Mattesini, Claudio Michelacci, Andrea Pozzi, Daniele Terlizzese, Ludo Visschers, and seminar participants at the UPenn Macro Club, EIEF and Tor Vergata Lunch Seminars, and University of Oxford, and conference participants at the Cambridge SaM Workshop, Annual SaM Conference 2016, and Spring 2016 Midwest Macro Meetings for helpful comments. For most recent version, please go to: `http://jeanflemming.com/research.html`

Though the large effect of duration on the job finding probability, or hazard rate of finding a job, has been thoroughly explored in the literature, there is less evidence on the effect of the length of the unemployment spell on starting wages. Instead, the empirical literature has mainly focused on long-term earnings losses due to unemployment. To understand the full effects of unemployment spells on future employment prospects and earnings, it is essential to think not only about the impact effect of entering unemployment but also the effect of duration.

This paper contributes to the literature in three ways. Theoretically, it develops a directed search model introducing a new channel that affects the outside options of the unemployed, skill accumulation in home production, that can account for the two features of the data discussed above. Quantitatively, the model is calibrated to standard labor market targets and is able to closely match the mild decline in reemployment wages. The model is then used to decompose the decline in the job finding probability and indicates a significant role for changes in workers' skills during unemployment, an effect known in the literature as "true duration dependence". Further, the predictions regarding business cycles imply a persistent decrease in labor force participation in response to temporary recessions due to a countercyclical outside option for the nonemployed. Empirically, this paper documents that the results previously shown using the Current Population Survey (CPS) also qualitatively hold in the Panel Study of Income Dynamics (PSID) and are robust to controlling both for observable heterogeneity as well as unobservable worker characteristics using individual fixed effects.

More precisely, this paper will address the facts above using a model of directed search with a home production sector. Each worker is endowed at the beginning of time with observable home and market productivities. Both of these skills evolve over time according to a worker's current employment status. Workers devote their labor to either home or market production, where workers at home are nonemployed and workers in the market are employed. Similar to the intuition of standard models of learning by doing, the skill that a worker uses in production appreciates and the skill not in use depreciates. The directed search framework implies that unemployed workers with different skills search for different jobs in different locations, or submarkets.

The driving force in the model is an evolving outside option for unemployed workers due to changes in both skills over time. Similar to with standard models of on the job learning, in this model unemployed workers lose market-related skills, causing the surplus created by a match to fall with duration. Unlike these models, skill accumulation in home production causes workers' outside options to increase and in turn implies that the worker's share of the surplus, if a match is created, is increasing with duration. Thus, firms find it less profitable to hire the long-term unemployed, decreasing the equilibrium job finding probability at long durations, but increasing the reemployment wages of those workers who match successfully.

The average job finding probability and reemployment wage over unemployment duration depend on two factors: the direct effect of individual skill changes with duration, and the indirect effect due to changes in the composition of unemployed workers. First, the evolution of an unemployed worker's skills influences her own optimal job search decisions. As workers spend more time in unemployment, the payoff from remaining in home production increases while market skills deteriorate simultaneously. These skill changes lead workers to become pickier about the jobs they find acceptable at the same time that they are losing market-related skills. When deciding whether to accept a job, unemployed workers face a tradeoff. By moving to employment, a worker expects to gain market skills but will give up future gains in home skills. Therefore the net utility gain that a worker expects when making her search decision depends on the effects of skill changes in both employment states in a complex way. When individual skills are persistent enough, the model implies that the changes in the two skills over the spell reinforce one another, causing the job finding probability to fall with duration, but offset each other in determining the reemployment wage.

Second, in the aggregate, unemployed workers who are relatively better at market production will tend to re-enter employment quickly, shifting the composition of the unemployed towards workers with lower market skills and higher home skills as duration increases. Together the direct effect, known in the literature as "true duration dependence", and the indirect effect due to compositional changes make up the decline in the job finding rate. The calibrated model implies a non-trivial role for true duration dependence, but a more important role for the composition effect in the decline of the hazard, as suggested by the previous literature.

The model is calibrated to match key labor market facts, including the average job finding and separation rates and the return to experience in market work estimated in US data. When skills in home production accumulate quickly during unemployment and depreciate quickly during employment, the model generates a large decline in the aggregate job finding probability but only a mild decrease in reemployment wages with duration. The effects of an aggregate market productivity shock are then studied with regard to the dynamics of the job finding, separation, participation, and unemployment rates. Changes in the composition of worker types across labor force statuses are essential to understand the model's cyclical predictions. Starting from a steady state in which the distribution of workers across types is constant, a recession caused by a temporary decline in aggregate productivity causes a temporary fall in the job finding rate for all job seekers, an increase in the unemployment rate, and a long-term decline in the participation rate.

Specifically, when the economy experiences a negative aggregate productivity shock, lower market productivity on the firm side decreases the relative value of working and increases the attractiveness of home work. This relative increase in the outside option implies that the unemployed trade off higher wages for lower matching probabilities, increasing unemployment duration for all types. If the shock is large enough, workers will leave the labor force: by strictly preferring home work, some workers will discontinue their job search. In the next period, the average skills of these individuals evolve to generate a persistent decline in the participation rate even after the temporary decline in aggregate productivity subsides. Therefore the model gives rise to hysteresis, resulting in a persistent fall in the participation rate in response to a temporary negative aggregate shock. Finally, the recession leads to an increase in average skills in home production as workers with longer unemployment spells have more time to improve their skills at home.

Empirically, this paper first replicates the analysis using the CPS of Fernández-Blanco and Preugschat (2015) to show a clear difference in the responsiveness of reemployment wages and the hazard rate out of unemployment to duration. This paper extends their analysis to data from the PSID to show that the results are robust to controlling for some unobservable heterogeneity contained in individual fixed effects. Finally, it exploits data from the American Time Use Survey to illustrate support for this model and rule out an alternative mechanism.

The paper proceeds as follows. The following section briefly reviews the related literature. Section 2 describes the model, beginning with the decentralized economy followed by the social planner's problem. Section 3 summarizes the theoretical results. Section 4 describes the data and summarizes the empirical results. Section 5 outlines the method for the calibration and report the quantitative results of the model, both in and out of steady state. Section 6 concludes.

Related Literature

Theoretically, this paper connects three branches of literature: directed search, learning by doing, and home production. Models of directed search (Montgomery 1991, Moen 1997, and Burdett et al. 2001) are able to generate efficient equilibria in the presence of search frictions. Under general assumptions, Menzio and Shi (2009) prove the existence of block recursive equilibria, a concept developed by Shi (2009), which makes the combination of aggregate uncertainty and individual heterogeneity tractable. This paper will exploit this feature of directed search models to study individual workers' transitions between employment states as well as the equilibrium responses to aggregate uncertainty.

This model builds on the framework developed by Menzio and Shi (2011). Rather than the match-specific productivities assumed in their model, the model in this paper incorporates worker-specific productivities in both the home and market sectors. The fact that these productivities are persistent and worker- rather than match-specific allows the model to generate the stylized facts described above.

This model uses a mechanism of learning by doing, one of the main approaches that has been shown to generate duration dependence in the job finding probability. Here, the mechanism of Ljungqvist and Sargent (1998) is extended from a one- to two-dimensional skill composed of market and home productivity[1]. More recently, Jarosch (2014) and Jung and Kuhn (2012) consider skill loss in unemployment in models of directed search to study wage scarring. The other approach generating a declining hazard rate out of unemployment is firm-side discrimination of applicants by duration, introduced by Blanchard and Diamond (1994) and studied recently by Doppelt (2015), Jarosch and Pilossoph (2015), and Fernández-Blanco and Preugschat (2015). Importantly, all of these papers focus on steady state outcomes, whereas here the model's business cycle implications are explored.

This paper's use of home production borrows greatly from the seminal papers by Benhabib et al. (1991) and Greenwood and Hercowitz (1991). In this model, unemployed workers accumulate productivity at home over time, unlike the fixed home production technology typically assumed in the literature.

Recent work by Fernández-Blanco and Preugschat (2015) documents the stylized facts discussed in the Introduction and proposes a directed search model in which firms discriminate across applicants based on unemployment duration. In their model, firms can observe duration but cannot observe workers' market productivity. Firms must post high wages for workers with longer durations in order to induce them to apply to jobs with a low matching probability. In contrast, the model in this paper assumes that skills are observable and changes in workers' outside options are the key element to reproduce the same features of the data. In this model, for a fixed level of home productivity, wages are increasing in a worker's market skill, that is, more productive workers receive higher wages. Empirically, Fernández-Blanco and Preugschat were the first to contrast the responsiveness of the average job finding probability with the average reemployment wage using the CPS. They document that for actively searching workers, in the first year of unemployment the decline in the job finding probability is much larger than the decline in real hourly wages at reemployment when controlling for observable heterogeneity.

Much of the empirical literature focuses on the long term consequences of job loss by comparing the pre- and post-unemployment wages, and often studies manufacturing plant closings[2]. Though less prone to selection issues, these studies may not be representative for all sectors and occupations. Papers that have studied the shorter-term effects of job loss, specifically the effects of the length of unemployment duration include Addison and Portugal (1989), Gregory and Jukes (2001), and Arulampalam (2001). The latter studies the wages of British men, and finds no evidence of declines in wages due to the length of the spell on top of those due to the incidence of unemployment. Unlike earlier studies, Arulampalam's data set covers only the 1990s, which saw a decrease in unemployment durations as long-term unemployed workers began to collect disability benefits on a larger scale. Further, a rise in early unemployment may have led to lower wage losses as some

[1] Other papers incorporating multi-dimensional skills include Lindenlaub (2014), Lise and Postel-Vinay (2015), and Guvenen et al. (2015).

[2] For a survey of the early literature on wage scarring in the US, see papers by Fallick (1996) and Kletzer (1998). For more recent evidence, see Couch and Placzek (2010).

older workers opted to retire rather than remain actively unemployed. Similar results are found by Albrecht et al. (1999) using Swedish data.

2 Model

The economy is populated by a continuum of workers of measure 1 and a continuum of firms with a positive measure. Time is discrete and the horizon is infinite. All agents are risk neutral and discount the future at rate $\beta \in (0,1)$. There is a single consumption good produced in the economy. Workers are ex ante heterogeneous and defined by their productivities in the market and at home, or their "type," respectively (z,h), where $z \in Z = \{z_1, z_2, \ldots, z_{N_z}\}$ and $h \in H = \{h_1, h_2, \ldots, h_{N_h}\}$. The values market productivity may take are scalars $0 < z_1 < \ldots < z_{N_z}$ with the integer $N_z \geq 2$, and the set of possible values for home productivity h is defined similarly. Each worker faces an exogenous probability of death $\lambda \in (0,1)$ in every period. I assume that workers have a bequest motive whereby they derive utility from future generations of newborn workers. Each period, the same fraction λ of workers is born with productivities drawn from a given stationary distribution F_0.

Each firm operates a constant returns to scale technology that turns 1 unit of labor from a worker of type (z,h) into zy units of output. Aggregate productivity y is common to all firms and each period lies in the set $Y = \{y_1, y_2, \ldots, y_{N_y}\}$, where $0 < y_1 < \ldots < y_{N_y}$ with the integer $N_y \geq 2$. Each firm maximizes its discounted expected sum of profits. When a worker of type (z,h) is matched with a firm, she is called employed and provides z units of effective labor inelastically. Unmatched workers are called unemployed, and produce h units of output through home production. The laws of motion for productivities are given by the conditional probability mass functions $f_U(z',h'|z,h)$ and $f_E(z',h'|z,h)$, where $f_i(z',h'|z,h) \in [0,1]$ is the probability that a worker of type (z,h) who is unemployed $(i = U)$ or employed $(i = E)$ will be of type (z',h') next period. Henceforth, for any aggregate variable ω, let $\hat{\omega}$ denote next period's value. For the remainder of the paper, assume that[3]:

Assumption 1. *Productivities z, h, and y evolve independently. That is, for $i \in \{E,U\}$,*

$$f_i(z',h',\hat{y}|z,h,y) = f_{iz}(z'|z)f_{ih}(h'|h)f(\hat{y}|y)$$

Moreover, denoting the transition matrix for skill $s = \{z,h\}$ in employment state $i = \{E,U\}$ as Γ_{is},

[3]Although Assumption 1 is unnecessary to show existence, uniqueness, and efficiency of the decentralized equilibrium, it is used in Theorem 2 and in Propositions 2 and 3 for results on monotonicity.

$$
\Gamma_{is} = \begin{bmatrix} f_{is}(s_1|s_1) & \dots & f_{is}(s_{N_s}|s_1) \\ & \vdots & \\ f_{is}(s_1|s_{N_s}) & \dots & f_{is}(s_{N_s}|s_{N_s}) \end{bmatrix}
$$

and

$$
\Gamma_{y} = \begin{bmatrix} f(y_1|y_1) & \dots & f(y_{N_y}|y_1) \\ & \vdots & \\ f(y_1|y_{N_y}) & \dots & f(y_{N_y}|y_{N_y}) \end{bmatrix}
$$

assume that Γ_y, Γ_{Eh}, Γ_{Uz}, Γ_{Ez} and Γ_{Uh} are monotone matrices:

$$
\sum_{k=1}^{\ell} \Gamma(y_k, y_j) - \Gamma(y_k, y_{j+1}) \geq 0 \quad j = 1, ...N_y - 1, \; \ell = 1, ...N_y - 1
$$

$$
\sum_{k=1}^{\ell} \Gamma_{is}(s_k, s_j) - \Gamma_{is}(s_k, s_{j+1}) \geq 0
$$

$$
i = E, U, \; s = z, h \text{ and } j = 1, ...N_s - 1, \; \ell = 1, ...N_s - 1
$$

with a strict inequality for some ℓ.

Assumption 1 states that given a worker's type is (z, h) today, the current employment state determines how each component of the productivity pair will evolve. Writing the transition matrices Γ in increasing order of productivity, the matrices are monotone if the conditional expectation of tomorrow's productivity is an increasing function of today's productivity level. I assume that the aggregate productivity y is independent of the individual productivities (z, h) to exclude the presence of human capital externalities as discussed by Lucas (1988).

In any period, the economy is characterized by the aggregate state $\psi \equiv (y, u, e)$, with the set of possible values that the ψ may take denoted by Ψ. The first element of ψ is the aggregate productivity $y \in Y$. The second element is a function $u : Z \times H \to [0, 1]$, describing the distribution of unemployed workers across productivities, where $u(z, h)$ denotes the mass of workers who are unemployed of type (z, h). Similarly, the third element is a function $e : Z \times H \to [0, 1]$, where $e(z, h)$ denotes the mass of employed workers of type (z, h).

2.1 Decentralized Economy

At the beginning of each period, agents die with probability λ and agents of the same mass are born into unemployment with skills drawn from the distribution F_0 with probability mass function denoted f_0. Nature draws new productivities according to f_U and f_E for the surviving agents, and next period's aggregate productivity $\hat{y}$ is drawn from $f(\hat{y}|y)$. After these draws, the timing in any period is as follows: production, separation, search and matching. During the production stage, employed worker-firm pairs use the firm's technology to produce output zy and unemployed workers produce and consume h units of output through home production[4]. Employed workers consume their labor income, which is a piece rate α of production in the match. In the separation stage, with probability $d \in [\delta, 1]$ an employed worker separates from his match and enters unemployment, where $\delta \in (0, 1)$ is the exogenous separation probability and the choice of d is determined by the worker's employment contract.

The labor market is defined by submarkets in which workers and vacancy-posting firms meet. Submarkets are indexed by (x, z, h, ψ), where $x \in \mathbb{R}$ is the value in terms of the worker's lifetime utility of the match and (z, h) is the type of worker for which the vacancy is intended. In the search stage, firms choose submarkets in which to post vacancies and workers observe the distribution of offers before choosing one submarket in which to search. A firm may post a vacancy by paying a constant cost $k > 0$. Each vacancy in a submarket offers the same value x, and firms commit to this value as well as the type of worker they will hire if a match occurs. In this model with identical firms and deterministic wages, there is no incentive for on-the-job search; therefore without loss of generality I assume that only unemployed workers have the opportunity to search in each period[5]. Employed workers may search for a new job only after separating and entering unemployment for one period.

In the matching stage, the number of hires in a submarket is determined by a constant returns to scale technology $M(a, v)$ where a is the number of applicants in the submarket and v is the number of vacancies. Market tightness in submarket (x, z, h, ψ) when the aggregate state is ψ is denoted $\theta(x, z, h, \psi)$ and is defined as the ratio of vacancies to applicants. The probability that a vacancy meets a worker is $q(\theta) \equiv \frac{M(a,v)}{v}$, where $q : \mathbb{R}_+ \to [0, 1]$ is a twice continuously differentiable, strictly decreasing and convex function with $q(0) = 1$ and $q'(0) < 0$. Similarly, the probability that a worker meets a vacancy is given by $p(\theta) = q(\theta)\theta$, where $p : \mathbb{R}_+ \to [0, 1]$ is twice continuously differentiable, strictly increasing and strictly concave with $p(0) = 0,\ p(\infty) = 1$ and $p'(0) < \infty$.

The value function for an unemployed worker of type (z, h) is

$$V_U(z, h, \psi) = \sup_x \Bigg\{ h + \beta(1-\lambda)\Bigg[(1 - p(\theta(x, z, h, \psi)))\mathbb{E}_U(V_U(z', h', \hat{\psi})|z, h, \psi) + p(\theta(x, z, h, \psi))x\Bigg] + \beta\lambda\mathbb{E}(V_U(\overline{z}, \overline{h}, \hat{\psi})|\psi)\Bigg\} \quad (1)$$

where the policy function is denoted $x(z, h, \psi)$ and the implied market tightness is

[4]For an extension where workers choose their effort level in home production, see Appendix D.

[5]If market productivities were match-specific, workers would find it optimal to search when the productivity in the current match is sufficiently low.

denoted $\theta(x,z,h,\psi)$. The expectation operator $\mathbb{E}_i$, $i \in \{U,E\}$ denotes the expectation taken with respect to distribution Γ_i conditional on the current type (z,h) and aggregate state ψ, while the expectation operator $\mathbb{E}$ denotes the expectation taken with respect to the aggregate state and distribution F_0. For notational convenience, henceforth let $\mathbb{E}_i(V_i(z',h',\hat{\psi}))$ denote $\mathbb{E}_i(V_i(z',h',\hat{\psi})|z,h,\psi)$ for $i \in \{U,E\}$ and $\mathbb{E}(V_U(\overline{z},\overline{h},\hat{\psi}))$ denote $\mathbb{E}(V_U(\overline{z},\overline{h},\hat{\psi})|\psi)$.

To describe employed workers and firms, I assume that employment contracts are complete in the sense that they specify the wage and separation probability as a function of tenure t and history of types $\{z^t,h^t;y^t\}$ over tenure in the match, t. As shown in Menzio and Shi (2011), this contractual environment results in bilaterally efficient contracts which maximize the sum of the firm's expected profits and the worker's expected utility. This result follows from the fact that firms must guarantee the expected value x to any worker with whom it matches, forcing the firm to internalize the optimal choices of the worker when choosing the contract. Hence the optimal choice of the separation probability is the solution to the joint value of the match, which is equal to the sum of the worker's and firm's value functions:

$$V_M(z,h,\psi) = zy + \beta(1-\lambda)\max_{d\in[\delta,1]}\left\{d\mathbb{E}_U(V_U(z',h',\hat{\psi})) + (1-d)\mathbb{E}_E(V_M(z',h',\hat{\psi}))\right\} + \beta\lambda\mathbb{E}(V_U(\overline{z},\overline{h},\hat{\psi})) \quad (2)$$

where the policy function is denoted $d(z,h,\psi)$. The wage is absent from (2) because it is simply a transfer from the firm to the worker, leaving the value of the match unchanged. It can be shown that the solution to (2) is $d(z,h,\psi) = \delta$ if and only if $\mathbb{E}_U(V_U(z',h',\hat{\psi})) < \mathbb{E}_E(V_M(z',h',\hat{\psi}))$ and $d(z,h,\psi) = 1$ otherwise.

To close the model, there is free entry into vacancy posting in every submarket, so that the firm's benefit of vacancy creation in a non-empty submarket is equal to the cost:

$$k \geq \beta(1-\lambda)q(\theta(x,z,h,\psi))(\mathbb{E}_E(V_M(z',h',\hat{\psi})) - x(z,h,\psi)) \quad \text{and} \quad \theta(x,z,h,\psi) \geq 0 \quad (3)$$

with complementary slackness. Since the timing of the model is such that matches are made at the end of the period and production occurs at the beginning of the period, a firm offers lifetime utility x which is known one period before production takes place. Therefore the firm discounts the expected value of the match by $\beta(1-\lambda)$. The expectation is taken with respect to the conditional distribution of productivities while employed, which depend only on the current type of the worker that the firm commits to hire, and not on the entire distribution of workers across types. By Assumption 1, the firm's expected profits are increasing in the current productivity of a new hire even though production does not occur until after new productivities are drawn.

Looking at equation (3), it is clear that the present model is not equivalent to a model with one relative skill, say z/h, that increases during employment and decreases during unemployment. Suppose two unemployed workers have the same ratio z/h, one with high z and high h and one with low z and low h. The model defined by a relative skill implies that all workers with a given z/h will search in the same submarket. However, in general the value of the match is different for the high (z, h) worker and for the low (z, h) worker even though their relative skills are equal, since only one of the two skills is useful in market production. Constant vacancy costs imply that it cannot be the case that any firm will be indifferent between these two workers, violating free entry condition (3).

Since each firm posting a vacancy for value x commits to hire a single type, the firm knows for certain the type of worker that it will hire in any submarket. For any worker of a type different than (z, h), it is not optimal to search in submarket (x, z, h, ψ) since there is zero probability that she will be hired. Thus, the firm's decision to post a vacancy does not depend on the distribution of searching workers. Zero expected profits in equilibrium imply that firms are indifferent as to which submarket they post vacancies.

Following the literature, equilibria are restricted to those in which the market tightness satisfies complementary slackness condition (3) in every submarket. This implies that firms must be indifferent between posting vacancies in any submarket, whether or not it is active in equilibrium, so that market tightness is always pinned down by the free entry condition. I now turn to the definition of equilibrium.

Definition 1. *A block recursive equilibrium (BRE) consists of a market tightness function* $\theta : \mathbb{R} \times Z \times H \times Y \to \mathbb{R}_+$, *a value function for the unemployed worker* $V_U : Z \times H \times Y \to \mathbb{R}$, *a policy function for the unemployed worker* $x : Z \times H \times Y \to \mathbb{R}$, *a value function for the employed worker-firm match* $V_M : Z \times H \times Y \to \mathbb{R}$, *and a policy function for the match* $d : Z \times H \times Y \to [\delta, 1]$, *where:*

(i) $V_U(z, h, y)$ *satisfies (1)* $\forall (z, h, \psi) \in Z \times H \times \Psi$ *and* $x(z, h, y)$ *is the associated policy function.*

(ii) $V_M(z, h, y)$ *satisfies (2)* $\forall (z, h, \psi) \in Z \times H \times \Psi$ *and* $d(z, h, y)$ *is the associated policy function.*

(iii) $\theta(x, z, h, y)$ *satisfies (3)* $\forall (x, z, h, \psi) \in \mathbb{R} \times Z \times H \times \Psi$

In any BRE, agents' value and policy functions are independent of the distributions of workers across employment and unemployment as functions of their types. Given the market tightness function θ, Condition (i) ensures that unemployed workers' search strategies are optimal and condition (ii) ensures that employed worker-firm pairs' separation strategies are optimal. Condition (iii) states that the market tightness function θ is consistent with firms' incentives to create vacancies.

Given the infinite horizon programming problem faced by individuals in the decentralized economy, the analysis of equilibrium proceeds as follows. First, a lemma is stated showing that there exists a functional equation for all agents equivalent to solving equilibrium conditions (1), (2), and (3). Then, Theorem 1 shows that the functional equation from the lemma satisfies boundedness and continuity restrictions and therefore admits a unique solution. Further, by the recursive structure of the functional equation, the solutions of the problem are independent of the distributions (u, e) and satisfy Definition 1, therefore the unique decentralized equilibrium is a BRE. All proofs are left to Appendix B.

Lemma 1. *An equilibrium exists if and only if it solves the following problem:*

$$\begin{aligned} V(a,z,h,\psi) = a\Big(zy + \beta(1-\lambda) \max_{d\in[\delta,1]} [d\mathbb{E}_U(V(0,z',h',\hat{\psi})) + (1-d)\mathbb{E}_E(V(1,z',h',\hat{\psi}))]\Big) \\ + (1-a)\max_{\theta}\big\{h + \big[-k\theta + \beta(1-\lambda)\big((1-p(\theta))\mathbb{E}_U(V(0,z',h',\hat{\psi})) \\ + p(\theta)\mathbb{E}_E(V(1,z',h',\hat{\psi}))\big)\big]\big\} + \beta\lambda\mathbb{E}(V(0,\overline{z},\overline{h},\hat{\psi})) \end{aligned} \tag{4}$$

$$s.t. \quad \theta \in [0,\overline{\theta}], \quad \beta \in (0,1)$$

$$\text{where } V(0,z,h,\psi) \equiv V_U(z,h,\psi), \quad V(1,z,h,\psi) \equiv V_M(z,h,\psi)$$

the period payoff function, $azy + (1-a)(h-k\theta)$, *is bounded and continuous, and*

$$\mathbb{E}_U(V(0,z',h',\hat{\psi})) = \sum_{\hat{y}\in Y}\sum_{z'\in Z}\sum_{h'\in H} f(\hat{y}|y)f_U(z',h'|z,h)V_U(z',h',\hat{\psi})$$

$$\mathbb{E}_E(V(1,z',h,\hat{\psi})) = \sum_{\hat{y}\in Y}\sum_{z'\in Z}\sum_{h'\in H} f(\hat{y}|y)f_E(z',h'|z,h)V_M(z',h',\hat{\psi})$$

$$\mathbb{E}(V(0,\overline{z},\overline{h},\hat{\psi})) = \sum_{\hat{y}\in Y}\sum_{z'\in Z}\sum_{h'\in H} f(\hat{y}|y)f_0(z',h')V_U(z',h',\hat{\psi})$$

Theorem 1. *(i) All equilibria are block recursive. (ii) There exists a unique BRE.*

Part (i) of Theorem 1 comes from the assumptions of directed search and complete contracts. Given a fixed aggregate productivity y, if there are two submarkets committed to hire a worker of type (z, h), the worker faces a trade off between a higher probability of matching and a higher expected value of the match. The higher is the value offered in a submarket committed to (z, h), the more applicants of type (z, h) relative to vacancies it will attract, decreasing the probability for an individual worker to find a match. Since the firm commits to hire a certain type of worker, it knows which type of worker it will hire if the vacancy is filled. Therefore the firm's probability of matching will depend only on one worker type rather than the distribution of searching workers across productivities. This feature of directed search is not present in random search models, in which the firm's choice depends on its expectation of the type of worker it will meet, and thus the entire distribution of searching workers across types.

The existence of type-specific submarkets acts to complete the labor market in the sense that market tightness is specific to each productivity pair and therefore provides a "price" for each type. The contracting assumption along with firm commitment allows me to restrict attention to the value of a match and pin down the lifetime value to the employed worker, x, as a function of the market tightness and the match value. Due to the two-dimensional heterogeneity of workers, without the restriction of commitment it is possible that two types of workers will find it optimal to search in the same submarket, causing the block recursive property of the equilibrium to break down. In this case, equilibria will still exist, although they will not be explored here.

2.2 Planner's Problem

The planner's problem is to maximize aggregate consumption in the economy by choosing how to allocate workers and vacancies across submarkets. Specifically, in the search stage the planner chooses how many vacancies firms post in each submarket and in which submarkets unemployed workers search. The formulation of the planner's problem is shown in Appendix A. In the same Appendix, Theorem 2 states that the planner's problem has a unique solution which is monotone in the individual and aggregate productivities and independent of distributions (u, e). Denote the optimal market tightness and separation probabilities chosen by the planner for workers of type (z, h) when the aggregate productivity is y as $\theta^*(z, h, y)$ and $d^*(z, h, y)$, respectively.

Several elements of the model complicate the analysis of the planner's problem relative to those analyzed in the previous literature. First, in a model with one-dimensional heterogeneity the planner will find it optimal to send workers with different productivities to search in different locations, however with two dimensions this result is not obvious. Here, if the planner finds it optimal to assign the same market tightness to two types of unemployed workers, it is equivalent in terms of welfare to create two type-specific submarkets with the same tightness. Therefore, I assume that there is one submarket per type in each period.

Second, the planner's decisions in terms of market tightness and separation rates affect the endogenous distributions of worker types across employment states in a nontrivial way. Unlike models with *iid* draws of match-specific productivity, here the persistence of workers' productivities when transitioning between unemployment and employment causes the planner's choices to not only affect the level of employment, but also to dynamically affect the distribution of types across employment and unemployment. This distributional dependence interacts with the uncertainty about the aggregate productivity. However, as Theorem 2 shows, the planner's objective of maximizing aggregate consumption is equivalent to maximizing each worker type's consumption separately. Intuitively, the law of large numbers implies that the matching and separation probabilities exactly determine the endogenous distributions of worker types next period. Since aggregate consumption is the sum of consumption of each type, it is equivalent to maximize the sum of utilities jointly or maximize each element separately. This greatly simplifies the analysis, by allowing me to focus only on the simple problem that maximizes consumption type by type to show uniqueness, monotonicity, and independence of decision functions from the distributions of worker types.

2.3 Efficiency of the Decentralized Equilibrium

The following proposition states that the equilibrium described in Section 2.1 is efficient in the sense that the value and policy functions satisfying the BRE are identical to those that solve the planner's problem discussed in the previous section.

Proposition 1. *The unique BRE in the decentralized economy is efficient in the sense that* $\theta(x, z, h, y) = \theta^*(z, h, y)$ *and* $d(z, h, y) = d^*(z, h, y)$.

The unique decentralized equilibrium is efficient because of the presence of type-specific submarkets and the assumptions of complete contracts and firm commitment to types. As discussed regarding Theorem 1, the presence of submarkets in which only one worker type (z, h) searches forces firms to internalize the externalities that are typically present in other models. Since the planner values home productivity as much as workers in the decentralized economy, it is easy to show that the planner's value of unemployment, $W_U(z, h, y)$, satisfies (1). Without complete contracts, a firm and worker would not necessarily divide the surplus optimally, and the joint value function (2) would not be solved by the value of a match to the planner $W_E(z, h, y)$. However, when the surplus is maximized by restricting the contract space, it can be shown that the value of a match in the decentralized equilibrium and the value of an employed worker to the planner both solve (2).

3 Theoretical Results

This section discusses several theoretical results. First, the optimal separation probability d is equal to the lower bound, δ, whenever the optimal market tightness is strictly positive. The first order conditions of the social planner's problem give the following conditions:

$$p'(\theta^*(z, h, y))\beta(1 - \lambda)(\mathbb{E}_E W_E(z', h', \hat{y}) - \mathbb{E}_U W_U(z', h', \hat{y})) \leq k \tag{5}$$

with equality if $\theta^*(z, h, y) > 0$, and

$$\beta(1 - \lambda)(\mathbb{E}_E W_E(z', h', \hat{y}) - \mathbb{E}_U W_U(z', h', \hat{y})) \geq 0 \tag{6}$$

with equality if $d > \delta$. If a worker of type (z, h) chooses to search in a submarket in which he has a positive probability of matching, then it must be the case that $d = \delta$ since k and

$p'(\theta)$ are both strictly positive. This is because no worker of the same type who is currently in a match would like to separate, since after spending one period in unemployment she will optimally choose to search again for work in the market. Since search is costly to the planner in terms of aggregate consumption, it is optimal for a currently matched worker of type (z, h) to separate from his match with the lowest possible probability whenever unemployed workers of the same type find it optimal to search.

Second, the market tightness $\theta^*(z, h, y)$ is strictly decreasing in x. Appendix B shows that efficiency implies $W_E = V_M$ and $W_U = V_U$. Rearranging the free entry condition for a firm in a submarket with $\theta > 0$ gives:

$$\frac{k}{\beta(1-\lambda)(\mathbb{E}_E(W_E(z', h', \hat{y})) - x)} = q\big(\theta^*(z, h, y)\big)$$

Since W_E does not depend on x and q is strictly decreasing, $\theta^*(z, h, y)$ is strictly decreasing in x. Therefore the higher the lifetime utility offered to a worker, the higher the market tightness in that submarket since there are fewer vacancies and more applicants. By properties of the matching function, this implies that the probability of a worker meeting a vacancy is decreasing in the lifetime value of the match to the worker, the key trade off present in directed search models.

The final two results relate to the motivating facts discussed in the Introduction and in more detail in Section 4, in particular, the job finding probability, given by $p(\theta(z, h, y))$, and the unemployed worker's reemployment value, $x(z, h, y)$. I first show monotonicity of the optimal market tightness θ. An additional assumption simplifies the exposition:

Assumption 2. *Let the evolutions of productivities obey:*

$$h' = \begin{cases} \min\{h_{s+1}, h_{N_h}\} & \textit{with probability } \pi_{Uh} \textit{ if } U \textit{ and } h = h_s,\ s = 1, ...N_h \\ h & \textit{with probability } 1 - \pi_{Uh} \end{cases}$$

$$z' = \begin{cases} \min\{z_{s+1}, z_{N_z}\} & \textit{with probability } \pi_{Ez} \textit{ if } E \textit{ and } z = z_s,\ s = 1, ...N_z \\ z & \textit{with probability } 1 - \pi_{Ez} \end{cases}$$

$$z' = \begin{cases} \max\{z_{s-1}, z_1\} & \textit{with probability } \pi_{Uz} \textit{ if } U \textit{ and } z = z_s,\ s = 1, ...N_z \\ z & \textit{with probability } 1 - \pi_{Uz} \end{cases}$$

$$h' = \begin{cases} \max\{h_{s-1}, h_1\} & \textit{with probability } \pi_{Eh} \textit{ if } E \textit{ and } h = h_s,\ s = 1, ...N_h \\ h & \textit{with probability } 1 - \pi_{Eh} \end{cases}$$

$$\hat{y} = \begin{cases} y_i & \textit{with probability } \pi_{iy} \textit{ for } y_i \neq y,\ i = 1, ..., N_y \\ y & \textit{with probability } 1 - \sum_{y_i \neq y} \pi_{iy} \textit{ for } y_i = y \end{cases}$$

Proposition 2. *The planner's policy correspondence $\theta^*(z,h,y)$ is single-valued. Further, if Assumptions 1 and 2 hold, then $\theta^*(z,h,y)$ is strictly decreasing in h and strictly increasing in z and y when the probability that productivities change is small: for some $\epsilon(N_h)$, $\epsilon(N_z)$, and $\epsilon(N_y) > 0$.*

$$\pi_{Uh}, \pi_{Eh} < \epsilon(N_h) \quad \textit{and} \quad \pi_{Uz}, \pi_{Ez} < \epsilon(N_z) \quad \textit{and} \sum_{y_i \neq y} \pi_{iy} < \epsilon(N_y)$$

The precise definitions of $\epsilon(N_h)$, $\epsilon(N_z)$ and $\epsilon(N_y)$ can be found in Appendix B. It can easily be shown that when productivities are constant, the difference between the value functions in employment and unemployment is strictly increasing in z and y and strictly decreasing in h. However, when skills evolve the job finding probability depends on the difference between the expected values of employment and unemployment, both of which are conditional on current productivities. It is the opposing directions of skill change in the two employment states that complicates the proof. For skills with high enough persistence, the additional effect of any expected change is negligible.

Proposition 2 says that the more productive is a worker at home, the less likely she is to find market work. Conversely, when an unemployed worker's market productivity is high, she would like to maximize her probability of matching so that she may re-enter the market quickly, as this implies no further depreciation of her market skill, and an expected increase in skill if she matches. Therefore the planner finds it optimal to make workers with high market skills search for jobs with a higher matching probabilities, and workers with high home skills search for jobs with a lower matching probability. In addition, the higher is the aggregate productivity in the economy, the higher is the matching probability for all searching workers, implying a pro-cyclical aggregate job finding probability.

Workers with high z and low h choose a submarket with high tightness and a relatively low expected value of employment. Workers with low z and high h choose a low tightness submarket with a high expected value of employment. Since on average home skills appreciate while market skills depreciate when a worker is unemployed, the optimal market tightness for a searching worker will be increasing in the duration of unemployment. Thus, the longer an agent is unemployed, the higher is the lifetime utility required to make the planner indifferent between assigning him to home or market production. Since market productivity depreciates during unemployment, the expected marginal product in market work of an unemployed worker is decreasing over the spell. These two forces imply that as the unemployment duration increases, the probability that an individual worker finds a job decreases.

Lastly, it can be shown that under an additional assumption for the functional form of the matching probability, x is increasing in all three productivities. In particular:

Proposition 3. *If the job finding probability is isoelastic, that is,*

$$\gamma = \frac{\theta p'(\theta)}{p(\theta)} \quad \textit{where } \gamma \in (0,1) \textit{ is a constant,}$$

then the equilibrium lifetime value of a match $x(z, h, y)$ is strictly increasing in all three of its arguments.

An isoelastic matching probability arises with the standard Cobb-Douglas matching function. A value of $\gamma \in (0,1)$ satisfies the restriction that p is strictly increasing and strictly concave. Under this additional restriction on the functional form of p, the match value to the worker is a convex combination of the expected values of employment and unemployment. Monotonicity follows from the fact that both W_E and W_U are increasing, shown in Theorem 2. The characterization of piece rates α is not straightforward and is left to Section 5.

4 Empirical Evidence

This section summarizes the empirical evidence on the motivating facts discussed in the Introduction. Section 4.1 computes reemployment wages and job finding probabilities for the unemployed using micro data from the CPS and PSID and studies their responses to duration in a series of regressions. Section 4.2 describes evidence in support of learning in home production and contrary to an alternative mechanism, namely habit formation in leisure.

4.1 Duration Dependence in the CPS and PSID

In the CPS, one can identify those workers who report being unemployed and actively searching for a job in month t and who report being employed in month $t+1$. The sample includes all individuals making this transition for whom both weekly unemployment duration and hourly earnings are recorded between February 1994 and December 2015. In the PSID, monthly duration is reconstructed using the monthly employment history of the head of household available in each annual interview between 1984 and 1996. Transitions are identified as observations of individuals reporting being unemployed in at least one month of the year prior to the interview who have transitioned to employment by the time of the interview. The main samples are restricted to individuals aged 18 to 65 to reduce potential issues with education and retirement, and include workers with unemployment durations up to one year. Details about the data and robustness checks are contained in Appendix E.

In the baseline regressions, the effect of duration on the job finding probability is estimated using a linear probability model. The dependent variable of interest is a binomial variable equal to one if an unemployed worker transitioned from unemployment to employment and zero otherwise. The independent variables are unemployment duration and controls for observable heterogeneity across workers and time. Results are contained in Table 1, with robust standard errors reported in parentheses. The results indicate a strong negative correlation between duration and the job finding probability, for instance the coefficient in column (1) indicates a decline of 0.28 percentage points in the probability for each additional week spent unemployed.

Table 1: Linear Probability Model: Job Finding Probability on Unemployment Duration

	CPS		PSID	
	(1)	(2)	(3)	(4)
duration	-.0028***	-.0314***	-.5512***	-.4421***
	(.0001)	(.0014)	(.0377)	(.0713)
duration^2		.0018***	.1183***	.0998***
		(.0001)	(.0127)	(.0248)
duration^3		-.4.56e-05***	-.0109***	-.0010***
		(3.41e-06)	(.0016)	(.0032)
duration^4		3.97e-07***	.0004***	.0004***
		(3.30e-08)	(6.64 e-05)	(.0001)
R^2	.1146	.1232	.2721	.1925
N	147,736	147,736	10,773	10,803

Notes: CPS: January 1994-December 2015, monthly; duration reported in weeks. Universe: workers unemployed in at least one month of the CPS with reported duration up to 52 weeks, ages 18-65. PSID: 1984-1996, annual; duration reported in months. Universe: heads of household unemployed in at least one month of the PSID employment history with reported duration up to 12 months, ages 18-65. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and a quadratic term in total labor market experience. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

An obvious concern about the estimates in columns 1 and 2 in Table 1 is that there may be some unobserved heterogeneity across workers driving the results. For instance, the pool of workers with short unemployment durations may be very different from the pool of workers with long durations in a way that is unobservable to the econometrician. Since individuals participate in the CPS for only a short time[6], it is unlikely that they experience more than one unemployment spell. The longer panel structure of the PSID allows for the inclusion of these variables. Column 4 uses individual fixed effects to control for any potential unobservable heterogeneity that is fixed over time. The main insight of

[6]Respondents in the CPS are interviewed 8 times over a period of 16 months: interviews are conducted for 4 months consecutively, followed by 8 months of no interviews, and finally again for 4 consecutive months.

this regression supports those in the other columns of Table 1: even within individuals, duration has a strong negative effect on the probability of finding a job.

Results for regressions of the reemployment wage on duration are reported in Table 2. Reemployment wages are defined as real reported hourly wages in logs, deflated using the US city average CPI. Columns 1 and 2 report results using weekly duration reported in the CPS, and columns 3 and 4 report results using monthly duration in the PSID, without and with individual fixed effects, respectively. Column 1 shows that wages decline with duration, controlling for observable characteristics, however after controlling for long term unemployment, the effect of duration on wages disappears[7].

Table 2: Regression of log Reemployment Wage on log Duration

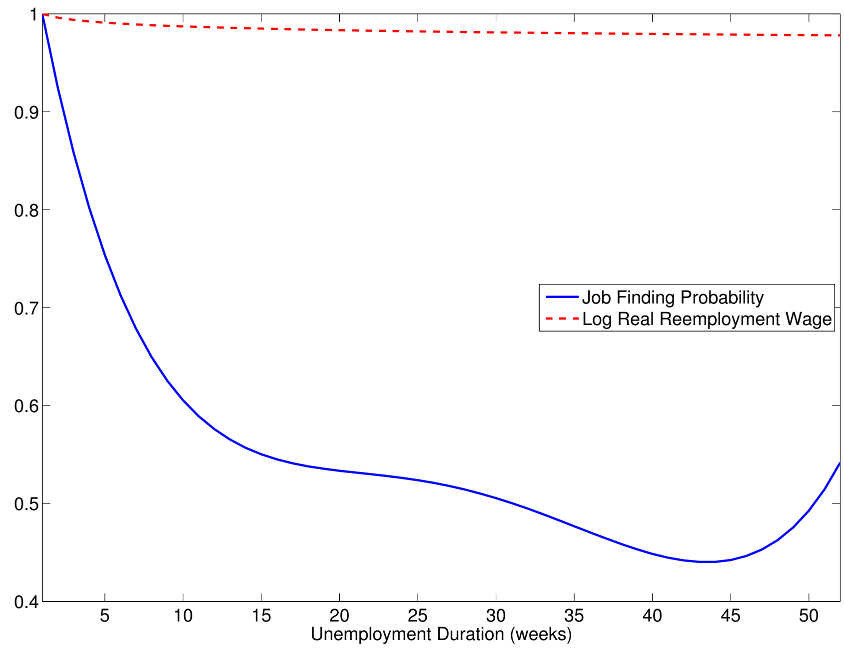
	CPS		PSID	
	(1)	(2)	(3)	(4)
log duration	-.0090***	-.0059	-.0055	-.0007
	(.0029)	(.0038)	(.0161)	(.0250)
dummy, > 6 mo	N	Y	Y	Y
FE	N	N	N	Y
R^2	.3917	.3918	.2200	.0112
Root MSE	.3375	.3375	.6391	
N	17,552	17,552	10,536	10,565

Notes: CPS Sample: January 1994-December 2015, monthly. Universe: respondents aged 18-65 who transitioned from U to E excluding those for whom the CPS allocated the hourly wage, with durations up to 52 weeks. PSID: 1984-1996, annual; duration reported in months. Universe: heads of household unemployed in at least one month of the PSID employment history with reported duration up to 12 months, ages 18-65. Controls for observables include the aggregate unemployment dummies for the interview year and month, the log of the aggregate unemployment rate, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and total labor market experience. Column 1 reports results for the regression of workers at all durations with no long term unemployment dummy in the CPS, column 2 is the same regression with the long term dummy. Column 3 is identical to column 2 using the PSID sample, and column 4 includes individual fixed effects in the PSID sample. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

To visually compare the effect of unemployment duration on the job finding probability and the wage, Figure 1 shows that in the CPS duration negatively affects both the probability of transitioning from unemployment to employment, plotted as a solid line, as well as the reemployment wages of those workers who do transition, plotted as a dashed line. Normalizing the wage and job finding probability to one at the shortest reported duration, the lines in the figure are the predicted values over duration using the estimates

[7]It is worth noting that the relatively small effect of duration on wages is not an artifact of all workers entering employment at the minimum wage. Of all workers reporting reemployment wages between the ages of 18 and 65 in the CPS sample, less than 10% report nominal hourly wages at or below the federal minimum wage.

Figure 1: Mean Job Finding Probability and Reemployment Wage by Duration



Notes: Predicted values of the mean job finding probability and log reemployment wage as functions of weekly reported unemployment duration, controlling for observables. Sample: CPS, 1994-2015, workers reporting unemployment and employment in two consecutive months, ages 18-65, with unemployment durations up to 1 year. Marginal effect of duration on the job finding probability is estimated in column (2) of Table E.13. Effect of log duration on log reemployment wage is estimated in column (1) of Table 2. Footnotes to Tables E.13 and 2 list control variables used in predictions.

in column 1 of Table 2 and the probit regression estimates in column 2 of Table E.13, setting all control variables used in the regressions to their population averages. The figure clearly shows that duration has a much stronger effect on the job finding rate than on wages.

4.2 Empirical Support for Evolving Outside Options

In addition to the facts documented above, incorporating a home-specific skill into this model has implications for agents' time use over the unemployment spell. If productivity changes with duration, one expects to see individual workers adjust their allocations of time spent in different activities over the course of an unemployment spell, as long as the income and substitution effects do not perfectly offset. The American Time Use Survey (ATUS) is used to explore this possibility in the data. The survey began in 2003 and respondents are a subset of recent CPS interviewees. As above, the respondents are restricted to those who complete the ATUS survey between 18 and 65 years old with unemployment durations up to 1 year. Unless otherwise stated, time use categories are defined as in Aguiar et al. (2013). Simple linear regressions of categories such as non-market work, core home production, and childcare on duration and controls show no effect of duration on time use at home. However, if one considers higher order specifications, duration affects individuals' allocations of time, both in core home production (mostly cooking and cleaning) as well as childcare activities. Results are shown in Tables 3 and E.24 in Appendix E. Both tables show that there is a significant effect of duration on time use in home production for the population (column 1), which is strongly driven by females (column 3). Column (2) suggests that there is little effect of duration on time use in home production for males.

Table 3: Regression: Minutes "core" home production plus childcare on duration

	(1)	(2)	(3)
duration	-8.243***	-5.021	-10.13**
	(2.905)	(3.520)	(4.551)
duration^2	.3837***	.2409	.4713**
	(.1226)	(.1491)	(.1930)
duration^3	-.0049***	-.0031*	-.0060**
	(.0015)	(.0018)	(.0024)
N	80,545	39,314	41,231
R^2	.1128	.0300	.0847

Notes: ATUS: January 2003-December 2013, monthly. Universe: respondents with no "unclassified" time use, ages 18-65, with imputed durations up to 52 weeks. Controls for observables include dummy variables for the year and month of the interview, race, age, gender (column 1 only), state of residence, education level, presence of an employed partner, and labor force status. Column 1 reports results for all workers and columns 2 and 3 report results for the subsamples of males and females, respectively. * denotes $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

An alternative explanation of the mechanism in this model is that the preferences for leisure activities are driving the results shown in the CPS and PSID. Results similar to those in this model would occur in a framework where workers develop a habit for leisure over the unemployment spell, implying that the outside option changes not through productivity but through preferences. Since for any specification of preferences, leisure has no income effect, in a model in which nonemployed workers allocate their time between home production and leisure, one would unambiguously expect an increase in leisure time with unemployment duration as the substitution effect dominates individuals' choices. The results of the ATUS indicate that the allocation of time in "home" activities changes over the unemployment spell, but the average time spent in leisure activities does not, as shown in Table E.25 in Appendix E.4. While the direction of the change in time spent doing home production depends on the specification of preferences, the data suggest that one can rule out the alternative explanation of a growing habit for leisure.

5 Quantitative Results

5.1 Calibration

The model is calibrated to match moments in the US data, with one period set to one month. The model is solved in steady state, normalizing the aggregate productivity y to one. Several parameters are chosen exogenously. First, the number of states that the market productivity may take is 7, and the number of states for home productivity is 10. Second, the state vectors for z and h are equally spaced, with $s_{i+1} - s_i = \Delta_s$, for $i = 1, \ldots N_s - 1,\ s = \{z, h\}$, with $\Delta_z = \Delta_h = .04$. The value of z_1 is normalized to 1.

The transition matrices for individual skills take the same form as Assumption 2, though without the restrictions on the probabilities necessary for the proof of Proposition 2. When a worker is employed and has market skill z, with probability π_{Ez} the worker will have skill $z' = \min\{z + \Delta_z, z_{N_z}\}$ next period, and with probability $1 - \pi_{Ez}$ the worker's skill does not change: $z' = z$. Similarly, the probability that an unemployed worker's home skill increases is π_{Uh}, and the probability that an employed worker's home skill falls and an unemployed worker's market skill falls are denoted π_{Eh} and π_{Uz}, respectively. Finally, the probability of death is chosen such that the expected lifetime of a worker is 40 years and the distribution from which newborn workers' skills are drawn, F_0, is equal to the stationary distribution of unemployed, u.

The parameters to be calibrated are summarized in Table 4 and the targets and model-implied values are shown in Table 5. The discount factor β implies an annual interest rate of 5%. Following Menzio and Shi (2011), the functional form for the probability that a worker matches with a firm is given by $p(\theta) = \min\{\theta^\gamma, 1\}$. The matching function parameter γ is set to the standard value 0.4. The remaining 7 parameters are calibrated jointly to minimize the distance between the model and the calibration targets.

The model is simulated in steady state for 1,000 workers over 500 periods, where initial productivities are drawn from the ergodic distribution u. For comparison with moments in the data, worker types with a job finding probability greater than 5% are considered to be actively searching, and comprise the pool of unemployed. Henceforth, this threshold will be referred to as the labor force cutoff. Several robustness checks for important parameters including the choice of this threshold are discussed in Appendix C.2. The lowest home skill, h_1, is chosen such that the expected value of an unemployed worker's home production in the steady state is equal to the estimate of the relative value of nonmarket to market activity by Hall and Milgrom (2008), $\frac{\mathbb{E}_U(h)}{\mathbb{E}_E(z)} = .71$. The parameters driving the accumulation and depreciation of market skills, π_{Ez} and π_{Uz}, are chosen to match the average wage increase after one year of employment and the lifetime earnings losses due to displacement, respectively. Wage increases are estimated by Kambourov and Manovskii (2009) as the regression coefficient representing the annual return to experience in terms of real wages for white male household heads in the PSID between 1981 and 1992.

Table 4: Parameters

Parameter	Value	Description
β	.9959	Discount factor
λ	.0021	Death probability
γ	.4	Job finding probability $p(\theta) = min\{\theta^{\gamma}, 1\}$
Δ_h	.04	Step in h: $\Delta_h = h_k - h_{k-1}$
Δ_z	.04	Step in z: $\Delta_z = z_j - z_{j-1}$
h_1	.83	Lowest home skill
z_1	1	Lowest market skill
π_{Ez}	.25	$z' = \min\{z + \Delta_z, z_{N_z}\}$ with prob π_{Ez} if E, z otherwise
π_{Eh}	.31	$h' = \max\{h - \Delta_h, h_1\}$ with prob π_{Eh} if E, h o.w.
π_{Uh}	.36	$h' = \min\{h + \Delta_h, h_{N_h}\}$ with prob π_{Uh} if U, h o.w.
π_{Uz}	.14	$z' = \max\{z - \Delta_z, z_1\}$ with prob π_{Uz} if U, z o.w.
k	3.65	Vacancy cost
δ	.023	Separation probability

Table 5: Targets

Description	Target	Model
Annual interest rate	5%	5%
Average working lifetime	40 years	40 years
Matching function elasticity w.r.t v	.4	.4
Relative value of nonmarket work	.71	.72
Change in earnings with 1 year of market experience	2.30%	5.56%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.39%
Lifetime earnings losses due to unemployment	−11.9%	−10.5%
Annual decline, hazard out of U	44.5%	44.8%
Quarterly average EU rate	.023	.023
Quarterly average UE rate	.328	.230

In the model, the annual implied wage increase from experience is equal to the average monthly increase in wages over the cross section of employed workers for each period of the model simulation, compounded to obtain the annual return. Average lifetime earnings losses are taken from estimates by Davis and von Wachter (2011) and represent the loss in the present value of earnings of workers with at least 3 years of tenure who experienced a mass layoff relative to a counterfactual had the layoff not occurred. In the calibration, earnings losses are computed as aggregate lifetime wages following a displacement relative to the counterfactual lifetime wages were the workers never to have entered unemployment for workers with at least 3 years of tenure.

The probabilities dictating the evolution of home skills, π_{Uh} and π_{Eh} are chosen to match two features of the job finding probability estimated using the CPS, controlling for observable characteristics: the percentage decline in the job finding probability between 1 and 12 months of unemployment duration and the average change in the job finding probability at one month of duration as a function of years of tenure in the previous job. Given the process for skill loss in unemployment predicted by the estimated earnings losses discussed above, the evolution of home skills for individuals in combination with compositional changes in the pool of unemployed determines the percentage decline in the aggregate job finding rate, estimated in the data in Section 4. The decline in the job finding probability in the model is the ratio of the average job finding probability in the cross section of unemployed workers, conditional on duration.

In the data, the change in the job finding probability over tenure is the estimated marginal effect of an additional year of pre-unemployment tenure on the job finding probability in the first month of unemployment, controlling for observables[8], for workers with up to 15 years of tenure. In the model, this moment corresponds to the average change in the job finding probability in the first period of unemployment with an additional year of pre-unemployment tenure, computed in each simulated month in the cross section of newly unemployed workers with tenure between 1 and 15 years.

The employment to unemployment and unemployment to employment (EU and UE, respectively) rates in the data are computed at quarterly frequency following Shimer (2005). The UE transition rate in the model is computed as the average number of individual transitions to employment in the simulations divided by the total number of unemployed over each 3-month span, and similarly for the EU rate. In the model, the UE rate is the fraction of workers with job finding probabilities above the active search threshold who enter employment in any period. The model's EU rate is equal to the involuntary separation probability δ, as voluntary separation indicates an optimal job finding probability of 0 by equations (5) and (6), and therefore corresponds to an employment to nonparticipation (EN) transition.

Table 6 summarizes some of the untargeted moments. All moments in the table are computed in the data over the period 1994-2015 to be consistent with the analysis in Section 4. Since wages are not pinned down in the model due to efficiency, one must take

[8]See Appendix C.1 for details.

Table 6: Untargeted Moments, Steady State

Description	Data	Model
% change, log reemployment wage (1-12 months)	-1.7%	-7.2%
Unemployment rate	6.0%	7.9%
Labor force participation rate	65.8%	88.3%
Initial job finding probability (1 month)	.402	.317
Percent long term unemployed	23.0%	16.0%
Percent long term unemployed	16.9%	16.0%

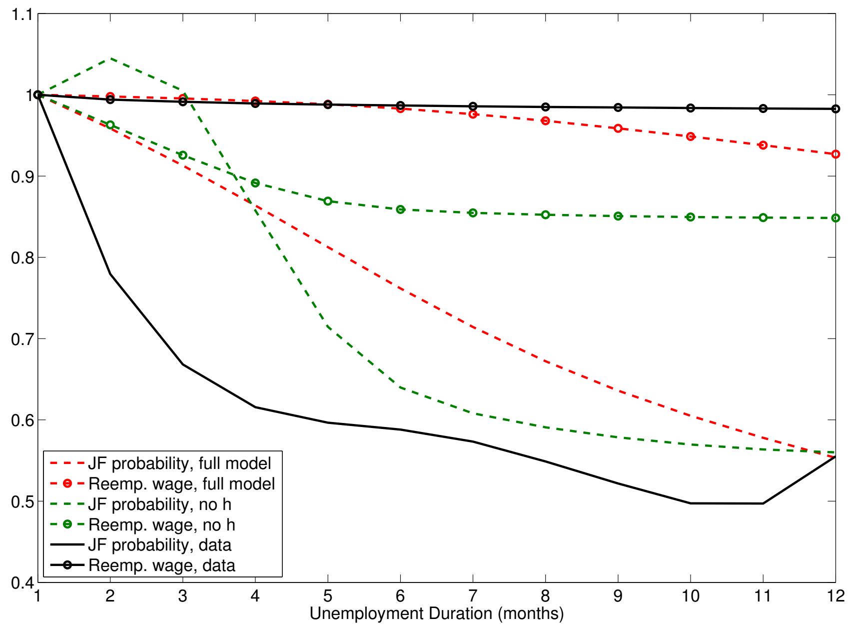
a stand on the form of equilibrium wages. Under the simple assumption that workers are paid a piece rate of the surplus α for the duration of the match, this piece rate is a function of the worker's type and the aggregate state at the time of the match, chosen to deliver the present discounted value of employment equal to the equilibrium value $x(z, h, y)$. For comparison to the log wages in the data, wages are scaled such that the average reemployment wage in the model simulations is equal to the average real hourly wage reported in the CPS. The predicted decline in the average log reemployment wage is 7.2% over the first year of unemployment, larger than the 1.7% drop estimated in Section 4. As is shown in Figure 2 below, the calibrated model in which the home skill for all unemployed workers is constant generates a wage that falls by over 15%, more than double the figure for the full model.

Although the model matches its percentage decline, the level of the job finding probability after 1 month of duration in the model is lower than in the data (31.9% compared to the estimate of 40.2% conditional on observables found in Section 4). The unemployment and labor force participation rates implied by the model are both above their empirical counterparts. Finally, the model prediction of the share of workers with unemployment spells over 6 months is slightly less than in the data over the full sample: in the model the share is about 16% whereas in the CPS the average share of long term unemployed between 1994 and 2015 is 23%. Leaving out the years since the recent recession, the average falls to 16.9%, in line with the model-implied value.

5.2 Steady State

Using the calibrated parameters described in the previous section, this section discusses the quantitative implications of the steady state model. Paths of the policy functions for individual worker types in steady state are shown in Figure 6. The aggregate job finding rate and reemployment wage are drawn as dashed red lines in Figure 2. The solid black lines correspond to the predicted data controlling for observables reproduced from Figure 1. In the model, the slope of the job finding rate is targeted, which is equivalent to targeting the value of the normalized job finding probability at 12 months. For comparison, the predictions of the analogous model with a fixed home skill are shown with green dotted lines. Calibrated parameters and model-implied moments for the fixed h model are shown in Tables C.12 and C.11. Though untargeted, the full model generates a relatively mild decline in the average reemployment wage, a large improvement over the model with no home skill accumulation.

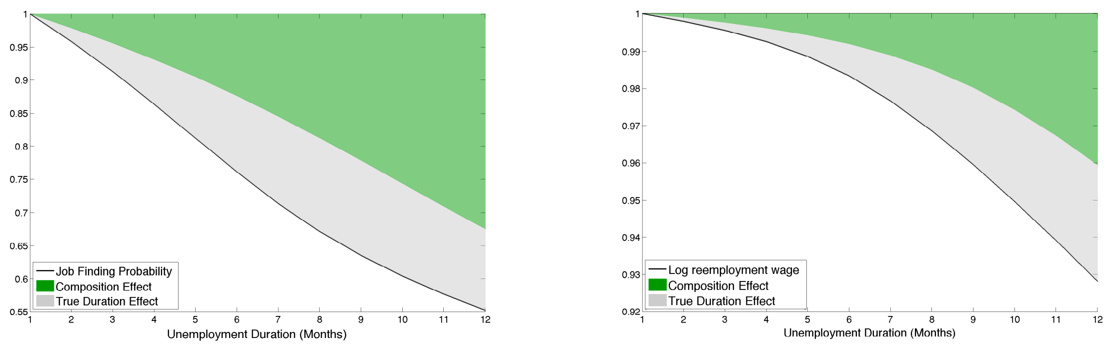
Figure 2: Normalized Job Finding Probability and Reemployment Wage



Notes: Model-implied values of the average job finding and log reemployment wages over unemployment duration, reported in months. Red and green dashed lines indicate values in the benchmark model and fixed h model respectively, and solid lines reproduce the predicted values from the data shown in Figure 1.

The decline in the job finding probability is decomposed into the true duration and composition effects, shown in the left panel of Figure 3. The composition effect is the relatively more important factor in explaining the drop in the hazard rate: accounting for about 60% of its total decline in the first year of unemployment. This result is roughly in line with recent findings by Alvarez et al. (2015) and Ahn and Hamilton (2015) who find that the majority of the decline is due to compositional changes. The composition effect in the model is computed as the average job finding probability holding the proportions of workers at each skill level constant at the initial values when entering unemployment. At short durations, the true duration and composition effects represent roughly equal proportions of the total decline in the job finding probability, but for longer durations, composition becomes relatively more important. This reflects the fact that those workers who remain unemployed for many months already had low job finding probabilities relative to the average worker upon entering unemployment. The true duration effect also grows over duration as workers' skills evolve in such a way that the average job finding probability falls. Results are similar for the average reemployment wage shown in the right panel of Figure 3, with the composition effect accounting for roughly half of the total decline in wages.

Figure 3: Decomposition: Job Finding Probability and Wage



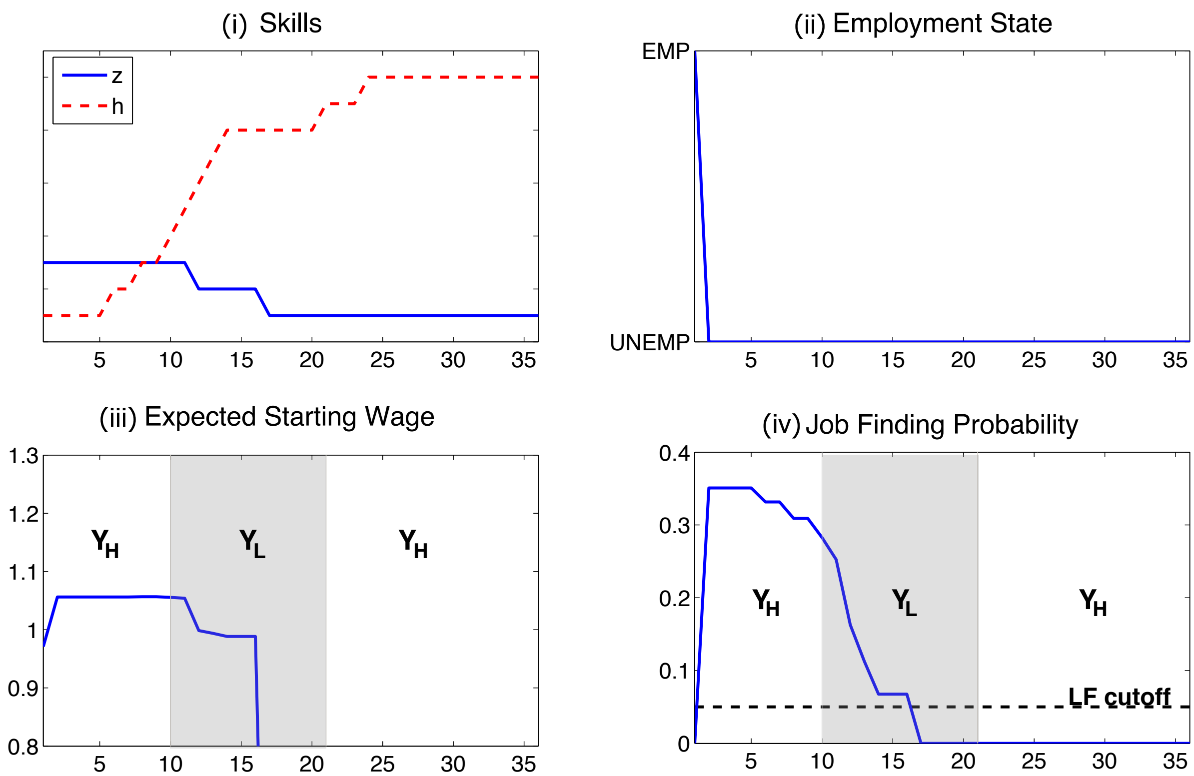
Notes: Left panel: decomposition of the model-implied average job finding probability. Right panel: decomposition of the average reemployment wage. The dark green shaded part of the figure indicates the proportion of the decline due to composition, and the light gray area indicates the remaining proportion of the decline, due to true duration dependence. Composition accounts for roughly 60% of the decline in the job finding probability and 53% of the decline in the wage over 12 months of unemployment duration.

5.3 Business Cycles

In this section aggregate uncertainty is incorporated into the calibrated model through the aggregate productivity y. In the data, aggregate productivity is chosen to match the seasonally adjusted real average output per worker in the nonfarm business sector constructed by the BLS. The process is discretized into a 5-state vector using the Rouwenhorst (1995) method, where the highest and lowest states correspond to two standard deviations above and below the mean, respectively. Normalizing the average aggregate productivity to one, the vector of aggregate productivities is given by $y = [0.9797 \quad 0.9899 \quad 1.0000 \quad 1.0101 \quad 1.0203]$.

For intuition on the effects of the recession on individual outcomes, Figure 4 shows a sample path of 36 months for one worker in response to a two-standard deviation decrease in aggregate productivity that lasts for 12 months, after which aggregate productivity permanently returns to its original level. Panel (ii) shows the worker's employment status: at the beginning of the sample the worker separates from his job and becomes unemployed. In panel (i), the worker's skills begin to react soon after his entry into unemployment, and are independent of the aggregate productivity by Assumption 1. When the recession arrives, this sample individual's expected reemployment wage falls in panel (iii), and his job finding probability falls in panel (iv). During the recession, the worker's home skills rise and market skills fall substantially, causing the job finding probability to fall below the labor force cutoff of 5%, after which the worker permanently drops out of the labor force.

Figure 4: Business Cycle: Sample Path of an Individual Worker

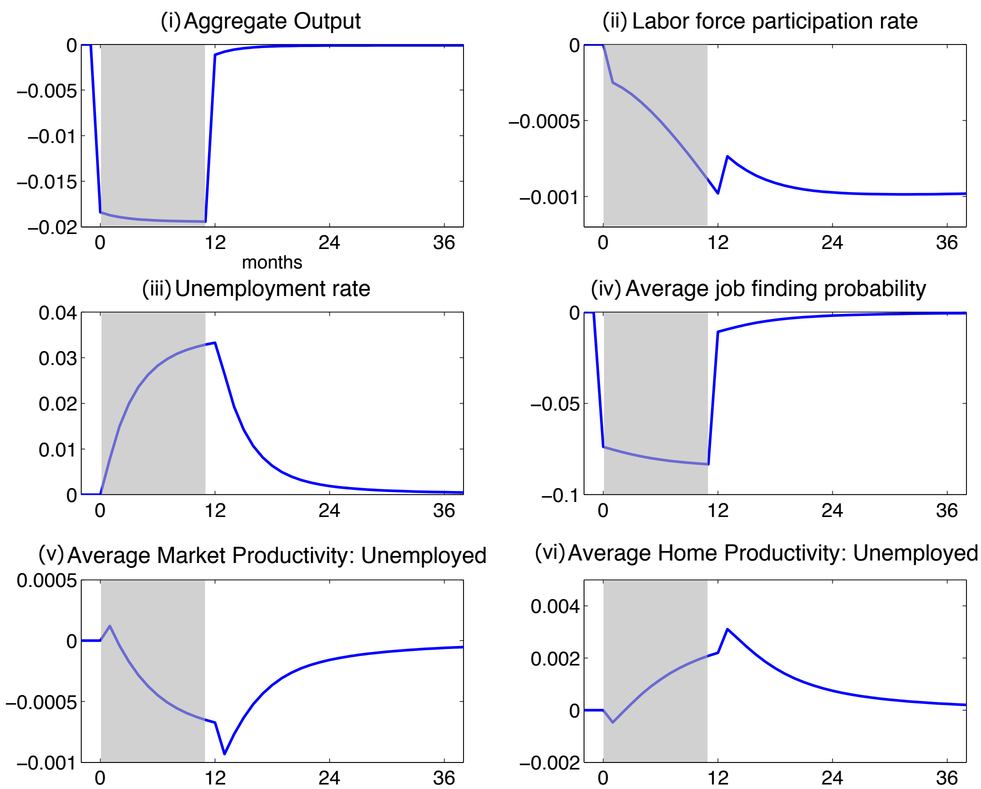


Notes: Paths of average skills, employment state, expected starting wage, and job finding probability for an individual worker over 3 years. y_H *and* y_L *denote normal times and times of aggregate productivity two standard deviations below its mean. The dashed line in the lower right panel denotes the* 5% *cutoff for the job finding probability, below which a worker is considered out of the labor force. For details on the choice of this cutoff, see Section C.2.*

In the aggregate, Figure 5 shows the responses of labor force participation, unemployment, the aggregate job finding probability, and average worker-specific productivities to the same two-standard deviation decrease in aggregate productivity. Responses are shown in percent deviations from steady state and are aggregated using the definition of the labor force introduced in the last section, that is, excluding unemployed workers below the labor force cutoff.

The 12-month negative aggregate shock is indicated in Figure 5 by the shaded region. Its effect on aggregate output is depicted in panel (i). The negative shock causes all unemployed workers' job finding probabilities to fall on impact because the relative value of market work falls, making home production more attractive for all workers. In panel

Figure 5: Responses to 12 Months of Low Aggregate Productivity



Notes: Responses of aggregate variables to a 12-month decline in aggregate productivity of 2 standard deviations below the mean. Responses measured in percentage deviations from the mean. Shaded region indicates periods of low aggregate productivity.

(ii), the participation rate decreases, reflecting the fact that some workers' optimal job finding probabilities fall below the cutoff used to define workers in the labor force, as seen in the individual sample path in Figure 4. However, this decline is more than offset by the increase in average durations for all workers above the cutoff, causing the unemployment rate to rise, shown in panel (iii). The initial decline in the average job finding probability in panel (iv) reflects the optimal policies of the unemployed workers.

The decline in the job finding probability implies longer expected unemployment durations for all workers, leading to the changes in the average worker-specific productivities of the unemployed shown in panels (v) and (vi). The impact effects in both panels reflect the effect of workers who drop out of the labor force when the recession begins. Since these workers are at the bottom (top) of the distribution of unemployed workers in terms of market (home) skills, their exit from the labor force causes the average market (home) skills to jump up (down) at the beginning of the recession. The jumps in both paths at the end of the recession reflects the re-entry to unemployment of some of these workers. During the intermediate months of the recession, average unemployment duration rises for the remaining unemployed workers, leaving more time for market (home) skills to depreciate (appreciate).

Regarding the dynamics after the recession, it is the changes in average market and home productivity that drives the hysteresis in labor force participation shown in panel (ii). Workers who choose to enter nonparticipation continue to evolve so that they do not return to the labor force even after the recession ends. The participation rate only returns to its mean as these workers are reborn with new productivities. The larger is the decline in aggregate productivity, the larger is the decrease in labor force participation and more persistent is its level.

6 Conclusion

This paper develops a model with learning by doing in home and market production in order to address two patterns seen in the data. The key mechanism is driven by the fact that during unemployment, workers learn in home production, affecting their outside options and job search strategies. The outside option of unemployed workers tends to increase over time as home-specific skills accumulate, giving rise to an average reemployment wage that is much less elastic than the job finding probability with respect to unemployment duration.

Quantitatively, when the model is calibrated to target the job finding probability it generates a reemployment wage that behaves similarly over duration to that in the data, a feature that is elusive in most models with a constant unemployment benefit and market-related skill loss. The model is then used to decompose the job finding probability into its true duration and compositional components, and implies a nontrivial role for true duration dependence due to individual skill changes over the unemployment spell. In order to understand the interaction between changes in the outside option and business cycles, aggregate shocks are added to the model. The model suggests that changes in workers' outside options during unemployment is an important force in generating the observed responses of aggregate labor market variables to shocks, and predicts a high degree of persistence in the labor force participation rate stemming directly from these changes.

References

Addison, J. T. and P. Portugal (1989). Job displacement, relative wage changes, and duration of unemployment. *Journal of Labor Economics*, 281–302.

Aguiar, M., E. Hurst, and L. Karabarbounis (2013). Time use during the great recession. *American Economic Review 103*(5).

Ahn, H. J. and J. Hamilton (2015). Heterogeneity and unemployment dynamics.

Albrecht, J. W., P.-A. Edin, M. Sundström, and S. B. Vroman (1999). Career interruptions and subsequent earnings: A reexamination using swedish data. *The Journal of Human Resources 34*(2), 294–311.

Alvarez, F., K. Borovičkovà, and R. Shimer (2015). Decomposing duration dependence in a stopping time model.

Arulampalam, W. (2001). Is unemployment really scarring? effects of unemployment experiences on wages. *The Economic Journal 111*(475), 585–606.

Benhabib, J., R. Rogerson, and R. Wright (1991). Homework in macro- economics: Household production and aggregate fluctuations. *Journal of Political Economy 99*, 1166–87.

Blanchard, O. J. and P. Diamond (1994). Ranking, unemployment duration, and wages. *The Review of Economic Studies 61*(3), 417–434.

Burdett, K., S. Shi, and R. Wright (2001). Pricing and Matching with Frictions. *Journal of Political Economy 109*, 1060–85.

Couch, K. A. and D. W. Placzek (2010). Earnings losses of displaced workers revisited. *The American Economic Review 100*(1), 572–589.

Davis, S. J. and T. von Wachter (2011). Recessions and the costs of job loss. *Brookings Papers on Economic Activity 43*(2), 1–72.

Doppelt, R. (2015). The hazards of unemployment.

Fallick, B. C. (1996). A review of the recent empirical literature on displaced workers. *Industrial & Labor Relations Review 50*(1), 5–16.

Fernández-Blanco, J. and E. Preugschat (2015). On the effects of ranking by unemployment duration.

Greenwood, J. and Z. Hercowitz (1991). The allocation of capital and time over the business cycle. *Journal of Political Economy 99*, 1188–1214.

Gregory, M. and R. Jukes (2001). Unemployment and subsequent earnings: Estimating scarring among british men 1984-94. *The Economic Journal 111*(475), 607–625.

Guvenen, F., B. Kuruscu, S. Tanaka, and D. Wiczer (2015). Multidimensional Skill Mismatch. (21376).

Hall, R. E. and P. R. Milgrom (2008). The limited influence of unemployment on the wage bargain. *American Economic Review 98*(4), 1653–74.

Jarosch, G. (2014). Searching for job security and the consequences of job loss.

Jarosch, G. and L. Pilossoph (2015). Statistical discrimination and duration dependence in the job finding rate.

Jung, P. and M. Kuhn (2012). Earnings losses and labor mobility over the lifecycle.

Kambourov, G. and I. Manovskii (2009). Occupational specificity of human capital. *International Economic Review 50*(1), 63–115.

Kletzer, L. G. (1998). Job displacement. *Journal of Economic Perspectives 12*(1), 115–136.

Krueger, A. B. and A. Mueller (2010). Job search and unemployment insurance: New evidence from time use data. *Journal of Public Economics 94*(3-4), 298–307.

Krueger, A. B. and A. I. Mueller (2016, February). A contribution to the empirics of reservation wages. *American Economic Journal: Economic Policy 8*(1), 142–79.

Lindenlaub, I. (2014). Sorting Multidimensional Types: Theory and Application.

Lise, J. and F. Postel-Vinay (2015). Multidimensional skills, sorting, and human capital accumulation.

Ljungqvist, L. and T. J. Sargent (1998). The European Unemployment Dilemma. *Journal of Political Economy 106*(3), 514–550.

Lucas, Robert E., J. (1988). On the Mechanics of Economic Development. *Journal of Monetary Economics 22*(1), 3–42.

Menzio, G. and S. Shi (2009). Efficient Search on the Job and the Business Cycle. (14905).

Menzio, G. and S. Shi (2011). Efficient Search on the Job and the Business Cycle. *Journal of Political Economy 119*(3), 468 – 510.

Moen, E. R. (1997). Competitive Search Equilibrium. *Journal of Political Economy 105*(2), 385–411.

Montgomery, J. D. (1991). Equilibrium Wage Dispersion and Interindustry Wage Differentials. *The Quarterly Journal of Economics 106*(1), 163–79.

Rouwenhorst, K. G. (1995). *Asset Pricing Implications of Equilibrium Business Cycle Models*. Princeton University Press.

Schmieder, J., T. Von Wachter, and S. Bender (2013). The causal effect of unemployment duration on wages: Evidence from unemployment insurance extensions.

Shi, S. (2009). Directed Search for Equilibrium Wage-Tenure Contracts. *Econometrica 77*(2), 561–584.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review 95*(1), 25–49.

Stokey, N., R. Lucas, and E. Prescott (1989). *Recursive Methods in Economic Dynamics*. Harvard University Press.

A Planner's Problem

At the beginning of each period, the planner observes aggregate state ψ and chooses θ in each submarket and d for each worker-firm pair. Given θ, aggregate consumption is given by

$$F(\theta|\psi) \equiv \sum_z \sum_h hu(z,h) + zye(z,h) - k\theta u(z,h) \tag{7}$$

The planner's problem is to solve:

$$W(\psi) = \sup_{\theta \in \mathbb{R}_+,\ d \in [\delta,1]} F(\theta|\psi) + \beta \mathbb{E} W(\hat{\psi}) \tag{8}$$

subject to the endogenous laws of motion for u and e, given by the following expressions:

$$\hat{u}(z',h') = \lambda f_0(z',h') + (1-\lambda) \sum_z \sum_h \left[f_U(z',h'|z,h)[(1-p(\theta(z,h,\psi)))u(z,h) + d(z,h,\psi)e(z,h)] \right] \tag{9}$$

$$\hat{e}(z',h') = (1-\lambda) \sum_z \sum_h \left[f_E(z',h'|z,h)[p(\theta(z,h,\psi))u(z,h) + (1-d(z,h,\psi))e(z,h)] \right] \tag{10}$$

where $\theta(z,h,\psi)$ is the market tightness for a given worker type when the aggregate state is ψ.

Equation (9) says that the distribution of unemployed workers of type (z',h') at the beginning of next period is given by a constant mass of newborn workers plus those surviving workers who were unemployed this period, did not match with a firm and drew productivities (z',h'), plus those employed workers who separated from their matches and drew new productivities (z',h'). Workers who match at the end of a period are counted as employed at the beginning of the following period. This timing assumption implies that new hires' market productivity is expected to increase before the first period of production in the match. Equation (10) is similar and gives the mass of surviving workers who will be employed at the beginning of next period with type (z',h'). The above formulation of the planner's problem leads to the following theorem.

Theorem 2. *(i) The following problem is equivalent to (8).*

$$\tilde{W}(\psi) = \sum_z \sum_h W_U(z,h,y)u(z,h) + W_E(z,h,y)e(z,h) \tag{11}$$

where

$$W_U(z,h,y) = \sup_{\theta\in\Re_+} \Bigg\{ h - k\theta + \beta(1-\lambda)\Big[(1-p(\theta))\mathbb{E}_U(W_U(z',h',\hat{y})) + p(\theta)\mathbb{E}_E(W_E(z',h',\hat{y}))\Big] + \beta\lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y}))\Bigg\}$$

$$W_E(z,h,y) = zy + \beta(1-\lambda)\max_{d\in[\delta,1]}\Bigg\{ d\mathbb{E}_U(W_U(z',h',\hat{y})) + (1-d)\mathbb{E}_E(W_E(z',h',\hat{y}))\Bigg\} + \beta\lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y}))$$

(ii) $\tilde{W}(\psi)$ is the unique solution to (11). (iii) W_U is strictly increasing in h and weakly increasing in z and y, and W_E is strictly increasing in z and y and weakly increasing in h if Assumption 1 holds. (iv) The policy correspondences θ^* and d^* associated with (11) depend on ψ only through y: $\theta^*(z,h,\psi) = \theta^*(z,h,y)$ and $d^*(z,h,\psi) = d^*(z,h,y)$.

The necessary assumption in part (*iii*) of Theorem 2 is intuitive. Monotonicity of the transition functions implies that the continuation values are nondecreasing in both productivity levels.

B Proofs

B.1 Proof of Lemma 1

($\Rightarrow$) Suppose there exists an equilibrium. Then $(V_U, V_M, \theta, x, e, d)$ satisfy (1), (2), and (3). If $\theta = 0$, x is not uniquely determined by the free entry condition, but the probability that a worker meets a vacancy in that submarket is zero, therefore following the literature[9] let $x = 0$ when $\theta = 0$.

If $\theta > 0$, solving (3) for x,

$$x = \mathbb{E}_E(V_M(z',h',\hat{\psi})) - \frac{k}{\beta(1-\lambda)q(\theta(x,z,h,\psi))}$$

[9] If this were not the case, then there would exist some inactive submarkets with a positive wage in which no matches would occur.

The combined value function for all agents in this equilibrium can be written as

$$V(a,z,h,\psi) = a\Big(zy + \beta(1-\lambda)[d\mathbb{E}_U(V(0,z',h',\hat{\psi})) + (1-d)\mathbb{E}_E(V(1,z',h',\hat{\psi}))]\Big)$$
$$+ (1-a)\Big(h + \beta(1-\lambda)\big[(1-p(\theta(x,z,h,\psi)))\mathbb{E}_U(V(0,z',h',\hat{\psi})) + p(\theta(x,z,h,\psi))x\big]\Big)$$
$$+ \beta\lambda\mathbb{E}(V(0,\overline{z},\overline{h},\hat{\psi}))$$

Plugging in for x from the free entry condition and noting that $\frac{p(\theta)}{q(\theta)} = \theta$:

$$V(a,z,h,\psi) = a\Big(zy + \beta(1-\lambda)[d\mathbb{E}_U(V(0,z',h',\hat{\psi})) + (1-d)\mathbb{E}_E(V(1,z',h',\hat{\psi}))]\Big)$$
$$+ (1-a)\Big(h + \big[-k\theta + \beta(1-\lambda)\Big((1-p(\theta))\mathbb{E}_U(V(0,z',h',\hat{\psi}))$$
$$+ p(\theta)\mathbb{E}_E(V(1,z',h',\hat{\psi}))\Big)\big]\Big) + \beta\lambda\mathbb{E}(V(0,\overline{z},\overline{h},\hat{\psi})) \quad (12)$$

Since x maximizes V_U and d maximizes V_M in the equilibrium, then the equilibrium θ must be the maximum of the above expression, giving us the value function in (4).

Since market tightness θ is bounded below by 0, it must be shown that $\exists\, \overline{\theta} < \infty$ such that $\theta \in [0, \overline{\theta}]\ \forall\ x, \psi$. Suppose not. By definition of the probability q, when $\theta \to \infty$, $q(\theta) \to 0$. By free entry,

$$k = \beta(1-\lambda)q(\theta(x,z,h,\psi))(\mathbb{E}_E(V_M(z',h',\hat{\psi})) - x)$$

since $k > 0$ and $\beta(E_E(V_M(z',h',\hat{\psi})) - x) \geq 0$, at $\theta = \infty$ the free entry condition is violated. Hence $\exists \overline{\theta} < \infty$ that solves

$$q(\overline{\theta}) = \frac{k}{\beta(1-\lambda)\mathbb{E}_E(V_M(z',h',\hat{\psi}))}$$

and the choice set for θ is $[0, \overline{\theta}]$.

In addition, $\beta < 1$ and the per-period payoff function $F(a,z,h,\psi) \equiv azy + (1-a)(h-k\theta)$ is continuous and bounded since all of its components are bounded. Therefore the equilibrium solves (4).

(⇐) Take any solution to (4). For $a = 0$,

$$V(0,z,h,\psi) = h + \max_{\theta}\big\{ -k\theta + \beta(1-\lambda)\big((1-p(\theta))\mathbb{E}_U(V(0,z',h',\hat{\psi})) + p(\theta)\mathbb{E}_E(V(1,z',h',\hat{\psi}))\big)\big\} + \beta\lambda\mathbb{E}(V(0,\overline{z},\overline{h},\hat{\psi}))$$

$$= h + \beta\max_{\theta}\big\{(1-p(\theta))\mathbb{E}_U(V_U(z',h',\hat{\psi})) + p(\theta)\big(\mathbb{E}_E(V_M(z',h',\hat{\psi})) - \frac{k}{\beta q(\theta)}\big)\big\} + \beta\lambda\mathbb{E}(V(0,\overline{z},\overline{h},\hat{\psi}))$$

if $x = (\mathbb{E}_E(V_M(z',h',\hat{\psi})) - \frac{k}{\beta q(\theta)})$, then

$$V_U(z,h,\psi) = h+\beta(1-\lambda)\sup_{x}\big\{(1-p(\theta))\mathbb{E}_U(V_U(z',h',\hat{\psi}))+p(\theta)x\big\}+\beta\lambda\mathbb{E}(V_U(\overline{z},\overline{h},\hat{\psi}))$$

Which satisfies (1) and (3) for $\theta > 0$.

Similarly, for $a = 1$,

$$V(1,z,h,\psi) = zy+\beta(1-\lambda)\max_{d\in[\delta,1]}\{d\mathbb{E}_U(V(0,z',h',\hat{\psi}))+(1-d)\mathbb{E}_E(V(1,z',h',\hat{\psi}))\} + \beta\lambda\mathbb{E}(V(0,\overline{z},\overline{h},\hat{\psi}))$$

Thus,

$$V_M(z,h,\psi) = zy + \beta(1-\lambda)\max_{d\in[\delta,1]}\{d\mathbb{E}_U(V_U(z',h',\hat{\psi})) + (1-d)\mathbb{E}_E(V_M(z',h',\hat{\psi}))\} + \beta\lambda\mathbb{E}(V_U(\overline{z},\overline{h},\hat{\psi}))$$

Which satisfies (2).

Finally, for $\theta = 0$, $p(0) = 0$, and by assumption $x = 0$, therefore $V(0,z,h,\psi)$ can be written as

$$V_U(z,h,\psi) = h + \beta\big((1-\lambda)\mathbb{E}_U(V_U(z',h',\hat{\psi})) + \lambda\mathbb{E}(V_U(\overline{z},\overline{h},\hat{\psi}))\big)$$

which is equivalent to $V_U(z,h,\psi)$ when $x = 0$. Thus any solution to (4) is an equilibrium.

B.2 Proof of Theorem 1

The following proof uses results from Stokey, Lucas, and Prescott (1989), Chapters 3 and 4, henceforth SLP.

Let (θ, V_U, V_M, x, d) be an equilibrium and let $V : [0,1] \times Z \times H \times \Psi$ be defined as

$$V(0, z, h, \psi) = V_U(z, h, \psi) \ \forall (z, h, \psi) \in Z \times H \times \Psi$$

$$V(1, z, h, \psi) = V_M(z, h, \psi) \ \forall (z, h, \psi) \in Z \times H \times \Psi$$

By Lemma 1, it remains to show that there exists a solution to the combined value function (4) and that it is unique. It is clear that the sets of feasible values for θ, and d are nonempty, compact, and continuous. The period utility function $F(a, z, h, \psi)$ is bounded and continuous, $\lambda \in [0,1]$ and $\beta \in (0,1)$. It immediately follows that a solution to (4) exists.

Let $\Omega = [0,1] \times Z \times H \times \Psi$ and $C(\Omega)$ be the space of continuous bounded functions $R : \omega \to \mathbb{R}$ for $\omega \in \Omega$ with the sup norm. Let $T : C(\Omega) \to C(\Omega)$ denote the operator associated with (4). It is easy to see that Blackwell's sufficient conditions for a contraction are satisfied by T, thus by Theorem 4.6 the mapping $TR = R$ admits a unique solution. Finally, note that since z, h, y, d, and θ are bounded, and $\mathbb{E}_U(V(0, z', h', \hat{\psi}))$, $\mathbb{E}(V(0, \overline{z}, \overline{h}, \hat{\psi}))$, and $\mathbb{E}_E(V(1, z', h', \hat{\psi}))$ are convex combinations of $V_U(z', h', \hat{\psi})$ and $V_M(z', h', \hat{\psi})$. Therefore $V(a, z, h, \psi)$ is bounded and by Theorem 4.3, the unique solution to $TR = R$ coincides with the solution to (4), therefore the equilibrium is unique.

I now show independence of the value and policy functions from (u, e). Note that if $R \in C(\Omega)$ depends on $\hat{\psi}$ only through $\hat{y}$ then TR depends on ψ only through y by the recursive structure of the operator T. Consider an arbitrary function $R \in C'(\Omega)$, denoting the set of continuous, bounded functions mapping $Z \times H \times Y \to \mathbb{R}$. It can easily be shown that $T : C'(\Omega) \to C'(\Omega)$, thus TR depends on ψ only through y. Therefore $V(a, z, h, \psi) = V(a, z, h, y)$ is the unique fixed point of (4).

Rewriting (3) by replacing ψ with y in V_M:

$$k \geq q(\theta(x, z, h, \psi))\beta(1-\lambda)(\mathbb{E}_E(V_M(z', h', \hat{y})) - x) \quad \text{and} \quad \theta(x, z, h, \psi) \geq 0 \quad (13)$$

Since q is strictly decreasing and convex, $\theta(x, z, h, \psi)$ is uniquely pinned down and therefore only depends on ψ through y.

Finally, rewriting (1) and (2) and replacing ψ with y:

$$V_U(z,h,y) = h + \beta(1-\lambda)\sup_x \Big\{(1-p(\theta(x,z,h,\psi)))\mathbb{E}_U(V_U(z',h',\hat{y})) + p(\theta(x,z,h,\psi))x\Big\} + \beta\lambda\mathbb{E}(V_U(\overline{z},\overline{h},\hat{y}))$$

$$V_M(z,h,y) = zy + \beta(1-\lambda)\max_{d\in[\delta,1]}\left\{d\mathbb{E}_U(V_U(z',h',\hat{y})) + (1-d)\mathbb{E}_E(V_M(z',h',\hat{y}))\right\} + \beta\lambda\mathbb{E}(V_U(\overline{z},\overline{h},\hat{y})) \quad (14)$$

Clearly both value functions depend on ψ only through y. Since the two value functions depend on $\hat{\psi}$ only through $\hat{y}$, the policy functions $x(z,h,\psi)$ and $d(z,h,\psi)$ depend on ψ only through y.

B.3 Proof of Theorem 2

(i) We want to show that the solutions to the following two problems are equivalent. Formulation 1 is given by:

$$W(\psi) = \sup_{\theta\in\mathbb{R}_+,\ d\in[\delta,1]}\left\{F(\theta|\psi) + \beta\mathbb{E}W(\hat{\psi})\right\} \quad (15)$$

where

$$F(\theta|\psi) \equiv \sum_z\sum_h hu(z,h) + zye(z,h) - k\theta u(z,h)$$

subject to the endogenous laws of motion for u and e, given by the following expressions:

$$\hat{u}(z',h') = \lambda f_0(z',h') + (1-\lambda)\sum_z\sum_h\Big[f_U(z',h'|z,h)[(1-p(\theta(z,h,\psi)))u(z,h) + d(z,h,\psi)e(z,h)]\Big] \quad (16)$$

$$\hat{e}(z',h') = (1-\lambda)\sum_z\sum_h\Big[f_E(z',h'|z,h)[p(\theta(z,h,\psi))u(z,h) + (1-d(z,h,\psi))e(z,h)]\Big]$$

Formulation 2 is given by:

$$\tilde{W}(\psi) = \sum_z\sum_h W_U(z,h,y)u(z,h) + W_E(z,h,y)e(z,h) \quad (17)$$

where

$$W_U(z,h,y) = h + \sup_{\theta\in\Re_+}\Big\{ -k\theta + \beta(1-\lambda)\big[(1-p(\theta))\mathbb{E}_U(W_U(z',h',\hat{y})|z,h,y)$$
$$+ p(\theta)\mathbb{E}_E(W_E(z',h',\hat{y})|z,h,y)\big]\Big\} + \beta\lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y})|y)$$

$$W_E(z,h,y) = zy + \beta(1-\lambda)\max_{d\in[\delta,1]}\Big\{ d\mathbb{E}_U(W_U(z',h',\hat{y})|z,h,y)$$
$$+ (1-d)\mathbb{E}_E(W_E(z',h',\hat{y})|z,h,y)\Big\} + \beta\lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y})|y)$$

Henceforth, I will write $\mathbb{E}_U(W_U(z',h',\hat{y})) = \mathbb{E}_U(W_U(z',h',\hat{y})|z,h,y)$, $\mathbb{E}_E(W_E(z',h',\hat{y})) = \mathbb{E}_E(W_E(z',h',\hat{y})|z,h,y)$, and $\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y})) = \mathbb{E}(W_U(\overline{z},\overline{h},\hat{y})|y)$ for notational convenience.

Denote the set of solutions to formulation 1 given by (15) as $\mathcal{A}_1$ and the set of solutions to formulation 2 given by (17) as $\mathcal{A}_2$.
($\Leftarrow$) Suppose $(\theta^*(z,h,y), d^*(z,h,y), W_U(z,h,y), W_E(z,h,y)) \in \mathcal{A}_2$. Then

$$\tilde{W}^*(\psi) = \sum_z\sum_h (h - k\theta^*(z,h,y))u(z,h) + zye(z,h)$$
$$+\beta(1-\lambda)\Big[(1-p(\theta^*(z,h,y)))\mathbb{E}_U(W_U(z',h',\hat{y}))+p(\theta^*(z,h,y))\mathbb{E}_E(W_E(z',h',\hat{y}))\Big]u(z,h)$$
$$+ \beta(1-\lambda)\Big[d^*(z,h,y)\mathbb{E}_U(W_U(z',h',\hat{y})) + (1-d^*(z,h,y))\mathbb{E}_E(W_E(z',h',\hat{y}))\Big]e(z,h)$$
$$+ \beta\lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y}))\big[u(z,h)+e(z,h)\big]$$

where

$$\mathbb{E}_U(W_U(z',h',\hat{y})) = \mathbb{E}\Big(\sum_{z'\in Z}\sum_{h'\in H} f_U(z',h'|z,h)W_U(z',h',\hat{y})\Big)$$
$$\mathbb{E}_E(W_E(z',h',\hat{y})) = \mathbb{E}\Big(\sum_{z'\in Z}\sum_{h'\in H} f_E(z',h'|z,h)W_E(z',h',\hat{y})\Big)$$

and

$$\mathbb{E}(W_U(z',h',\hat{y})) = \mathbb{E}\Big(\sum_{z'\in Z}\sum_{h'\in H} f_0(z',h')W_U(z',h',\hat{y})\Big).$$

We can rewrite $\tilde{W}^*(\psi)$ as

$$\begin{aligned}\tilde{W}^*(\psi) = \sum_z \sum_h (h - k\theta^*(z,h,y))u(z,h) + zye(z,h) \\ + \beta(1-\lambda)\Big[(1-p(\theta^*(z,h,y)))\mathbb{E}\big(\sum_{z'}\sum_{h'} f_U(z',h'|z,h)W_U(z',h',\hat{y})\big) \\ + p(\theta^*(z,h,y))\mathbb{E}\big(\sum_{z'}\sum_{h'} f_E(z',h'|z,h)W_E(z',h',\hat{y})\big)\Big]u(z,h) \\ + \beta(1-\lambda)\Big[d^*(z,h,y)\mathbb{E}\big(\sum_{z'}\sum_{h'} f_U(z',h'|z,h)W_U(z',h',\hat{y})\big) \\ + (1-d^*(z,h,y))\mathbb{E}\big(\sum_{z'}\sum_{h'} f_E(z',h'|z,h)W_E(z',h',\hat{y})\big)\Big]e(z,h) \\ + \beta\lambda\mathbb{E}\big(W_U(\overline{z},\overline{h},\hat{y})\big)\big[u(z,h)+e(z,h)\big]\end{aligned}$$

Let

$$F(\theta^*|\psi) = \sum_z \sum_h hu(z,h) + zye(z,h) - k\theta^*(z,h,y)u(z,h)$$

Rearranging the summations in the equation for $\tilde{W}^*(\psi)$, we have

$$\begin{aligned}\tilde{W}^*(\psi) = F(\theta^*|\psi) + \beta\lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y})) + \\ \sum_z \sum_h \sum_{z'} \sum_{h'} \Bigg(\beta(1-\lambda)\Big[(1-p(\theta^*(z,h,y)))\mathbb{E}\big(f_U(z',h'|z,h)W_U(z',h',\hat{y})\big) \\ + p(\theta^*(z,h,y))\mathbb{E}\big(f_E(z',h'|z,h)W_E(z',h',\hat{y})\big)\Big]u(z,h) \\ + \beta(1-\lambda)\Big[d^*(z,h,y)\mathbb{E}\big(f_U(z',h'|z,h)W_U(z',h',\hat{y})\big) \\ + (1-d^*(z,h,y))\mathbb{E}\big(f_E(z',h'|z,h)W_E(z',h'\hat{y})\big)\Big]e(z,h)\Bigg)\end{aligned}$$

using the definitions for the laws of motion of $\hat{u}$ and $\hat{e}$, we have

$$\begin{aligned}\tilde{W}^*(\psi) &= F(\theta^*|\psi) + \beta E\Bigg(\sum_{z'}\sum_{h'} \hat{u}(z',h')W_U(z',h',\hat{y}) + \hat{e}(z',h')W_E(z',h',\hat{y})\Bigg) \\ &= F(\theta^*|\psi) + \beta E\tilde{W}^*(\hat{\psi})\end{aligned}$$

Since $\theta^*(z,h,y)$ maximizes $W_U(z,h,y)$ and $d^*(z,h,y)$ maximizes $W_E(z,h,y)$, we cannot improve upon the welfare for any one type because each type's value function was maximized separately in (17). Further, we cannot increase the value of more than one type by transferring utility from one type to another without making the latter type worse off. Therefore the maximum of $\tilde{W}^*(\psi)$ must also be $(\theta^*(z,h,y), d^*(z,h,y))$ for each $(z,h) \in Z \times H$, that is, $\mathcal{A}_1 \subset \mathcal{A}_2$.

($\Rightarrow$) Suppose $(\theta^*(z,h,\psi), d^*(z,h,\psi), W(\psi)) \in \mathcal{A}_1$.

$$W(\psi) = \sum_z \sum_h \big[h - k\theta^*(z,h,\psi))\big] u(z,h) + zye(z,h) + \beta \mathbb{E} W(\hat{\psi})$$

where for each $z' \in Z$, $h' \in H$,

$$\hat{u}(z',h') = \lambda f_0(z',h') + (1-\lambda) \sum_z \sum_h \Big[f_U(z',h'|z,h)[(1-p(\theta(z,h,\psi)))u(z,h) + d(z,h,\psi)e(z,h)] \Big]$$

$$\hat{e}(z',h') = (1-\lambda) \sum_z \sum_h \Big[f_E(z',h'|z,h)[p(\theta^*(z,h,\psi))u(z,h) + (1-d^*(z,h,\psi))e(z,h)] \Big] \tag{18}$$

We can write

$$\tilde{W}(\psi) = \sum_z \sum_h W_U(z,h,\psi)u(z,h) + W_E(z,h,\psi)e(z,h)$$

where

$$\begin{aligned} W_U(z,h,\psi) = h - k\theta^*(z,h,\psi) + \beta(1-\lambda)\Big[&(1-p(\theta^*(z,h,\psi)))\mathbb{E}_U(W_U(z',h',\hat{\psi})) \\ &+ p(\theta^*(z,h,\psi))\mathbb{E}_E(W_E(z',h',\hat{\psi}))\Big] + \beta\lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{\psi})) \end{aligned}$$

$$\begin{aligned} W_E(z,h,y) = zy + \beta(1-\lambda)\Big(&d^*(z,h,\psi)\mathbb{E}_U(W_U(z',h',\hat{\psi})) \\ &+ (1-d^*(z,h,\psi))\mathbb{E}_E(W_E(z',h',\hat{\psi}))\Big) + \beta\lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{\psi})) \end{aligned}$$

and where

$$\begin{aligned} \mathbb{E}_U(W_U(z',h',\hat{\psi})) &\equiv \mathbb{E}_U(W_U(z',h',\hat{\psi})|z,h,\psi) \\ &= \mathbb{E}\Big(\sum_{z' \in Z} \sum_{h' \in H} f_U(z',h'|z,h) W_U(z',h',\hat{\psi})\Big) \end{aligned}$$

$$\mathbb{E}(W_U(\overline{z}, \overline{h}, \hat{\psi})) \equiv \mathbb{E}_U(W_U(\overline{z}, \overline{h}, \hat{\psi})|\psi)$$

$$= \mathbb{E}\bigg(\sum_{z' \in Z} \sum_{h' \in H} f_0(z', h') W_U(z', h', \hat{\psi})\bigg)$$

$$\mathbb{E}_E(W_E(z', h', \hat{\psi})) \equiv \mathbb{E}_E(W_E(z', h', \hat{\psi})|z, h, \psi)$$

$$= \mathbb{E}\bigg(\sum_{z' \in Z} \sum_{h' \in H} f_E(z', h'|z, h) W_E(z', h', \hat{\psi})\bigg)$$

Since $W(\psi)$ is additive in W_U and W_E, the sum of the element-by-element maximization is equal to the maximum of the sum. Under the additional constraint that $\theta^*(z, h, \psi) = \theta^*(z, h, y)$ and $d^*(z, h, \psi) = d^*(z, h, y)$, the solution to (15) is also a solution to (17). Therefore $\mathcal{A}_2 \subseteq \mathcal{A}_1$. We have shown $\mathcal{A}_1 \subseteq \mathcal{A}_2$ and $\mathcal{A}_2 \subseteq \mathcal{A}_1$; it follows that $\mathcal{A}_2 = \mathcal{A}_1$.

(ii) The optimality conditions for $W_u(z, h, y)$ imply that if $\theta^*(z, h, y) > 0$,

$$k = \beta(1-\lambda) p'(\theta^*(z, h, y)) \Big[\mathbb{E}_E(W_E(z', h', \hat{\psi})) - \mathbb{E}_U(W_U(z', h', \hat{\psi}))\Big]$$

Since $k > 0$ and $\mathbb{E}_E(W_E(z', h', \hat{\psi})) - \mathbb{E}_U(W_U(z', h', \hat{\psi})) > 0$ whenever $\theta > 0$, $p'(\theta) \to 0$ as $\theta \to \infty$ implies that $\exists \overline{\theta} < \infty$ such that the optimality condition holds. Therefore it is equivalent to write $W_U(z, h, y)$ as

$$W_U(z, h, y) = \max_{\theta \in [0, \overline{\theta}]} \bigg\{ h - k\theta + \beta(1-\lambda) \Big[(1 - p(\theta)) \mathbb{E}_U(W_U(z', h', \hat{y})) + p(\theta) \mathbb{E}_E(W_E(z', h', \hat{y}))\Big] + \beta\lambda \mathbb{E}(W_U(\overline{z}, \overline{h}, \hat{y})) \bigg\}$$

Therefore the constraint set for θ is nonempty, compact, and continuous. Since h and θ are bounded, current period utility $h - k\theta$ is bounded and continuous, $\lambda \in [0, 1]$, and $\beta \in (0, 1)$ implies that a solution to $W_U(z, h, y)$ exists by SLP Theorem 4.2. One can easily show that Blackwell's sufficient conditions for a contraction hold, therefore the operator associated with $W_U(z, h, y)$ has a unique solution in the space of continuous bounded functions on $Z \times H \times Y$. Similarly, the same results hold for $W_E(z, h, y)$.

Since all variables are bounded and $\mathbb{E}_U(W_U(z', h', \hat{y}))$, $\mathbb{E}(W_U(\overline{z}, \overline{h}, \hat{y}))$ and $\mathbb{E}_E(W_E(z', h', \hat{y}))$ are convex combinations of $W_U(z, h, y)$ and $W_E(z, h, y)$, respectively, the value functions are bounded, therefore by SLP Theorem 4.3, the unique solution to the operators associated with $W_U(z,h,y)$ and $W_E(z,h,y)$ coincide with the solutions to

$$W_U(z,h,y) = h + \max_{\theta\in[0,\bar{\theta}]}\Big\{ - k\theta + \beta(1-\lambda)\Big[(1-p(\theta))\mathbb{E}_U(W_U(z',h',\hat{y})) + p(\theta)\mathbb{E}_E(W_E(z',h',\hat{y}))\Big]\Big\} + \beta\lambda\mathbb{E}(W_U(\bar{z},\bar{h},\hat{y}))$$

and

$$W_E(z,h,y) = zy + \beta(1-\lambda)\max_{d\in[\delta,1]}\Big\{d\mathbb{E}_U(W_U(z',h',\hat{y})) + (1-d)\mathbb{E}_E(W_E(z',h',\hat{y}))\Big\} + \beta\lambda\mathbb{E}(W_U(\bar{z},\bar{h},\hat{y}))$$

Since (17) is a sum over W_U and W_E for each $(z,h)\in Z\times H$ and u and e are predetermined, the solution to (17) is unique.

(iii) Define the space of bounded, continuous functions that are nondecreasing in each of their arguments $B''(\Psi)\subset B(\Psi)$, and the space of bounded, continuous functions that are strictly increasing in h as $B''_z(\Psi)\subset B''(\Psi)\subset B(\Psi)$. Define the operator T as

$$(Tf_U)(z,h,y) = \max_{\theta\in[0,\bar{\theta}]}\Big\{h - k\theta + \beta(1-\lambda)\Big[(1-p(\theta))\mathbb{E}_U(f_U(z',h',\hat{y})) + p(\theta)\mathbb{E}_E(f_E(z',h',\hat{y}))\Big] + \beta\lambda\mathbb{E}(f_U(\bar{z},\bar{h},\hat{y}))\Big\}$$

for f_U and f_E nondecreasing functions in each of their arguments. Suppose $\theta^*(z,h,y)$ achieves the maximum of the above equation, and take any $\tilde{h}>h$. Then

$$(Tf_U)(z,h,y) = h - k\theta^*(z,h,y) + \beta(1-\lambda)\Big[(1-p(\theta^*(z,h,y)))\mathbb{E}_U(f_U(z',h',\hat{y})|z,h,y) + p(\theta^*(z,h,y))\mathbb{E}_E(f_E(z',h',\hat{y})|z,h,y)\Big] + \beta\lambda\mathbb{E}(f_U(\bar{z},\bar{h},\hat{y}))$$

$$
\begin{aligned}
< \;& \tilde{h} - k\theta^*(z,h,y) + \beta(1-\lambda)\Big[(1-p(\theta^*(z,h,y)))\mathbb{E}_U(f_U(z',h',\hat{y})|z,\tilde{h},y) \\
& + p(\theta^*(z,h,y))\mathbb{E}_E(f_E(z',h',\hat{y})|z,\tilde{h},y)\Big] + \beta\lambda\mathbb{E}(f_U(\overline{z},\overline{h},\hat{y})) \\
\leq \;& \max_{\theta\in[0,\overline{\theta}]}\Big\{\tilde{h} - k\theta + \beta(1-\lambda)\Big[(1-p(\theta))\mathbb{E}_U(f_U(z',h',\hat{y})|z,\tilde{h},y) \\
& + p(\theta)\mathbb{E}_E(f_E(z',h',\hat{y})|z,\tilde{h},y)\Big] + \beta\lambda\mathbb{E}(f_U(\overline{z},\overline{h},\hat{y}))\Big\} \\
= \;& (Tf_U)(z,\tilde{h},y)
\end{aligned}
$$

Where the inequality on the second line follows from the fact that Γ_U and Γ_E are monotone transition functions by Assumption 1. Since $B''(\Psi)$ is a closed subset of $B(\Psi)$ and $T(B''(\Psi)) \subset B''_z(\Psi)$, it follows from SLP Theorem 4.7 that $W_U \subset B''_z(\Psi)$. Similarly, it can be shown that $W_U(z,h,y)$ is weakly increasing in z and y, and $W_E(z,h,y)$ is strictly increasing in z and y and weakly increasing in h.

(iv) From part (ii), the policy correspondence $\theta^*(z,h,\psi)$ and $d(z,h,\psi)$ solve the maximization problems $W_U(z,h,y)$ and $W_E(z,h,y)$. Since neither the expression to be maximized nor the constraint depends on (u,e), $\theta^*(z,h,\psi)$ and $d(z,h,\psi)$ depend on ψ only through y and not on (u,e).

B.4 Proof of Proposition 1

This proof shows that the equilibrium allocation and efficient allocation coincide, that is, $\theta(x,z,h,y) = \theta^*(z,h,y)$. Recall the component functions of the planner:

$$
\begin{aligned}
W_U(z,h,y) = h + \max_{\theta\in\Re_+}\Big\{ & -k\theta + \beta(1-\lambda)\Big[(1-p(\theta))\mathbb{E}_U(W_U(z',h',\hat{y})) \\
& + p(\theta)\mathbb{E}_E(W_E(z',h',\hat{y}))\Big]\Big\} + \beta\lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y}))
\end{aligned}
$$

$$
\begin{aligned}
W_E(z,h,y) = zy + \beta(1-\lambda)\max_{d\in[\delta,1]}\Big\{ & d\mathbb{E}_U(W_U(z',h',\hat{y})) + (1-d)\mathbb{E}_E(W_E(z',h',\hat{y}))\Big\} \\
& + \beta\lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y}))
\end{aligned}
$$

Define $W'(0,z,h,y) = W_U(z,h,y)$ and $W'(1,z,h,y) = W_E(z,h,y)$. Then we can write the combined value function of the planner as

$$W'(a,z,h,y) = a\Bigg(zy + \beta(1-\lambda) \max_{d\in[\delta,1]} \big\{ d\mathbb{E}_U(W'(0,z',h',\hat{y})) + (1-d)\mathbb{E}_E(W'(1,z',h',\hat{y}))\big\}\Bigg) + (1-a)\Bigg(h + \max_{\theta\in\mathbb{R}_+} \big\{ -k\theta + \beta(1-\lambda)\big[(1-p(\theta))\mathbb{E}_U(W(0,z',h',\hat{y})) + p(\theta)\mathbb{E}_E(W(1,z',h',\hat{y}))\big]\big\}\Bigg) + \beta\lambda\mathbb{E}(W'(0,\overline{z},\overline{h},\hat{y})) \quad (19)$$

From (19) it is clear that $W'(a,z,h,y)$ satisfies (4). Since V is unique, it must be the case that

$$V_U(z',h',\hat{y}) = W_U(z',h',\hat{y})$$

$$V_M(z',h',\hat{y}) = W_E(z',h',\hat{y})$$

By definition, the allocation that solves (19) is the solution to the planner's problem, $(\theta^*(z,h,y), d^*(z,h,y))$, and the allocation that solves (4) corresponds to the decentralized equilibrium, $(\theta(x,z,h,y), e(h), d(z,h,y))$. Thus, $\theta(x,z,h,y) = \theta^*(z,h,y)$ and $d(z,h,y) = d^*(z,h,y)$.

B.5 Proof of Proposition 2

For any aggregate productivity level $y \in Y$, $\theta^*(z,h,y)$ solves

$$\max_{\theta\in[0,\overline{\theta}]} \Bigg\{ -k\theta + \beta(1-\lambda)\Bigg[(1-p(\theta))\mathbb{E}_U(W_U(z',h',\hat{y})) + p(\theta)\mathbb{E}_E(W_E(z',h',\hat{y}))\Bigg]\Bigg\} \quad (20)$$

The derivative with respect to θ is

$$-k + \beta(1-\lambda)p'(\theta^*(z,h,y))[\mathbb{E}_E(W_E(z',h',\hat{y})) - \mathbb{E}_U(W_U(z',h',\hat{y}))]$$

Since p is strictly increasing and strictly concave, p' is strictly decreasing. At $\theta = \overline{\theta}$, $p'(\overline{\theta}) = 0$ and the derivative is strictly negative. Therefore the unique solution to (20) is given by

$$\beta(1-\lambda)p'(\theta^*(z,h,y))\big(\mathbb{E}_E(W_E(z',h',\hat{y})) - \mathbb{E}_U(W_U(z',h',\hat{y}))\big) \leq k \text{ and } \theta^*(z,h,y) \geq 0$$

with complementary slackness. To show that $\theta^*(z,h,y)$ is strictly increasing in z and y and strictly decreasing in h, suppose $\theta^*(z,h,y)>0$. Then

$$\beta(1-\lambda)p'(\theta^*(z,h,y))[\mathbb{E}_E(W_E(z',h',\hat{y}))-\mathbb{E}_U(W_U(z',h',\hat{y}))]=k$$

Solving for $\theta^*(z,h,y)$:

$$p'(\theta^*(z,h,y))=\frac{k}{\beta(1-\lambda)[\mathbb{E}_E(W_E(z',h',\hat{y}))-\mathbb{E}_U(W_U(z',h',\hat{y}))]}$$

Where p' is strictly decreasing and continuous. Then $\theta^*(z,h,y)$ is increasing in z and y if and only if $\mathbb{E}_E(W_E(z',h',\hat{y}))-\mathbb{E}_U(W_U(z',h',\hat{y}))$ is increasing in z and y, and $\theta^*(z,h,y)$ is decreasing in h if and only if $\mathbb{E}_E(W_E(z',h',\hat{y}))-\mathbb{E}_U(W_U(z',h',\hat{y}))$ is decreasing in h when $\theta^*(z,h,y)$ is strictly positive.

By parts (ii) and (iii) of Theorem 2, $W_U(z,h,y)$ and $W_E(z,h,y)$ are bounded and continuous, $W_U(z,h,y)$ is strictly increasing in h and weakly increasing in z and y, and $W_E(z,h,y)$ is strictly increasing in z and y and weakly increasing in h.

I first show that the value of $\theta^*(z,h,y)$ is strictly increasing in z in the special case of fixed productivities. Then we show that under the specification for the laws of motion for skill changes in Assumption 2, the same monotonicity of $\theta^*(z,h,y)$ holds for a small probability of skill appreciation and depreciation. The parts of the proof showing that $\theta^*(z,h,y)$ is strictly increasing in y and strictly decreasing in h are omitted, but closely follow the argument below.

When productivities (z,h,y) are fixed, the planner's component value functions are given by

$$\begin{aligned}W_U(z,h,y)=h+\max_{\theta\in\Re_+}\Big\{-k\theta+\beta(1-\lambda)\Big[(1-p(\theta))W_U(z,h,y)+p(\theta)W_E(z,h,y)\Big]\Big\}\\+\beta\lambda W_U(\overline{z},\overline{h},y)\end{aligned}$$

$$W_E(z,h,y)=zy+\beta(1-\lambda)\max_{d\in[\delta,1]}\Big\{dW_U(z,h,y)+(1-d)W_E(z,h,y)\Big\}+\beta\lambda W_U(\overline{z},\overline{h},y)$$

Denoting $W_D(z,h,y)\equiv W_E(z,h,y)-W_U(z,h,y)$, and for arbitrary $\tilde{z},z\in Z$, $h\in H$, $y\in Y$ such that $\theta^*(z,h,y)>0$, $\theta^*(\tilde{z},h,y)>0$ and $\tilde{z}>z$,

$$\begin{aligned}W_D(\tilde{z},h,y)-W_D(z,h,y)&=(\tilde{z}-z)y+k\big(\theta^*(\tilde{z},h,y)-\theta^*(z,h,y)\big)\\&+\beta(1-\delta-p(\theta^*(\tilde{z},h,y)))W_D(\tilde{z},h,y)-\beta(1-\delta-p(\theta^*(z,h,y)))W_D(z,h,y)\end{aligned}$$

which is equivalent to the expression

$$\big(W_D(\tilde{z},h,y) - W_D(z,h,y)\big)(1-\beta(1-\delta-p(\theta^*(z,h,y)))) = (\tilde{z}-z)y$$
$$+\big(\theta^*(\tilde{z},h,y)-\theta^*(z,h,y)\big)\left(k-\beta\frac{p(\theta^*(\tilde{z},h,y))-p(\theta^*(z,h,y))}{\theta^*(\tilde{z},h,y)-\theta^*(z,h,y)}W_D(\tilde{z},h,y)\right)$$

Since $p(\cdot)$ is continuous on $[0,\overline{\theta}]$ and differentiable on $(0,\overline{\theta})$ and θ^* is continuous in each of its arguments, by the Mean Value Theorem there exists a point $\hat{z}\in(z,\tilde{z})$ such that

$$p'(\theta^*(\hat{z},h,y)) = \frac{p(\theta^*(\tilde{z},h,y))-p(\theta^*(z,h,y))}{\theta^*(\tilde{z},h,y)-\theta^*(z,h,y)}$$

For fixed z_1 and z_{N_z}, since z and $\tilde{z}$ were arbitrary, fix N_z large enough such that

$$-(\theta^*(\tilde{z},h,y)-\theta^*(z,h,y))\big(p'(\theta(\tilde{z},h,y))-p'(\theta(\hat{z},h,y))\big)W_D(\tilde{z},h,y) \geq (\tilde{z}-z)y$$

Using the planner's optimality conditions,

$$\begin{aligned}
&\big(W_D(\tilde{z},h,y) - W_D(z,h,y)\big)(1-\beta(1-\delta-p(\theta^*(z,h,y))))\\
=\ &(\tilde{z}-z)y+\beta(\theta^*(\tilde{z},h,y)-\theta^*(z,h,y))\big(p'(\theta(\tilde{z},h,y))-p'(\theta(\hat{z},h,y))\big)W_D(\tilde{z},h,y)\\
\geq\ &(1-\beta)(\tilde{z}-z)y
\end{aligned}$$

$$W_D(\tilde{z},h,y)-W_D(z,h,y) = \frac{(1-\beta)(\tilde{z}-z)y}{1-\beta(1-\delta-p(\theta^*(z,h,y)))} > 0$$

If the productivities evolve as specified in Assumption 2 and the productivities $z,\tilde{z}\in Z$, $h\in H$ and $y\in Y$ again satisfy the restrictions $\theta^*(z,h,y)>0$, $\theta^*(\tilde{z},h,y)>0$, and $\tilde{z}>z$, then we can again write the change $W_D(\tilde{z},h,y)-W_D(z,h,y)$ as

$$\big(W_D(\tilde{z},h,y) - W_D(z,h,y)\big)(1-\beta(1-\delta-p(\theta^*(z,h,y)))) = (\tilde{z}-z)y$$
$$+\big(\theta^*(\tilde{z},h,y)-\theta^*(z,h,y)\big)\left(k-\beta\frac{p(\theta^*(\tilde{z},h,y))-p(\theta^*(z,h,y))}{\theta^*(\tilde{z},h,y)-\theta^*(z,h,y)}W_D(\tilde{z},h,y)\right)$$
$$+\mathcal{G}(\pi_{iy},\pi_{Ez},\pi_{Eh},\pi_{Uz},\pi_{Uh})$$

As in the case with fixed productivities, fix N_z large enough such that

$$-(\theta^*(\tilde{z},h,y)-\theta^*(z,h,y))\big(p'(\theta(\tilde{z},h,y))-p'(\theta(\hat{z},h,y))\big)W_D(\tilde{z},h,y) \geq (\tilde{z}-z)y \quad (21)$$

Then

$$\big(W_D(\tilde{z},h,y) - W_D(z,h,y)\big)(1-\beta(1-\delta-p(\theta^*(z,h,y)))) > (1-\beta)(\tilde{z}-z)y \\ + \mathcal{G}(\pi_{iy},\pi_{Ez},\pi_{Eh},\pi_{Uz},\pi_{Uh})$$

Where the term $\mathcal{G}(\pi_{iy},\pi_{Ez},\pi_{Eh},\pi_{Uz},\pi_{Uh})$ is composed of all terms in which at least one productivity evolves, and which is omitted here for brevity.

Since the component value functions are bounded, $\exists\ B_E, B_U$ such that $B_i = \max_{(z,h,y)\in Z\times H\times Y}\{W_i(z,h,y)\},\ i = E, U$. It can be shown that $\mathcal{G}(\pi_{iy},\pi_{Ez},\pi_{Eh},\pi_{Uz},\pi_{Uh})$ is strictly greater than

$$\sum_{y_i\neq y}\pi_{iy}\Big[\pi_{Ez}\pi_{Eh}(-4B_E) + (1-\pi_{Ez})\pi_{Eh}(-4B_E) + \pi_{Ez}(1-\pi_{Eh})(-4B_E) \\ + \pi_{Uz}\pi_{Uh}(-4B_U) + (1-\pi_{Uz})\pi_{Uh}(-4B_U) + \pi_{Uz}(1-\pi_{Uh})(-4B_U)\Big]$$

If π_{Eh} and $\pi_{Uh} < \epsilon(N_h)$, π_{Ez} and $\pi_{Uz} < \epsilon(N_z)$ and $\sum_{y_i\neq y}\pi_{iy} < \epsilon(N_y)$ for some arbitrary fixed N_z and N_h, and N_z satisfying (21), then

$$\mathcal{G}(\pi_{iy},\pi_{Ez},\pi_{Eh},\pi_{Uz},\pi_{Uh}) > -4\epsilon(N_y)(B_E+B_U)(\epsilon(N_z)\epsilon(N_h)+\epsilon(N_h)+\epsilon(N_z))$$

Then

$$W_D(\tilde{z},h,y) - W_D(z,h,y) > \frac{(1-\beta)(\tilde{z}-z)y + \mathcal{G}(\pi_{iy},\pi_{Ez},\pi_{Eh},\pi_{Uz},\pi_{Uh})}{1-\beta(1-\delta-p(\theta^*(z,h,y)))}$$

Which is positive if and only if

$$(1-\beta)(\tilde{z}-z)y > -\mathcal{G}(\pi_{iy},\pi_{Ez},\pi_{Eh},\pi_{Uz},\pi_{Uh})$$

Since $\epsilon(N_h), \epsilon(N_z)$, and $\epsilon(N_y)$ were arbitrary, for any N_z satisfying (21) and for any N_h and N_y we can always find values of $\epsilon(\cdot)$ small enough such that

$$(1-\beta)(\tilde{z}-z)y > 4\epsilon(N_y)(B_E+B_U)(\epsilon(N_z)\epsilon(N_h)+\epsilon(N_h)+\epsilon(N_z))$$

which satisfies the above condition.

Since the optimal market tightness is a function of the expected values $\mathbb{E}_E(W_E(z',h',\hat{y})|z,h,y)$ and $\mathbb{E}_U(W_U(z',h',\hat{y})|z,h,y)$ it remains to show that $W_D(z,h,y) = W_E(z,h,y) - W_U(z,h,y)$ strictly increasing in z implies

$\hat{W}_D(z,h,y) = \mathbb{E}_E(W_E(z',h',\hat{y})|z,h,y) - \mathbb{E}_U(W_U(z',h',\hat{y})|z,h,y)$ is strictly increasing in z.

Rewriting $\hat{W}_D(z,h,y)$ as

$$\mathbb{E}_E\big(W_E(z',h',\hat{y}) - W_U(z',h',\hat{y})|z,h,y\big) + \mathbb{E}_E(W_U(z',h',\hat{y})|z,h,y) \\ - \mathbb{E}_U(W_U(z',h',\hat{y})|z,h,y)$$

The first term, $\mathbb{E}_E\big(W_E(z',h',\hat{y}) - W_U(z',h',\hat{y})|z,h,y\big)$ is increasing in z by monotonicity of the transition matrix Γ_{Ez} in Assumption 1. Therefore $\hat{W}_D(\tilde{z},h,y) - \hat{W}_D(z,h,y) > 0$ if

$$\mathbb{E}_E\big(W_E(z',h',\hat{y}) - W_U(z',h',\hat{y})|\tilde{z},h,y\big) - \mathbb{E}_E\big(W_E(z',h',\hat{y}) - W_U(z',h',\hat{y})|z,h,y\big) \\ > \mathbb{E}_U(W_U(z',h',\hat{y})|\tilde{z},h,y) - \mathbb{E}_E(W_U(z',h',\hat{y})|\tilde{z},h,y) \\ - \mathbb{E}_U(W_U(z',h',\hat{y})|z,h,y) + \mathbb{E}_E(W_U(z',h',\hat{y})|z,h,y) \quad (22)$$

Under Assumption 2, the left hand side of the expression can be shown to be strictly less than

$$8B_U\epsilon(N_y)(\epsilon(N_z)\epsilon(N_h) + \epsilon(N_h) + \epsilon(N_z))$$

Therefore Equation (22) holds if

$$\mathbb{E}_E\big(W_E(z',h',\hat{y}) - W_U(z',h',\hat{y})|\tilde{z},h,y\big) - \mathbb{E}_E\big(W_E(z',h',\hat{y}) - W_U(z',h',\hat{y})|z,h,y\big) \\ > 8B_U\epsilon(N_y)(\epsilon(N_z)\epsilon(N_h) + \epsilon(N_h) + \epsilon(N_z))$$

It is always possible choose a combination of ϵ small enough to ensure that W_D was increasing in z such that the above condition holds.

B.6 Proof of Proposition 3

If $\theta(z,h,y) > 0$, the free entry condition implies that the equilibrium value of employment to a searching worker of type (z,h,y) is given by

$$x(z,h,y) = \mathbb{E}_E(V_M(z',h',\hat{y})) - \frac{k}{\beta(1-\lambda)q(\theta(z,h,y))}$$

By Proposition 1, we know that $\mathbb{E}_E(V_M(z',h',\hat{y})) = \mathbb{E}_E(W_E(z',h',\hat{y}))$. If p is isoelastic, $\exists$ a constant γ such that

$$\gamma = \frac{\theta p'(\theta)}{p(\theta)}$$

where $\gamma \in (0,1)$ by the strict concavity of p. Further, by properties of the CRS matching function, $q(\theta) = \frac{p(\theta)}{\theta}$, and by isoelasticity, $q(\theta) = \frac{p'(\theta)}{\gamma}$.

Then

$$x(z,h,y) = \mathbb{E}_E(W_E(z',h',\hat{y})) - \frac{k\gamma}{\beta(1-\lambda)p'(\theta(z,h,y))}$$

Plugging in the expression for $p'(\theta(z,h,y))$ from the optimality conditions, we are left with

$$x(z,h,y) = (1-\gamma)\mathbb{E}_E(W_E(z',h',\hat{y})) + \gamma\mathbb{E}_U(W_U(z',h',\hat{y}))$$

By part (iii) of Theorem 2 and the monotonicity of transition matrices, $\mathbb{E}_E(W_E(z',h',\hat{y})) = \mathbb{E}_E(W_E(z',h',\hat{y})|z,h,y)$ is strictly increasing in z and y and weakly increasing in h, and $\mathbb{E}_U(W_U(z',h',\hat{y})) = \mathbb{E}_U(W_U(z',h',\hat{y})|z,h,y)$ is strictly increasing in h and weakly increasing in z and y. Since $\gamma \in (0,1)$ it follows that $x(z,h,y)$ is strictly increasing in each of its arguments.

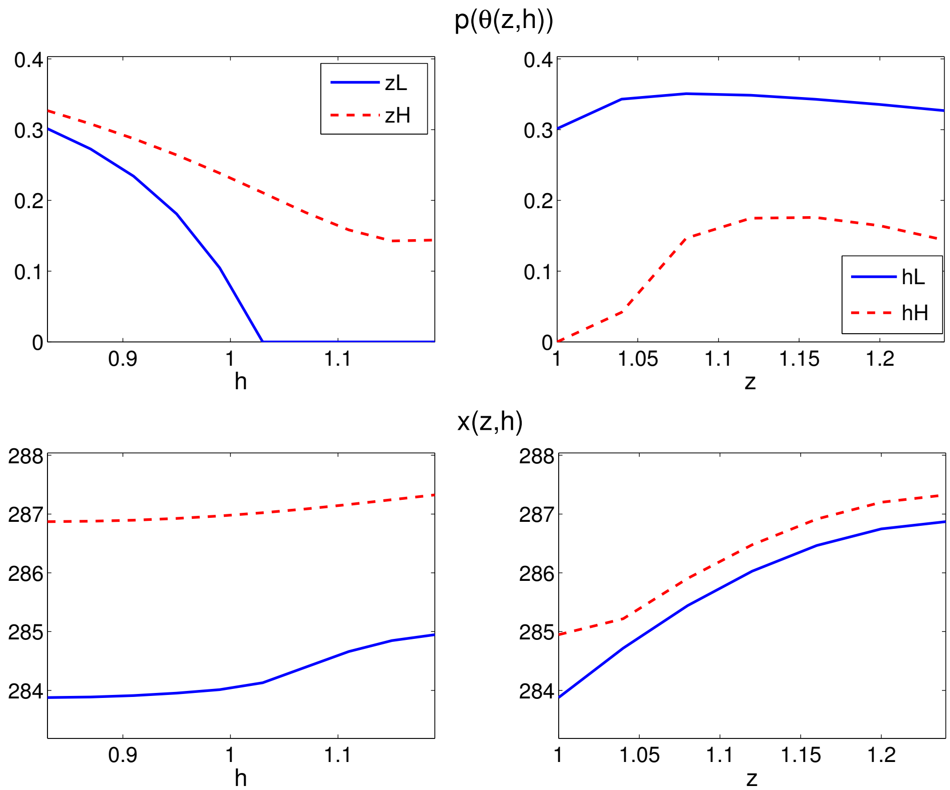
C Calibration: Details and Robustness

C.1 Details and Figures

To obtain the estimate of the job finding probability over tenure in the previous job, I identify workers who transitioned from unemployment to employment over two consecutive months of CPS interviews, and who also participated in the Displaced Worker Survey (DWS) the month they reported being unemployed. The DWS is conducted biannually; the sample used here covers the period from 1994 to 2010. According to the BLS, the DWS includes all CPS respondents who are "20 years of age and older who lost or left jobs because their plant or company closed or moved, there was insufficient work for them to do, or their position or shift was abolished." There are 5,634 workers who appeared in two consecutive months of the CPS and participated in the DWS in the first month in the 9 DWS interviews between 1994 and 2010. Of these, 898 transitioned to employment in the second month, or about 16% of the DWS participants considered here.

In order to identify the depreciation of home skills during employment, I consider only those workers with unemployment spells shorter than one month at the time of the DWS. In the model, these short duration workers are those whose productivities most resemble their productivities at the time of separation. I regress a dummy variable equal to 1 if the worker transitioned from U to E and 0 if they reported being unemployed in both months on months of tenure in the previous job reported

Figure 6: Steady State: Optimal Job Finding Probability and Implied Value for Unemployed Worker



Notes: Paths of individual job finding probability $p(\theta)$ and implied lifetime value of a match x in steady state holding z (first column) or h (second column) constant.

in the DWS for workers with up to 15 years of tenure, and controls for year, month, the log of the unemployment rate, gender, race, age, education, marital status, occupation, industry, and a quadratic term in total labor market experience. The estimated marginal effect of 1 additional month of pre-unemployment tenure is .03%, which is equivalent to an increase in the hazard out of unemployment of .41% for each additional year of tenure.

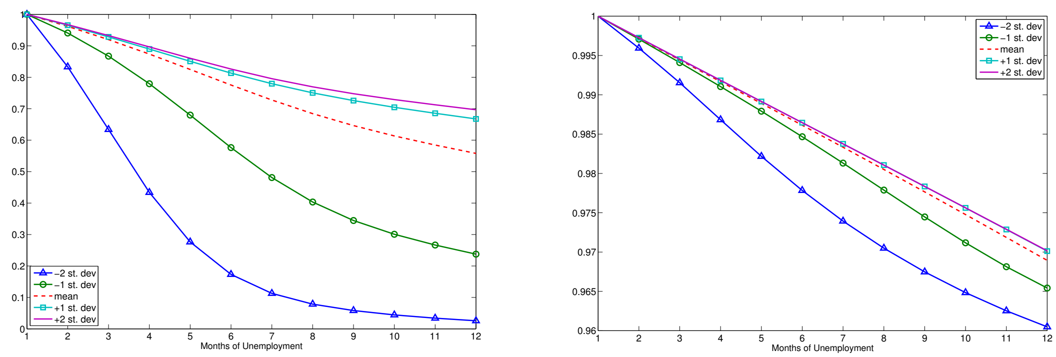
The rest of this section contains additional figures relating to the model in steady state varying the (fixed) aggregate productivities.

C.2 Robustness

This section performs several alternative calibrations of the steady state model and shows their cyclical properties in order to evaluate the robustness of the model's mechanism. There are three main parameters of interest which may alter the predictions of the model, namely, the probability of death λ, the active search cutoff for

Figure 7: Steady State: Effects of Aggregate Productivity

(a) Job Finding Probability (b) Reemployment wage



Notes: Paths of average job finding probability (a) and reemployment wage (b) in steady states defined by varying levels of the aggregate productivity.

workers in the labor force, and the elasticity of the matching function with respect to vacancies, γ. The first robustness check is to increase the probability of death, corresponding to an expected lifetime of 30 years, rather than the 40 year expected lifetime assumed in the benchmark. The second decreases the active search cutoff for unemployed workers to be in the labor force to match the 99th percentile of durations reported by the unemployed in the CPS. The final two robustness checks vary the elasticity of the matching function, first decreasing its value to match the value estimated in the CPS from 1951-2003 by Shimer (2005) of .28, and then to study the implications of increasing its value to .5.

This section concludes with a version of the model comparable to Ljungqvist and Sargent (1998), in which the distribution of home skills is degenerate. In order to match the targets in the calibration, this model implies a strong decline in the reemployment wage due to a constant, rather than an improving, outside option with unemployment duration.

C.2.1 Changes in the Death Probability

The calibration of the model with a higher probability of death implies a lower value of h_1 and π_{Uh} and a higher value of π_{Uz}, shown in Table C.1. An increase in the probability of death decreases the effective discount factor, making agents more impatient since future payoffs are less likely to be realized. Since there is a cost to search, k, the relative attractiveness of unemployment increases, making the matches less likely. To amend this change, the value of home production must fall, implying a decrease in the vector of home skills, a decrease in the probability of accumulating home skills, and an increase in the probability of losing market skills when nonemployed. These changes make the tradeoff faced during unemployment less attractive, causing more workers to search. Targeted and untargeted parameters are shown in Tables C.2 and C.3.

Table C.1: Parameters: Higher Death Probability

Parameter	Value	Description
β	.9959	Discount factor
λ	.0027	Death probability
γ	.4	Job finding probability $p(\theta) = min\{\theta^\alpha, 1\}$
Δ_h	.04	Step in h: $\Delta_h = h_k - h_{k-1}$
Δ_z	.04	Step in z: $\Delta_z = z_j - z_{j-1}$
h_1	.829	Lowest home skill
z_1	1	Lowest market skill
π_{Ez}	.25	$z' = \min\{z + \Delta_z, z_{N_z}\}$ with prob π_{Ez} if E, z otherwise
π_{Eh}	.31	$h' = \max\{h - \Delta_h, h_1\}$ with prob π_{Eh} if E, h o.w.
π_{Uh}	.33	$h' = \min\{h + \Delta_h, h_{N_h}\}$ with prob π_{Uh} if U, h o.w.
π_{Uz}	.19	$z' = \max\{z - \Delta_z, z_1\}$ with prob π_{Uz} if U, z o.w.
k	3.65	Vacancy cost
δ	.023	Separation probability

C.2.2 Changes in the Active Search Cutoff

This section considers different cutoffs to define workers who are in the labor force. Table C.4 shows selected percentiles of unemployment duration in the empirical distribution using the CPS and PSID samples described in Sections 4 and E. In the calibration presented in the paper, a 5% cutoff for the job finding probability was used, below which unemployed workers are considered out of the labor force. This corresponds to an expected duration of 20 months, which is equivalent to roughly 86 weeks[10], corresponding to the 93rd percentile in the CPS sample, but well beyond the 99th percentile of the PSID sample. Due to possible measurement error in the PSID sample from the annual nature of the data, the remainder of this section considers only the CPS.

[10]Following convention in the literature, one month is equal to roughly 4.3 weeks.

Table C.2: Targets: Higher Death Probability

Description	Target	Model
Annual interest rate	5%	5%
Matching function elasticity w.r.t v	.4	.4
Relative value of nonmarket work	.71	.715
Change in earnings with 1 year of market experience	2.30%	5.29%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.61%
Lifetime earnings losses due to unemployment	−11.9%	−8.33%
Annual decline, hazard out of U	44.5%	47.1%
Quarterly average EU rate	.023	.023
Quarterly average UE rate	.328	.231

Table C.3: Untargeted Moments: Higher Death Probability

Description	Data	Model
% change, log reemployment wage (1-12 months)	−2.2%	−4.1%
Unemployment rate	6.0%	7.7%
Labor force participation rate	65.8%	92.3%
Initial job finding probability (1 month)	.402	.326
Percent long term unemployed	23.0%	15.2%
Percent long term unemployed	16.9%	15.2%

Since there is a nonnegligible mass of unemployed workers with durations above the 20 month threshold, it is reasonable to decrease the minimum job finding probability to understand how this changes the results. For robustness, I use the 99th percentile from the empirical distribution of the CPS, recalibrating the model in steady state and then simulating the responses to aggregate productivity shocks.

A monthly job finding probability of 3.6% represents the cutoff corresponding to the 99th percentile of the CPS duration distribution. Tables C.5 and C.6 show the calibrated parameters and targeted moments of the model using this cutoff for

Table C.4: Deciles of Unemployment Duration

Percentile	CPS	PSID
50	12 weeks	2 months
60	18 weeks	3 months
70	27 weeks	3 months
80	44 weeks	4 months
90	61 weeks	6 months
95	104 weeks	9 months
98	113 weeks	12 months
99	119 weeks	13 months

Notes: CPS: January 1994-December 2015, monthly; duration reported in weeks. PSID: 1984-1996, annual; duration reported in months.

labor force participation.

Table C.5: Parameters: 3.6% Labor Force Cutoff

Parameter	Value	Description
β	.9959	Discount factor
λ	.0021	Death probability
γ	.4	Job finding probability $p(\theta) = min\{\theta^\alpha, 1\}$
Δ_h	.04	Step in h: $\Delta_h = h_k - h_{k-1}$
Δ_z	.04	Step in z: $\Delta_z = z_j - z_{j-1}$
h_1	.835	Lowest home skill
z_1	1	Lowest market skill
π_{Ez}	.25	$z' = \min\{z + \Delta_z, z_{N_z}\}$ with prob π_{Ez} if E, z o.w.
π_{Eh}	.31	$h' = \max\{h - \Delta_h, h_1\}$ with prob π_{Eh} if E, h o.w.
π_{Uh}	.35	$h' = \min\{h + \Delta_h, h_{N_h}\}$ with prob π_{Uh} if U, h o.w.
π_{Uz}	.2	$z' = \max\{z - \Delta_z, z_1\}$ with prob π_{Uz} if U, z o.w.
k	2.5	Vacancy cost
δ	.023	Separation probability

Table C.6: Targets: 3.6% Labor Force Cutoff

Description	Target	Model
Annual interest rate	5%	5%
Matching function elasticity w.r.t v	.4	.4
Relative value of nonmarket work	.71	.714
Change in earnings with 1 year of market experience	2.30%	5.50%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.53%
Lifetime earnings losses due to unemployment	−11.9%	−9.27%
Annual decline, hazard out of U	44.5%	43.9%
Quarterly average EU rate	.023	.023
Quarterly average UE rate	.328	.269

The calibrated model with an active search cutoff of 3.6% implies a higher value of h_1 and π_{Uz} and a lower value of the vacancy cost k relative to that with a cutoff of 5%. When unemployed workers are more likely to be counted as actively searching, the hazard rate out of unemployment will decline by more since some of those workers between the two cutoffs are present at long durations. In order to incentivize the planner to send more unemployed workers to search, the cost of searching, k, must be decreased, and the cost of remaining unemployed, π_{Uz}, must be increased. By expanding the definition of the labor force, those workers who were previously "marginally attached" are now counted as unemployed, increasing the average home skills and decreasing the average market skills of the pool of unemployed workers. By increasing the vector of home skills, more workers leave the labor force, decreasing the fraction of active searchers close to its value in the baseline model.

C.2.3 Changes in the Matching Function Elasticity

This section explores how changes in the elasticity of the matching function with respect to vacancies, denoted γ in Section 5.1, affect the results of the calibrated model. Two possibilities are explored in addition to the standard value: one below and one above the baseline calibration of .4. First, Shimer (2005) estimates an elasticity of 0.28 using CPS data from 1951-2003. Second, a higher value of .5 is shown to retain many of the model's predictions.

An elasticity of .28 implies that the number of hires increases by less in response to an increase in vacancies relative to the elasticity of .4 used in the benchmark. In this case, the job finding probability is less responsive to changes in workers' skills over the unemployment spell, and therefore the calibration requires stronger incentives for workers to remain unemployed in order to match the observed decline in the hazard rate in the data. To achieve this, the calibration with $\gamma = .28$ has a higher initial value for home skills h_1, lower rates of skill evolution when employed, π_{Ez} and π_{Eh}, and higher rates of skill evolution when unemployed, π_{Uz} and π_{Uh}. Parameters and targets for the low elasticity calibration are shown in Tables C.7 and C.8.

Table C.7: Parameters: Matching Elasticity .28

Parameter	Value	Description
β	.9959	Discount factor
λ	.0021	Death probability
γ	.28	Job finding probability $p(\theta) = min\{\theta^{\alpha}, 1\}$
Δ_h	.04	Step in h: $\Delta_h = h_k - h_{k-1}$
Δ_z	.04	Step in z: $\Delta_z = z_j - z_{j-1}$
h_1	.838	Lowest home skill
z_1	1	Lowest market skill
π_{Ez}	.18	$z' = \min\{z + \Delta_z, z_{N_z}\}$ with prob π_{Ez} if E, z o.w.
π_{Eh}	.21	$h' = \max\{h - \Delta_h, h_1\}$ with prob π_{Eh} if E, h o.w.
π_{Uh}	.38	$h' = \min\{h + \Delta_h, h_{N_h}\}$ with prob π_{Uh} if U, h o.w.
π_{Uz}	.2	$z' = \max\{z - \Delta_z, z_1\}$ with prob π_{Uz} if U, z o.w.
k	3.7	Vacancy cost
δ	.023	Separation probability

Conversely, an elasticity of .5 implies that hires are more responsive to changes in vacancies than in the benchmark calibration. The implications of a higher elasticity are opposite of those discussed in the paragraph above. In particular, this calibration requires a lower initial value for home skills h_1, lower values for π_{Uh} and the vacancy cost k, and a higher value for π_{Uz}. All of these parameters counteract the stronger incentive for workers to remain unemployed when the matching elasticity is higher

Table C.8: Targets: Matching Elasticity .28

Description	Target	Model
Annual interest rate	5%	5%
Matching function elasticity w.r.t v	.28	.28
Relative value of nonmarket work	.71	.719
Change in earnings with 1 year of market experience	2.30%	5.53%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.38%
Lifetime earnings losses due to unemployment	−11.9%	−6.9%
Annual decline, hazard out of U	44.5%	42.5%
Quarterly average EU rate	.023	.023
Quarterly average UE rate	.328	.276

than in the benchmark. Tables C.9 and C.10 contain the parameters and targets for the high elasticity calibration, respectively.

The cyclical patterns of all of the robustness checks discussed in this section are largely identical to those of the baseline model. Specifically, the small but highly persistent decline in labor force participation, the large asymmetric response of the unemployment rate, the fall in the job finding probability, and the countercyclical behavior of the average home productivity of the unemployed. For brevity, Figures similar to 5 with the alternative parameters discussed in this section are omitted.

C.2.4 Fixed Home Productivity

In this section, workers' market skills evolve as in the benchmark model, but all workers have the same, constant, home productivity $\overline{h}$. This model most clearly resembles that of Ljungqvist and Sargent (1998) in the directed, rather than random, search context. Tables C.11 and C.12 summarize the cailbrated parameters and targeted moments used in the calibration. Figure **??** is the analog to 1 for the full model.

The model implies that market skills must depreciate much more quickly when home skills are fixed, resulting in a brief increase in the job finding probability at short durations. This reflects the fact that at the beginning of the unemployment

Table C.9: Parameters: Matching Elasticity .5

Parameter	Value	Description
β	.9959	Discount factor
λ	.0021	Death probability
γ	.5	Job finding probability $p(\theta) = min\{\theta^{\alpha}, 1\}$
Δ_h	.04	Step in h: $\Delta_h = h_k - h_{k-1}$
Δ_z	.04	Step in z: $\Delta_z = z_j - z_{j-1}$
h_1	.821	Lowest home skill
z_1	1	Lowest market skill
π_{Ez}	.25	$z' = \min\{z + \Delta_z, z_{N_z}\}$ with prob π_{Ez} if E, z o.w.
π_{Eh}	.31	$h' = \max\{h - \Delta_h, h_1\}$ with prob π_{Eh} if E, h o.w.
π_{Uh}	.32	$h' = \min\{h + \Delta_h, h_{N_h}\}$ with prob π_{Uh} if U, h o.w.
π_{Uz}	.2	$z' = \max\{z - \Delta_z, z_1\}$ with prob π_{Uz} if U, z o.w.
k	2.85	Vacancy cost
δ	.023	Separation probability

Table C.10: Targets: Matching Elasticity .5

Description	Target	Model
Annual interest rate	5%	5%
Matching function elasticity w.r.t v	.5	.5
Relative value of nonmarket work	.71	.706
Change in earnings with 1 year of market experience	2.30%	5.57%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.56%
Lifetime earnings losses due to unemployment	−11.9%	−8.12%
Annual decline, hazard out of U	44.5%	45.6%
Quarterly average EU rate	.023	.023
Quarterly average UE rate	.328	.256

spell, the average worker has much to lose from remaining unemployed, leading to an increasing matching probability (and decreasing offer x) for the first few months of unemployment. Once the average worker has lost enough skills, both the job finding probability and reemployment wage monotonically decrease.

Table C.11: Parameters: Fixed Home Skill

Parameter	Value	Description
β	.9959	Discount factor
λ	.0021	Death probability
γ	.4	Job finding probability $p(\theta) = min\{\theta^\alpha, 1\}$
Δ_h	.05	Step in h: $\Delta_h = h_k - h_{k-1}$
Δ_z	.05	Step in z: $\Delta_z = z_j - z_{j-1}$
h_1	1.12	Lowest home skill
z_1	1	Lowest market skill
π_{Ez}	.22	$z' = \min\{z + \Delta_z, z_{N_z}\}$ with prob π_{Ez} if E, z otherwise
π_{Eh}	0	$h' = \max\{h - \Delta_h, h_1\}$ with prob π_{Eh} if E, h o.w.
π_{Uh}	0	$h' = \min\{h + \Delta_h, h_{N_h}\}$ with prob π_{Uh} if U, h o.w.
π_{Uz}	.65	$z' = \max\{z - \Delta_z, z_1\}$ with prob π_{Uz} if U, z o.w.
k	4	Vacancy cost
δ	.023	Separation probability

D Extension: Effort in Home Production

Suppose that each unemployed worker chooses how much effort to use at cost $c(e)$ to produce eh units of output through home production, where $c'(e) > 0$ and $c''(e) > 0$ and $e \in [0, \bar{e}]$. Assume that workers have a bequest motive, whereby they derive utility from the future generation of newborn workers. Timing in the extended model is as follows: at the beginning of each period, agents die with probability λ and agents of the same mass are born into unemployment with draws from distribution F_0. The surviving agents draw new productivities according to their current employment state. In the production stage, unemployed workers first choose effort e in order to produce eh units of the good, and production of the employed worker-firm pairs is unchanged. Timing in the second half of the period is identical to the baseline model: some workers separate from their matches and search and matching takes place. For notational clarity, denote the mass of employed workers g, where

Table C.12: Targets: Fixed Home Skill

Description	Target	Model
Annual interest rate	5%	5%
Average working lifetime	40 years	40 years
Matching function elasticity w.r.t v	.4	.4
Relative value of nonmarket work	.71	.88
Change in earnings with 1 year of market experience	2.30%	9.38%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	1.34%
Lifetime earnings losses due to unemployment	−11.9%	−11.4%
Annual decline, hazard out of U	44.5%	44.0%
Quarterly average EU rate	.023	.023
Quarterly average UE rate	.328	.245

$g(z,h)$ is the current fraction of employed workers of type (z,h).

The value functions and free entry condition in the decentralized equilibrium are given by

$$V_U(z,h,\psi) = \sup_{e,x} \Big\{ eh - c(e) + \beta(1-\lambda)\big[(1-p(\theta(\gamma,z,h,\psi)))\mathbb{E}_U(V_U(z',h',\hat{\psi})) + p(\theta(\gamma,z,h,\psi))x\big] + \beta\lambda\mathbb{E}_U(V_U(\overline{z},\overline{h},\hat{\psi}))\Big\}$$

$$J(z,h,\psi) = yz - w + \beta(1-\lambda)(1-d)\mathbb{E}_E\big(J(z',h',\hat{\psi})\big)$$

$$V_E(z,h,\psi) = w + \beta(1-\lambda)\Big(d(z,h,\psi)\mathbb{E}_U(V_U(z',h',\hat{\psi})) + (1-d(z,h,\psi))\mathbb{E}_E(V_E(z',h',\hat{\psi})\Big) + \beta\lambda\mathbb{E}_U(V_U(\overline{z},\overline{h},\hat{\psi}))$$

$$k \geq \beta(1-\lambda)q(\theta(\gamma,z,h,\psi))(\mathbb{E}_E(V_M(z',h',\hat{\psi})) - x(z,h,\psi)) \quad \text{and} \quad \theta(\gamma,z,h,\psi) \geq 0$$

The bequest term is $\mathbb{E}(V_U(\overline{z}, \overline{h}, \hat{\psi}))$, where the expectation operator $\mathbb{E}$ is taken over the distribution of next period's aggregate state $\hat{\psi}$ and the distribution F_0. Contracts are bilaterally efficient, implying that it is equivalent to write the joint value of the match, V_M in place of the firm and employed workers' values, respectively J and V_E. Following Lemma 1, the combined value function is:

$$V(a, z, h, \psi) = a\Bigg(zy + \beta(1-\lambda) \max_{d\in[\delta,1]}[d\mathbb{E}_U(V(0, z', h', \hat{\psi})) + (1-d)\mathbb{E}_E(V(1, z', h', \hat{\psi}))]$$

$$+\beta\lambda\mathbb{E}_U(V(0, \overline{z}, \overline{h}, \hat{\psi}))\Bigg) + (1-a)\Bigg(\max_{\theta,e}\big\{eh - c(e) - k\theta + \beta(1-\lambda)\big[(1-p(\theta))\mathbb{E}_U(V(0, z', h', \hat{\psi}))$$

$$+ p(\theta)\mathbb{E}_E(V(1, z', h', \hat{\psi}))\big]\big\} + \beta\lambda\mathbb{E}_U(V(0, \overline{z}, \overline{h}, \hat{\psi}))\Bigg)$$

$$s.t. \quad \theta \in [0, \overline{\theta}], \quad \beta \in (0, 1), \quad e \in [0, 1]$$

$$\text{where } V(0, z, h, \psi) \equiv V_U(z, h, \psi), \quad V(1, z, h, \psi) \equiv V_M(z, h, \psi)$$

the period payoff function, $azy + (1-a)(eh - c(e) - k\theta)$, is bounded and continuous, and

$$\mathbb{E}_U(V(0, z', h', \hat{\psi})) = \sum_{\hat{y}\in Y}\sum_{z'\in Z}\sum_{h'\in H} f(\hat{y}|y) f_U(z', h'|z, h) V_U(z', h', \hat{\psi})$$

$$\mathbb{E}(V(0, z', h', \hat{\psi})) = \sum_{\hat{y}\in Y}\sum_{z'\in Z}\sum_{h'\in H} f(\hat{y}|y) f_0(z', h') V_U(z', h', \hat{\psi})$$

$$\mathbb{E}_E(V(1, z', h, \hat{\psi})) = \sum_{\hat{y}\in Y}\sum_{z'\in Z}\sum_{h'\in H} f(\hat{y}|y) f_E(z', h'|z, h) V_M(z', h', \hat{\psi})$$

To analyze the planner's problem in the extended model, we must consider the laws of motion for (u, g), given by:

$$\hat{u}(z', h') = \lambda f_0(z', h') + (1-\lambda)\sum_z\sum_h\Big[f_U(z', h'|z, h)[(1 - p(\theta(z, h, \psi)))u(z, h) + d(z, h, \psi)g(z, h)]\Big]$$

$$\hat{g}(z', h') = (1-\lambda)\sum_z\sum_h\Big[f_E(z', h'|z, h)[p(\theta(z, h, \psi))u(z, h) + (1-d(z, h, \psi))g(z, h)]\Big]$$

The measure of workers in $\hat{u}(z', h')$ is a constant for each $(z', h') \in Z \times H$, thus it can easily be shown that the results in Theorem 2 hold, with the planner's component value functions written as

$$W_U(z,h,y) = \sup_{\theta\in\Re_+, e\in[0,1]} \Big\{ eh - c(e) - k\theta + \beta\big((1-\lambda)\Big[(1-p(\theta))\mathbb{E}_U(W_U(z',h',\hat{y})) + p(\theta)\mathbb{E}_E(W_E(z',h',\hat{y}))\Big] + \lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y}))\big)\Big\}$$

$$W_E(z,h,y) = zy + \beta\big((1-\lambda)\max_{d\in[\delta,1]} \Big\{ d\mathbb{E}_U(W_U(z',h',\hat{y})) + (1-d)\mathbb{E}_E(W_E(z',h',\hat{y}))\Big\} + \lambda\mathbb{E}(W_U(\overline{z},\overline{h},\hat{y}))\big)$$

Total consumption today is independent of the mass of newborn workers at the beginning of the period, therefore the first term is independent of λ for all types. A fraction $1-\lambda$ of each type of worker will survive to produce next period, and the evolutions of their employment states and productivities is identical to the baseline model. The remaining fraction λ of workers will die and be reborn as unemployed. By the timing assumption stated at the beginning of this section, newborn workers' productivities will not evolve immediately, and their choices next period depend on the aggregate productivity $\hat{y}$, therefore the expectations $\mathbb{E}$ appearing in the last term of each equation is the expectation over $\hat{y}$ only. It is easily shown that the efficiency result of Proposition 1 holds, with $W_U(z,h,y) = V_U(z,h,y)$ and $W_E(z,h,y) = V_M(z,h,y)\ \forall\ (z,h,y) \in Z \times H \times Y$.

The equilibrium value of $\theta^*(z,h,y)$ in the baseline and extended models, respectively, are determined by the equations:

$$p'(\theta^*(z,h,y)) = \frac{k}{\beta\big(\mathbb{E}_E W_E(z',h',\hat{y}) - \mathbb{E}_U W_U(z',h',\hat{y})\big)}$$

$$p'(\theta^*(z,h,y)) = \frac{k}{\beta(1-\lambda)\big(\mathbb{E}_E W_E(z',h',\hat{y}) - \mathbb{E}_U W_U(z',h',\hat{y})\big)}$$

Since $\lambda \in (0,1)$, the denominator of the right hand side in the second equation is smaller than in the first, and it follows that the equilibrium market tightness in the extended model is lower $\forall(z,h,y)$ than in the baseline model. This is because a probability of death decreases the expected continuation value, making a match less productive in expectation because the effective probability of continuation in the match is $(1-\lambda)(1-d)$ rather than $1-d$ in the baseline.

The effect on the decentralized equilibrium value of employment x is ambiguous: the probability of death decreases the expected continuation value $\mathbb{E}_E(W_E(z',h',\hat{y}))$, and decreases the equilibrium value θ^*:

$$x = \mathbb{E}_E(W_E(z', h', \hat{y})) - \frac{k}{\beta(1-\lambda)q(\theta^*(z, h, y))}$$

If p is isoelastic as assumed in Proposition 3, then the expression for the value of employment x is the same as in the baseline model:

$$x(z, h, y) = (1-\alpha)\mathbb{E}_E(W_E(z', h', \hat{y})) + \alpha\mathbb{E}_U(W_U(z', h', \hat{y}))$$

This implies that the value of employment is strictly increasing in each of its arguments.

E Data

This section summarizes the data sources used in the regressions in Section 4 and checks the robustness of the benchmark results shown in Tables 1 and 2.

E.1 Job Finding Probability Using Probit Marginal Effects

Table E.13 shows the marginal effects from probit regressions corresponding to the linear probability model shown in Table 1. The last column is omitted due to the bias present in probit estimates when including fixed effects.

E.2 Panel Study of Income Dynamics (PSID)

The PSID sample begins the first time the monthly employment histories for the year previous to the interview year are available, 1984, and ends in 1996, the year before the PSID became biannual. In this sample, job transitions are identified as workers reporting being unemployed for at least one month in the current year or the year prior to the interview and employed at the time of the interview. Durations are computed from the employment histories and therefore are recorded in months. In Table 1, the PSID sample is further restricted to heads of household age 18-65 with duration up to 12 months, though the results for "wives" are similar. Annual family weights are used in all regressions. Table E.16 shows estimated marginal effects for heads of household of all ages in Column (1), all durations in Column (2), all ages and all durations in Column (3). Table E.17 shows estimated marginal effects for the subsamples of males in Column (1), females in Column (2), workers with durations up to 6 months in Column (3), and durations between 6 and 12 months in Column

Table E.13: Probit: Job finding probability on unemployment duration

	CPS (1)	(2)	PSID (3)
duration	-.0041***	-.0333***	-.3801***
	(.0001)	(.0017)	(.0543)
duration^2		.0019***	.0497**
		(.0002)	(.0203)
duration^3		-4.75e-05***	-.0024
		(4.72e-06)	(.0027)
duration^4		4.18e-07***	3.61 e-05
		(4.72e-08)	(.0001)
Pseudo R^2	.0685	.0755	.2624
N	121,142	121,142	10,772

Notes: CPS: January 1994-December 2015, monthly; duration reported in weeks. Universe: workers unemployed in at least one month of the CPS with reported duration up to 52 weeks, ages 18-65. PSID: 1984-1996, annual; duration reported in months. Universe: heads of household unemployed in at least one month of the PSID employment history with reported duration up to 12 months, ages 18-65. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and a quadratic term in total labor market experience. Results reported are the estimated marginal effect of duration on the job finding probability. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

(4). Over the 13-year sample period, there are 14,334 unique heads of household who respond to the survey. The total number of families responding to the survey in 1984 was 6,918 and in 1996 was 8,511. The average number of years that the head of household participates in the PSID over the sample is 9.7.

There are several drawbacks to using the PSID. First, duration is measured in months due to the structure of the PSID employment histories. Therefore we may see little effect due to time aggregation bias. Second, wages in the PSID are annual income divided by annual hours, therefore if wages grow in the first months of employment or if a worker was unemployed only in the middle of the year but employed at the beginning and end in two different jobs, this average does not represent the true reemployment wage. Finally, the PSID is a less representative sample relative to the CPS. However, since the PSID follows individuals over many years, it may be used to control for some individual fixed effect unobservable to the econometrician.

Since duration is measured in weeks in the CPS and in months in the PSID, the values of the coefficients in the two surveys cannot be compared, but the results

Table E.14: Linear Probability Model: Job Finding Probability on Unemployment Duration, Recession and Boom Subsamples

	(1)	(2)	(3)	(4)
duration	-.0326***	-.0210***	-.0324***	-.0234***
	(.0014)	(.0038)	(.0014)	(.0039)
duration^2	.0019***	.0011***	.0019***	.0012***
	(.0001)	(.0003)	(.0001)	(.0003)
duration^3	-4.77e-05***	-2.77e-05***	-4.80e-05***	-2.71e-05***
	(3.67e-06)	(9.11e-06)	(3.62e-06)	(1.02e-05)
duration^4	4.15e-07***	2.41e-07***	4.19e-07***	2.23e-07**
	(3.57e-08)	(8.75e-08)	(3.50e-08)	(9.97e-08)
R^2	.1251	.1070	.1244	.1236
N	129,576	18,160	130,981	16,755

Notes: CPS: January 1994-December 2015, monthly; duration reported in weeks. Universe: workers unemployed in at least one month of the CPS with reported duration up to 52 weeks, ages 18-65. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and a quadratic term in total labor market experience. Column 1 reports results for the regression of workers at all durations in the CPS covering all months in 1994-2007 and 2010-2015, column 2 is the same regression over the period January 2008-December 2009, column 3 covers all non-recession months as indicated by the NBER, and column 4 covers ll months indicated as a recession. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

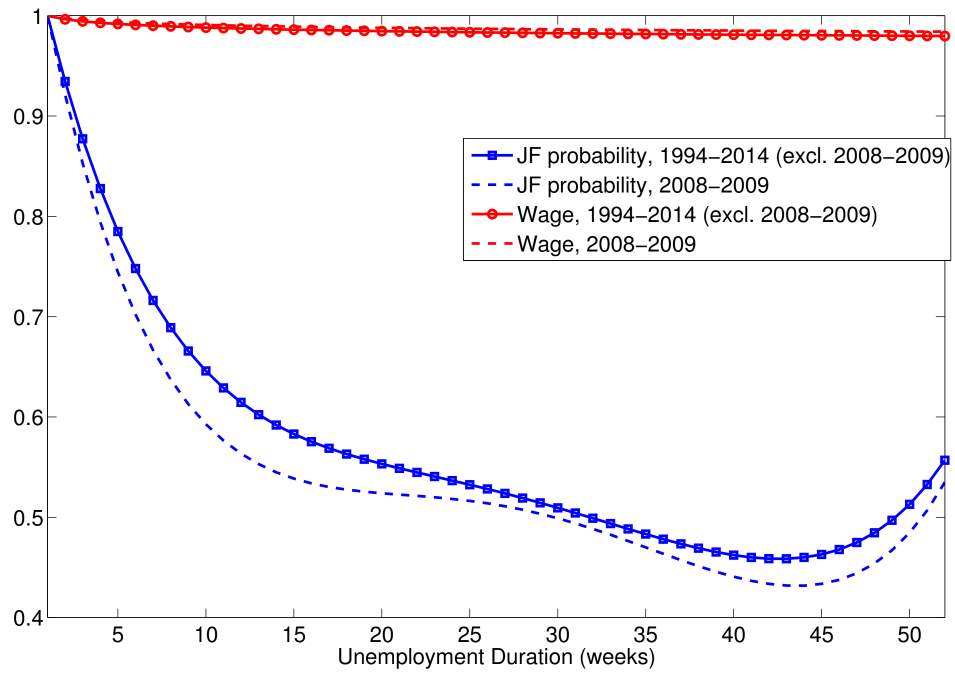
Table E.15: Regression of log Reemployment Wage on log Duration, Recession and Boom Subsamples

	(1)	(2)	(3)	(4)
log duration	-.0091***	-.0108	-.0091***	-.0110
	(.0031)	(.0091)	(.0032)	(.0084)
R^2	.3920	.4376	.3901	.4312
Root MSE	.3377	.3290	.3393	.3233
N	15,809	1,743	15,558	1,994

Notes: CPS: January 1994-December 2015, monthly. Universe: respondents aged 18-65 who transitioned from U to E excluding those for whom the CPS allocated the hourly wage, with durations up to 52 weeks. Controls for observables include the aggregate unemployment dummies for the interview year and month, the log of the aggregate unemployment rate, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and total labor market experience. Column 1 reports results for the regression of workers at all durations in the CPS covering all months in 1994-2007 and 2010-2015, column 2 is the same regression over the period January 2008-December 2009, column 3 covers all non-recession months as indicated by the NBER, and column 4 covers ll months indicated as a recession. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

contained in Table E.16 are qualitatively similar to those using the CPS in Table 1. There continues to be a negative effect of duration on the probability of exiting unemployment as seen in the baseline CPS results in Table 1. Running the same

Figure 8: Mean Job Finding Probability and Reemployment Wage by Duration, Recession and Boom Subsamples



Notes: Predicted values of the mean job finding probability and log reemployment wage as functions of weekly reported unemployment duration, controlling for observables. Sample: CPS, 1994-2015, workers reporting unemployment and employment in two consecutive months, ages 18-65, with unemployment durations up to 1 year. Solid lines plot the "boom" subsample (1994-2007, 2010-2014) and dashed lines correspond to the "recession" subsample (2008-2009).

regressions without restricting to age 18-65 gives similar results in all cases.

Real reemployment wages in the PSID are the wages reported for the year prior to the interview in which the respondent first reports being employed, deflated using the US city average CPI. Robust standard errors are reported in parentheses. Tables E.18 and E.19 report the log wage regressions introduced in Table 2 with the same subsamples as in Tables E.16 and E.17, respectively. It should be noted that in the PSID, if a male adult is present in the household he is typically assigned the role of "head". Therefore, over two thirds of heads of household in the sample are males. Though results are not reported here, all of the coefficients are insignificant at the 10% significance level when including fixed effects in the regressions shown in Tables E.18 and E.19 . The wage regressions below provide further evidence that the elasticity of starting wages with respect to unemployment duration is small. We can conclude that the fact that reemployment wages are less responsive to duration relative to the hazard rate of exiting unemployment is robust to the choice and timespan of the survey.

Table E.16: Probit Regressions: all ages and durations, PSID

	(1)	(2)	(3)
duration	-.3752***	-.3752***	-.3723***
	(.0540)	(.0342)	(.0342)
duration^2	.0479**	.0490***	.0481***
	(.0202)	(.0101)	(.0101)
duration^3	-.0021	-.0022**	-.0021**
	(.0027)	(.0010)	(.0010)
duration^4	2.82-05	2.04-05	1.79-05
	(.0001)	(3.40-05)	(3.40-05)
Pseudo R^2	.2630	.2666	.2671
N	10,823	10,905	10,956

Notes: PSID: 1984-1996, annual. Universe: heads of household unemployed in at least one month of the PSID employment history in the annual interview. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 2 show the regression of the job finding variable on duration plus all controls for heads of household of all ages with durations up to 12 months, and for workers ages 18-65 for all durations, respectively. Column 3 shows results for heads of household of all ages and all durations. Results reported are the estimated marginal effect of duration on the job finding probability. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

E.3 CPS: data description and robustness

The Bureau of Labor Statistics (BLS) conducts the Current Population Survey (CPS) on a monthly basis since 1940. Respondents participate in the CPS 8 times: they respond to the survey for 4 consecutive months, then do not participate for 8 months, and then participate again for 4 consecutive months. Respondents in the 4th and 8th interviews, known as the outgoing rotation group (ORG), are asked additional questions about their labor income. As in Fernández-Blanco and Preugschat (2015), I consider workers whose first month of employment after an unemployment spell is in the ORG to maximize the accuracy of the measure of reemployment wages. The sample considered here begins in January 1994 and ends in December 2014. June through September 1995 are excluded due to changes in the variables required to identify individuals over time. All regressions below use the monthly ORG weights.

Workers who were unemployed and actively searching in one month and employed the following month in the ORG are identified by tracing them over time using the household identifier, household number, and person line number. To check

Table E.17: Probit Regressions: by sex and long term spell, PSID

	(1)	(2)	(3)	4
duration	-.2896***	-.6549***	.1672	-.2514
	(.0610)	(.1123)	(.2561)	(.3804)
duration^2	.0153	.1521***	-.2602*	.0444
	(.0230)	(.0400)	(.1466)	(.0622)
duration^3	.0019	-.0148***	.0620*	-.0034
	(.0031)	(.0052)	(.0328)	(.0045)
duration^4	-.0001	.0005**	-.0043*	.0001
	(.0001)	(.0002)	(.0025)	(.0001)
Pseudo R^2	.2622	.3595	.2458	.4803
N	8,745	2,018	9,844	571

Notes: PSID: 1984-1996, annual. Universe: heads of household unemployed in at least one month of the PSID employment history in the annual interview, ages 18-65 and with duration up to 12 months. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 2 report results for the subsample of males and females, respectively. Column 3 reports results for the subsample of workers with durations less than 6 months, and column 4 for the subsample with durations between 6 and 12 months. Results reported are the estimated marginal effect of duration on the job finding probability.$*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

that matches are accurate, I compare the age and sex of the individuals across months. Workers who report being retired, disabled, actively serving in the armed forces, or in farming and agriculture are excluded. The job finding probability is computed as those workers who transition from actively searching to employed (UE switchers) as a fraction of active searchers. In the baseline results, "employed" includes both workers who were at work and workers who were absent the previous week. The absent employed workers make up less than 1 percent of all UE switchers and excluding these workers does not meaningfully affect the results. Real hourly wages are computed as the wages of workers who report being paid hourly deflated by the US CPI city average, 1982-84=100. In the baseline results, wages that are allocated to the respondent are omitted. The regression including these observations is summarized in Table E.21.

Table E.20 shows similar regressions to E.16, reporting estimated marginal effects for workers of all ages in column 1, all durations in column 2, all ages and all durations in column 3. Table E.21 shows the corresponding wage regressions to Table E.18 in the unrestricted age and duration sample using the unallocated wages only in columns 1-3, and repeats the same regression including allocated wages in

Table E.18: Wage Regressions: all ages and durations, PSID

	(1)	(2)	(3)
log duration	-.0070	-.0189	-.0196
	(.0161)	(.0145)	(.0145)
dummy, > 6 mo	Y	Y	Y
FE	N	N	N
R^2	.2193	.2219	.2212
Root MSE	.6407	.6393	.6410
N	10,586	10,662	10,712

Notes: PSID sample: 1984-1996, annual; duration reported in months. Universe: heads of household unemployed in at least one month of the PSID employment history with reported duration up to 12 months, ages 18-65. Controls for observables include the aggregate unemployment dummies for the interview year and month, the log of the aggregate unemployment rate, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and total labor market experience, and a dummy indicating whether the most recent unemployment spell was over 6 months. Columns 1 and 2 show the regression of log real reemployment wages on log duration plus all controls for heads of household of all ages with durations up to 12 months, and for workers ages 18-65 for all durations, respectively. Column 3 shows results for heads of household of all ages and all durations. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

Table E.19: Wage Regressions: by sex and long term spell, PSID

	(1)	(2)	(3)	(4)
log duration	-.0024	-.0266	.0096	-.1334
	(.0173)	(.0369)	(.0148)	(.1607)
dummy, > 6 mo	Y	Y	N	N
FE	N	N	N	N
R^2	.1926	.3027	.2140	.5042
Root MSE	.6469	.5705	.6322	.6174
N	8,566	1,970	9,711	825

Notes: PSID sample: 1984-1996, annual; duration reported in months. Universe: heads of household unemployed in at least one month of the PSID employment history with reported duration up to 12 months, ages 18-65. Controls for observables include the aggregate unemployment dummies for the interview year and month, the log of the aggregate unemployment rate, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and total labor market experience. Columns 1 and 2 report results for the subsample of males and females, respectively. Column 3 reports results for the subsample of workers with durations less than 6 months, and column 4 for the subsample with durations between 6 and 12 months. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

columns 4-6.

In the regressions of the job finding probability, the point estimate of the marginal

Table E.20: Probit Regressions: all ages and durations, CPS

	(1)	(2)	(3)
duration	-.0331***	-.0156***	-.0155***
	(.0016)	(.0006)	(.0006)
duration^2	.0019***	.0004***	.0004***
	(.0001)	(2.57e-05)	(2.52e-05)
duration^3	-4.75e-05***	-4.45e-06***	-4.43e-06***
	(4.64e-06)	(3.92e-07)	(3.86e-07)
duration^4	4.18e-07***	1.63e-08***	1.63e-08***
	(4.65e-08)	(1.86e-09)	(1.83e-09)
Pseudo R^2	.0843	.0852	.0942
N	128,860	140,825	149,423

Notes: CPS: January 1994-December 2015, monthly. Universe: workers unemployed in at least one month of the CPS with duration up to 52 weeks. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 2 show the regression of the job finding variable on duration plus all controls for respondents of all ages with durations up to 52 weeks, and for respondents ages 18-65 for all durations, respectively. Column 3 shows results for all ages and all durations. Results reported are the estimated marginal effect of duration on the job finding probability. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

Table E.21: Wage Regressions: all ages and durations, CPS

	(1)	(2)	(3)	(4)	(5)	(6)
log duration	-.0082***	-.0118***	-.0111***	-.0027	-.0058***	-.0048**
	(.0027)	(.0026)	(.0024)	(.0024)	(.0022)	(.0021)
dummy, > 6 mo	N	N	N	N	N	N
R^2	.4068	.3893	.4039	.3715	.3487	.3666
Root MSE	.3304	.3372	.3304	.3517	.3587	.3525
N	19,425	18,612	20,534	27,471	26,666	29,218

Notes: Sample: January 1994-December 2015, monthly. Universe: respondents aged 18-65 who transitioned from U to E. Controls for observables include the aggregate unemployment dummies for the interview year and month, the log of the aggregate unemployment rate, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and total labor market experience. Columns 1-3 exclude those workers for whom the CPS allocated the hourly wage. Columns 1 and 2 show the regression of the log real reemployment wage on log duration plus all controls for respondents of all ages with durations up to 52 weeks, and for respondents ages 18-65 for all durations, respectively. Column 3 shows results for all ages and all durations. Columns 4-6 repeat the regressions including workers with allocated hourly wages. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

effect of duration in column 1 of Table E.20 including workers of all durations are comparable in magnitude to the baseline results in column 2 of Table 1 focusing only on those workers between 18 and 65 years old. When we include workers with longer durations, the estimates of the marginal effect is only half as large as in the baseline regression. Again, including a dummy for long term unemployment causes all effects of duration on wages to disappear.

Table E.22 shows results for regressions identical to those in Table 1 for subsamples of males (columns 1-4) and females (columns 5-8). Table E.23 are the wage regressions corresponding to Table2 in the text. Results in both tables restrict the age of respondents to be between 18 and 65 as in the main results. Results in the baseline regressions are robust to splitting the sample by gender.

E.4 American Time Use Survey (ATUS)

Using the method of Krueger and Mueller (2010), duration is imputed for individuals who are unemployed in the CPS and the ATUS as the duration reported in the CPS plus the length of time in weeks between the CPS and ATUS interviews. For those individuals employed in the CPS and unemployed in the ATUS, duration is set to half the time in weeks between the two interviews. Finally, duration for individuals who are employed in the ATUS is set to 0. This ensures that workers who are out of the labor force have recorded duration only if they reported being unemployed in the CPS and recently transitioned out of the labor force.

Regression results using minutes of core home production excluding child care as the dependent variable are shown in Table E.24. Column 1 shows results from the regression of core home production on a cubic term for duration, plus controls for observable heterogeneity (see footnote below Table for details). Columns 2 and 3 repeat the regression of column 1 for the subsamples of males and females, respectively. Though the effect is nonlinear, the length of an individual's unemployment spell changes the allocation of time in core home production, providing some indirect evidence in favor of the model mechanism. These results are robust to controlling for the number of children, age of the youngest child, and family income. Table 3 shows the analogous regressions to those shown in

Table E.25 repeats the analysis using leisure time to show that for both men and women, the time spent in leisure activities does not vary with duration. As discussed in Section 4, these regressions provide strong evidence against an observationally equivalent mechanism of the model whereby workers build a habit in leisure over the unemployment spell.

Table E.22: Probit Regressions: Job finding probability on unemployment duration by gender

	Males				Females			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
duration	-.0042***	-.0359***	-.0577***	-.6741**	-.0038***	-.0302***	-.0397***	-.0715
	(.0002)	(.0023)	(.0076)	(.2761)	(.0001)	(.0024)	(.0076)	(.3029)
duration^2		.0021***	.0057***	.0277**		.0017***	.0033**	.0032
		(.0002)	(.0013)	(.0112)		(.0002)	(.0013)	(.0123)
duration^3		-5.35e-05***	-.0003***	-.0005**		-4.02e-05***	-.0002*	-6.42e-05
		(6.49e-06)	(8.32e-05)	(.0002)		(6.86e-06)	(8.53e-05)	(.0002)
duration^4		4.74e-07***	5.03e-06***	3.32e-06***		3.48e-07***	2.71e-06	4.85e-07
		(6.48e-08)	(1.76e-06)	(1.29e-06)		(6.89e-08)	(1.82e-06)	(1.42e-06)
Pseudo R^2	.0693	.0767	.0702	.0619	.0737	.0803	.0758	.0616
N	64,098	64,098	49,947	14,151	57,044	57,044	45,645	11,399

Notes: CPS: January 1994-December 2015, monthly. Universe: workers unemployed in at least one month of the CPS with reported duration up to 52 weeks, ages 18-65. Controls include the aggregate unemployment rate, plus dummies for the interview year and month, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 5 report results for the regression of workers on a linear term for duration, columns 2 and 6 is the same regression with a 4th degree polynomial of duration, columns 3 and 7 for workers with durations up to 6 months, and columns 4 and 8 for workers with durations between 6 and 12 months. Results reported are the estimated marginal effects of duration on the job finding probability. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

Table E.23: Regression of log reemployment wage on log unemployment duration by gender

	Males				Females			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log duration	-.0120***	-.0073	-.0077	.1142	-.0058	-.0073	-.0056	.0686
	(.0040)	(.0050)	(.0051)	(.0507)	(.0042)	(.0050)	(.0060)	(.0447)
dummy, > 6 mo	N	Y	N	N	N	Y	N	N
R^2	.3979	.3982	.4036	.4422	.3981	.3982	.3902	.5447
Root MSE	.3395	.3394	.3393	.3391	.3292	.3394	.3319	.3012
N	8,678	8,678	7,607	1,071	8,874	8,874	7,819	1,055

Notes: Sample: January 1994-December 2015, monthly. Universe: respondents aged 18-65 who transitioned from U to E excluding those for whom the CPS allocated the hourly wage, with durations up to 52 weeks. Reemployment wage is the reported hourly wage. Duration is reported duration (prunedur). Controls for observables include the aggregate unemployment dummies for the interview year and month, the aggregate unemployment rate, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and total labor market experience. Columns 1 and 5 report results for the regression of workers at all durations with no long term unemployment dummy, columns 2 and 6 are the same regression with the long term dummy, columns 3 and 7 for workers with durations up to 6 months, and columns 4 and 8 for workers with durations between 6 and 12 months. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

Table E.24: Regression: minutes of "core" home production on duration

	(1)	(2)	(3)
duration	-6.002***	-3.190	-8.241**
	(2.249)	(2.816)	(3.432)
duration^2	.2664***	.1505	.3554**
	(.0957)	(.1225)	(.1446)
duration^3	-.0033***	-.0020	-.0043**
	(.0012)	(.0015)	(.0018)
N	80,545	39,314	41,231
R^2	.0997	.0240	.0797

Notes: ATUS: January 2003-December 2013, monthly. Universe: respondents with no "unclassified" time use, ages 18-65, with imputed durations up to 52 weeks. Controls for observables include dummy variables for the year and month of the interview, race, age, gender (column 1 only), state of residence, education level, presence of an employed partner, and labor force status. Column 1 reports results for the regression for all workers and columns 2 and 3 report results for the subsamples of males and females, respectively. * denotes $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table E.25: Regression: Minutes leisure on duration

	(1)	(2)	(3)	(4)	(5)	(6)
duration	3.035	.3710	-5.373	9.385	-6.636	5.693
	(4.166)	(4.048)	(6.312)	(5.011)	(6.358)	(4.511)
duration^2	-.1009	-.0110	.2730	-.3942	.3130	-.2733
	(.1788)	(.1734)	(.2710)	(.2148)	(.2724)	(.1929)
duration^3	.0011	.0001	-.0036	.0047	-.0040	.0035
	(.0022)	(.0022)	(.0034)	(.0027)	(.0034)	(.0024)
N	80,545	80,545	39,314	41,231	39,314	41,231
R^2	.0842	.0586	.1002	.0726	.0697	.0489

Notes: ATUS: January 2003-December 2013, monthly. Universe: respondents with no "unclassified" time use, ages 18-65, with imputed durations up to 52 weeks. Controls for observables include dummy variables for the year and month of the interview, race, age, gender (column 1 only), state of residence, education level, presence of an employed partner, and labor force status. Columns (1) and (2) report results for all workers using definitions of leisure time including and excluding sleep, respectively. Columns (3) - (6) report results for the subsamples of males and females under each of the two definitions, respectively. * denotes $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$