Knowledge Spillovers in Competitive Search Equilibrium

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Abstract

Do firms have the right incentives to innovate in the presence of spillovers? This paper proposes an explicit channel of spillovers through labor flows within a framework of competitive search. Firms can choose to innovate or to imitate by hiring a worker from a firm that has already innovated. We show that with long-term wage contracts information spillovers caused by worker turnover are efficiently internalized. If innovating firms cannot commit to long-term wage contracts, there is too little innovation and too much imitation. A combination of a subsidy to innovation and a fee to imitation can restore efficiency. A stand-alone subsidy of innovating firms will also improve efficency. A fee on imitation has ambigous welfare effects, although it reduces imitation it also reduces innovation. Restrictions on labor mobility reduces welfare.

Key words: Competitive search, innovation, imitation, spillovers, worker flows, efficiency

JEL Codes: J31, J68, 031.

1 Introduction

Do firms have the right incentives to innovate when innovations cause information spillovers to other firms? This paper proposes an explicit channel of spillovers through labor flows within a framework of competitive search. Firms can choose to innovate or to imitate by hiring a worker from a firm that has already innovated. We show that if firms can commit to long-term wage contracts, the information spillovers are efficiently internalized. In the absence of such contracts, there is too little innovation and too much imitation, and hence a scope for policy.

We show that it is important to have the right mix of policy instruments to improve efficiency. A combination of a subsidy to innovation and a fee to imitation can restore efficiency.

For some innovations, knowledge spillovers can be restricted through patents. However, it is much more difficult to prevent spillovers stemming from innovations that are not necessarily linked to a specific output or to a patentable invention. Such innovations can take many forms, e.g. process improvements, managerial know-how, intellectual human capital (see Møen, 2005). It is both difficult to prevent workers from learning about such innovations and write enforceable contracts about such within-firm knowledge. Hence, labor flows between firms become an important channel for inter-firm knowledge transfers.

We model spillovers within a competitive search framework. In order to get knowledge of how to produce, firms may either invest in order to innovate, or hire a worker from another firm that has already innovated and who possesses the knowledge. The critical assumption is that it is sufficient that one party knows the innovation, either the entrepreneur (the firm) or the worker. If the firm has innovated, it can hire any worker from the pool of unemployed (and inexperienced) workers. If the firm chooses the imitation strategy, it is constrained by the availability of workers who have learned the innovation by his employer. The equilibrium of our model typically exhibits both innovation and imitation, although firms are identical ex-ante.

The focus of the analysis is on the efficiency properties of equilibrium. A social planner faces a trade-off between innovation costs and search costs. If a large fraction of the firms innovate, aggregate innovation costs are high, as the innovation costs are duplicated in many firms. On the other hand, the planner economizes on on-the job search and replacement search costs, as less on-the-job search is necessary in order to disseminate the information to imitating firms. The welfare maximum optimally balances the trade-off between search costs and innovation costs.

The question we address is whether competitive search equilibrium will deliver the optimal allocation of resources. We show that this depends on what restrictions we make on the contract space. If an innovating firm can commit to long-term wage contracts, it will commit to high wages for experienced workers with knowledge of the innovation, so that these workers have the right incentives when searching for an imitating firm. In particular, innovating firms will give the full net surplus of the second period to the worker. Hence the search behavior optimally trades off the net gain of remaining in the firm with the gain from leaving. Firms are willing to give the full surplus to the worker in the second period since they can extract profits from the worker by setting the wage in period 1. The recruiting decision in the first period (and all other recruiting decisions) are optimal from an aggregate perspective because the innovating firm is able to extract the entire social value of its innovation. The two elements together, full commitment and competitive search, yield efficiency of equilibrium.

From both a theoretical and an empirical perspective full commitment might be too strong an assumption. If firms cannot commit to future wages, it cannot credibly commit to leave the full second-period match surplus to the worker. Hence the firm can no longer separate the problems of rent extraction and optimal retention. The second-period wage will be lower, and more firms will find it worthwhile to imitate and search for underpaid informed workers. Through the participation constraint of the workers, the inefficient on-the-job search decisions are passed on to the innovating firms, who obtain a lower total surplus and hence weaker incentives to innovate than with full commitment. Hence the equilibrium delivers too little innovation and too much imitation compared with the socially optimal levels.

The allocation of the no-commitment equilibrium does not reach efficiency and therefore leaves room for welfare improving policies. An important conclusion from our policy analysis is that the right mix of policy instruments is crucial to increase efficiency. A subsidy to innovators together with a fee on imitation can implement the efficient allocation. A subsidy to innovating firms is welfare improving, while a fee on imitation by itself has countervailing effects as it reduces imitation but also innovation. The net effect is uncertain. Finally, we use the model to analyze the welfare effects of restrictions on worker mobility. In our set-up, this turns out to be welfare-detoriorating. At this point our findings contrast those of Marimon and Quadrini (2011).

There is a substantial empirical literature on R&D spillovers and worker flows. Møen analyses spillover effects by analyzing wage profiles of workers in R&D intensive firms, and compare them with wage profiles in ordinary firms. The hypothesis is that R&D intensive firms generate new knowledge which will be shared by the workers. As a result, workers in R&D intensive firms will receive a high wage later in their careers, and foreseeing this accept a low wage early on in their careers. This is confirmed by data. Møen finds that the technical staff in R&D-intensive firms receive a discount in the beginning of the career is 6.1 percent and the premium at the end of the career is 6.8 percent. Møen concludes that the potential externalities associated with labor mobility are, at least partially, internalized in the labor market.

Several papers study the effect of the mobility of engineers and scientists using patent citation data. Although our analysis regards non-patented ideas, one may hypothesize that the dissemination of knowledge of patentable innovations also shed light on the dissemination pattern of non-patentable ideas. Almeida and Kogut (1999) use US patent citation data to show that ideas spread through the mobility of patent holders. Jaffe, Trajtenberg and Henderson (1993) find that patent citations come disproportionately from geographical areas close to the cited patent. Brechi and Lissoni (2009) find that the movement of inventors between firms account for a very large fraction of the localized patent citations. Kim and Marschke (2005) obtain similar results.

Our notion of innovations is not only restricted to the outcome of R&D in the traditional sense, but captures all forms of improvement in production technology broadly defined. Greenstone, Hornbeck and Moretti (GHM) analyze productivity spillovers by comparing changes in total factor productivity on incumbent firms stemming from the opening of new large manufacturing plants in the US. They find that positive spillovers exist and are increasing in the worker flow between the incumbent plants' industry and the opening plants industry. GHM conclude that their finding is consistent with the notion that spillovers are embedded in workers that move between firms. A related strand of the empirical literature studies the effect of multinationals to non-multinational firms. In a recent paper, Balsvik (2011) analyzes labor mobility as a channel of spillovers from multinationals (MNEs) to non-MNE in Norway. She finds that profits of non-MNEs increases when hiring workers from MNEs, and attributes this to knowledge spillovers. See also Görg and Strobl (2005) who get similar results.

Within industrial organization there is a literature on spillovers through worker mobility. The seminal paper is Pakes and Nitzan (1983). A monopolist innovates, and as in our model an employee learns the innovation. If this employee moves to a potential competitor, the two firms will engage in duopoly competition. It is shown that the employee will quit if and only if aggregate duopoly profit is greater than the monopoly profit of the incumbent. Several other papers have developed these issues further, see Fosfuri and Rønde (2004), Combes and Duranton (2006), Kim and Marschke (2005). Cooper (2001) examines labor mobility as a source of knowledge spillovers in a competitive environment, and argues that firms have an incentive to free-ride on the average amount of R&D undertaken in society. In contrast with our paper, most of the contributions in the IO literature do not explicitly model the labor market, do not allow for entry of firms, and most importantly, do not model the choice between innovation and imitation.

In the labor-search litterature, we do not know any theoretical studies of innovation and imitation. There is a small literature on on-the-job investments in general human capital in the presence of search frictions, see Acemoglu and Pischke (1999) and Moen and Rosen (2004). The difference between innovations and aquisition of human capital is that the latter cannot costlessly be shared. Hence if the worker obtains general human capital, a firm's output falls if a trained worker leaves and is replaced by an untrained worker.

The paper proceeds as follows. The two next sections 2 and 3 describe the economy and analyze the equilibrium with commitment. Then, section 4 establishes efficiency of the equilibrium with commitment. Section 5 studies a variant of the model with no commitment. We characterize equilibrium and do a welfare and policy analysis. Conclusions are summarized in section 6.

2 Model Environment

The setting is a two-period competitive search model with a measure 1 of initially unemployed workers and an endogenously determined measure of firm entrants e_1 and e_2 for period 1 and period 2, respectively. A firm can have at most one worker. All agents in the economy are assumed to be risk-neutral. In the first period firms can enter to innovate and then search for a worker. Innovating costs k units of output. Posting a vacancy to attract a worker costs K units of output. If a match is formed, the firm and the worker produce y units of output in the first period and again y units in the second period if the match continues. In the second period there is entry of firms attempting to imitate the innovation by poaching a worker from a firm that has innovated. Offering a vacancy to employed workers has costs K^I. A firm that has successfully poached a worker produces y units of output. Finally, innovating firms that have lost a worker to an imitating firm can replace that worker by posting a vacancy at no additional costs to the pool of unemployed workers.[1] If the innovating firm successfully finds a replacement worker it produces y units of output. We assume that the leisure value for a worker is normalized to zero.

2.1 Timing

The following summarizes the timing protocol:

- First Period:

    1. Firms enter to innovate: pay innovation cost k.

[1] Vacacancy costs are usually comprised of (capital) costs of creating a job and (labor) costs for the hiring process. Here, we set the latter costs to zero for simplicity. The results would hold also for positive but relatively small hiring costs.

2. Firms then create a job at cost K and post a vacancy to fill the job with an unemployed worker.
3. A successfully matched firm produces y units of output, other firms exit.

- Second Period:

1. Firms enter to imitate by creating a job at cost K^I and posting a vacancy to employed workers.
2. Incumbent firms that have lost a worker can post a vacancy to replace the worker with an unemployed worker.
3. All matched firms produce y units of output, other firms exit.
4. Employed workers consumer their share of output, there is no consumption if unemployed.

Note that this structure of moves entails two simplifications: First, we rule out entry to innovation in the second period. This would complicate the algebra considerably without qualitatively changing the results or adding new insights. Secondly, we do not allow incumbent firms that have lost a worker in the second period to poach workers from other incumbent firms. Again this is a innocuous assumption to make the model more tractable.

2.2 Wage Posting, Matching Technology, and Contracts

The search frictions are modeled as in the competitive search framework by Moen (1997). Before matching takes place firms post wages/contracts that are observable by all workers in that market. Subsets of the market with the same wage form a submarket. Each worker selects one submarket to apply for a job. In each submarket matches are the outcome of a matching technology. The technology is the same across submarkets within a market, but we do not require the technology to be identical across markets. The model has three matching markets: One for innovating firms in the first period, one for imitating firms in the second period, and one for innovating firms that need to replace a worker in the second period. We index the imitating firm market with superscript I, the replacement market with superscript U, and omit the index for the first period matching market. We assume that matching is governed by a c.r.s. matching function $m^i(s^i, v^i) \leq min\{s^i, v^i\}$, where m^i is the total number of matches and s^i the measure of searching workers, and v^i the measure of searching firms, and i is the index for the market. In particular we assume a Cobb-Douglas matching function: $m^i(s,v) = A^i s^{\epsilon^i} v^{1-\epsilon^i}$. As it is common in the literature, we reformulate all matching-related variables in terms of the labor market tightness $\theta^i \equiv \frac{v^i}{s^i}$. We denote the probability of finding a worker by $q(\theta) \equiv m(\theta^{-1}, 1) = \frac{m(s,v)}{v}$ and the job finding probability as $p(\theta) \equiv \theta q(\theta) = \frac{m(s,v)}{s}$. Note that with a Cobb-Douglas matching function the elasticity of q with respect to θ, i.e. $-q' \frac{\theta}{q}$, is equal to the parameter ϵ (where by abuse of notation we define $q' \equiv \frac{\partial q}{\partial u}$).

Firms that successfully innovate in the first period can post contracts specifying wage payments for both periods. We do not allow firms to make counteroffers to offers from poaching firms.[2] We will always assume that workers cannot commit and thus may leave the firm in the second period if they receive an attractive offer from an imitating firm. On the firm side we analyze two different settings. In the benchmark case we assume that the innovating firm can fully commit to wages in the second period promised in period one. We then consider the case where the firm cannot commit to future wages.

3 Benchmark Model: The Case of Full Commitment

This section analyzes the model where firms can fully commit to wage contracts in period 1.

3.1 Workers' Decisions and Values

In period one workers receive an offer for a wage contract consisting of w_1 and w_2 for wages in period 1 and 2 respectively. In the second period Imitating firms offer a wage w^I, and workers hired from the unemployment pool (replacement market) are offered a wage w^U. Unemployed workers in period one receive a job offer with probability $p(\theta)$. Employed workers in period 2 receive an offer from an imitating firm with probability $p^I(\theta^I)$. Unemployed workers in period 2 get a job offer with probability $p^U(\theta^U)$.

For an employed worker in period 1, the value of a contract at the beginning of period 1 is given by

$$W_1 = w_1 + W_2,$$

where w_1 is the wage paid in period 1, and W_2, the value of the contract at the beginning of period 2, is defined as:

[2] In equilibriumwith full commitment they would not want to make a counteroffer in this model.

$$W_2 = w_2 + p^I(\theta^I)[w^I - w_2].$$

That is, the value of a worker at an innovating firm at the beginning of period 2 is the promised wage w_2 plus the the expected surplus from getting the chance to move to an imitating firm.

The value of an unemployed worker at the beginning of period 1 and period 2 then is

$$U_1 = p(\theta)[W_1 - U_2] + U_2 \tag{1}$$

and

$$U_2 = \ p^U(\theta^U)w^U, \tag{2}$$

respectively. An unemployed worker in period one picks the combination of $\{W_1, \theta\}$ with the highest value, and an unemployed worker in period 2 chooses $\{w^U, \theta^U\}$.

3.2 Firms' Decisions and Values

3.2.1 Imitating Firms

Given the market value of an employed worker, W_2, the imitating firm in period 2 chooses $\{w^I, \theta^I\}$ to maximize its profits:

$$\begin{aligned} V^I = & \max_{\theta^I, w^I} q^I(\theta^I)(y - w^I) - K^I \\ & s.to\ W_2 \leq w_2 + p^I(\theta^I)(w^I - w_2), \end{aligned} \tag{3}$$

As it is standard in competitive search models, the constraint entails how the firm forms rational expectations about the workers' trade-off between the job finding probability and the wage (implying the trade-off between the hiring probability and the wage). Anticipating equilibrium, this maximization problem implies a job finding probability function that is parametrized by the wage w_2 the incumbent firm offers: $\widehat{p}^I(w_2) \equiv p^I(\widehat{\theta}^I(w_2))$, where $\widehat{\theta}^I(w_2)$ is the optimal choice of θ^I if the outside wage is w_2 (for later use we will also denote $\widehat{w}^I(w_2)$ to be the wage that is optimal for the imitating firm given the wage the incumbent firm offers). This is discussed in more detail further below.

3.2.2 Innovating Firms

Innovating firms have to solve three different problems. First they have to recruit a worker in period 1 by setting a total value of the contract W_1. Second, they have to design a contract $\{w_1, w_2\}$ that optimally retains the worker, anticipating the behavior of imitating firms. Finally, in case they lose a worker in period 2 they have to set wage w^U to attract an unemployed worker. The last problem is a simple static problem. The firm has to maximize the value

$$V^U = \max_{\theta^U, w^U} \; q^U(\theta^U)(y - w^U)$$
$$s.to \; U_2 \leq p^U(\theta^U) w^U. \tag{4}$$

With full commitment the recruiting problem and the contract design problem can be formulated separately. Since the firm can commit to actions in period 2, the worker in period 1 only cares about the total value W_1 the contract implies at the beginning of period 1.

Given the a total value W_1 promised to the worker as well as θ^U and w^U determined in (4), the firm then solves the contracting problem by choosing a wage profile $\{w_1, w_2\}$ that balances costs and benefits of retaining a worker:

$$J(W_1) = \underset{w_1, w_2}{max} 2y - w_1 - w_2 + \widehat{p}^I(w_2)[q^U(y - w^U) - (y - w_2)]$$
$$s.to \, W_1 = w_1 + w_2 + \widehat{p}^I(w_2)[\widehat{w}^I(w_2) - w_2]$$
$$U_2 \leq W_2, \tag{5}$$

where $U_2 \leq W_2$ is the worker's participation constraint in period 2. The functions $\widehat{p}^I(w_2)$ and $\widehat{w}^I(w_2)$ make explicit that the firm has rational expectations of how the imitating firms will behave if they face an inside offer for the worker of w_2. More precisely, if a measure $\gamma > 0$ of firms offer w_2, then a submarket in the imitating firm market will form to respond to this wage. The imitating firms within this submarket face a trade-off between θ^I and w^I which depends on the equilibrium market value. Note, that this reasoning also holds off equilibrium and therefore can be used to form rational expectations about $\widehat{p}^I(w_2)$ and $\widehat{w}^I(w_2)$.

Finally, given the unemployment values U_1 and U_2, and the value of the optimal contract to the firm, $J(W_1)$, the firm has to solve the optimal recruiting problem:

$$V = \quad \underset{W_1, \theta}{max} \; q(\theta)J(W_1) - K - k$$
$$s.to \;\; U_1 \leq p(\theta)W_1 + (1 - p(\theta))U_2 \tag{6}$$

The above formulation includes the innovation stage, at which the firm pays a cost k to obtain an innovation with probability ρ.

3.3 Equilibrium

Definition 1 *An equilibrium is a vector of market tightness* $\{\tilde{\theta}, \tilde{\theta}^I, \tilde{\theta}^U\}$,*values for workers* $\{\tilde{W}_1, \tilde{W}_2, \tilde{U}_1, \tilde{U}_2\}$ *and values for firms* $\{\tilde{V}, \tilde{V}^I\}$, *a value function* $J(W_1)$, *functions* $\widehat{p}^I(w_2)$ *and* $\widehat{p}^I(w_2)$, *a contract* $\{\tilde{w}_1, \tilde{w}_2\}$ *and wages* $\{\tilde{w}^I, \tilde{w}^U\}$ *such that:*

1. *Optimal Contract and Profit Maximization:*
   (a) $\widehat{p}^I(w_2)$ *and* $\widehat{w}^I(w_2)$, $\{\tilde{w}_1, \tilde{w}_2\}$ *solve problem (5).*
   (b) $\{\tilde{\theta}^I, \tilde{w}^I\}$ *solve problem (3).*
   (c) $\{\tilde{\theta}^U, \tilde{w}^U\}$ *solve problem (4).*
   (d) $\{\tilde{W}_1, \tilde{\theta}\}$ *solve problem (6).*
2. *Zero Profit Conditions: At the equilibrium values,* $\tilde{V} = \tilde{V}^I = 0$, *where* V *and* V^I *are defined by (6) and (3), respectively.*
3. *The values* $\tilde{U}_1, \tilde{U}_2$ *are defined by equations (1) and (2), respectively.*
4. *Rational Expectations about imitating firm's behavior: Given* W_2 *and* w_2, $\widehat{w}^I(w_2) = w^{I*}$ *and is* $\widehat{p}^I(w_2) = p^I(\theta^{I*})$, *where* w^{I*} *and* θ^{I*} *is the solution to problem (3).*
5. *Market tightness in the replacement market:* $\tilde{\theta}^U = \frac{p(\tilde{\theta})p^I(\tilde{\theta}^I)}{1-p(\tilde{\theta})}$.

The first set of conditions involves the firms' maximization problems and the requirement that the profits are zero. Note that the market for replacement workers in the second period (with index U) has no free entry since only firms from the first period can post vacancies there. Therefore, in equilibrium the market tightness θ^U is predetermined by the market tightness in the other market (requirement 5). The fourth set of conditions pins down the rational expectations innovating firms have about how period two wages influence application behavior of employed workers and wage setting behavior of the imitating firms

3.4 Characterization of Equilibrium

In the following we focus on interior equilibria only. In particular we only look at the case where imitation occurs. To solve for equilibrium, we start from period 2 decisions. Consider first the replacement market in period 2. Substituting out the wage from the constraint in problem (4), we get

$$\max_{\theta^U} q^U(y - \frac{U_2}{\theta^U q^U}),$$

which gives

$$w^U = \epsilon^U y,$$

where ϵ^U is the elasticity of the job finding probability in the replacement market. This is the standard result in competitive search models that the total surplus (here y) is shared between the worker and the firm according to the elasticity of the job finding probability.

Next we turn to the imitating firms in period 2. Here it is more convenient to use the dual of problem (3) and impose zero profits:

$$\begin{array}{cc} \underset{\theta^I \geq 0, w^I \geq w_2}{max} & p^I(w^I - w_2) \\ s.to & q^I(y - w^I) = K^I. \end{array} \tag{7}$$

Substituting out w^I from the constraint and solving the first order condition this gives:

$$w^I = \epsilon^I y + (1 - \epsilon^I) w_2, \tag{8}$$

Given w_2, the zero profit condition implicitly determines θ^I :

$$q^I = \frac{K^I}{(1 - \epsilon^I)(y - w_2)}. \tag{9}$$

From here it follows that since $q^I(\theta^I)$ is decreasing in θ^I, θ^I is decreasing in w_2.

In order to characterize w_2, we have to derive the optimal long-term contract offered by innovating firms. We will first give an intuitive argument of the form of the contract, and then derive it algebraically for the interested reader. For any given NPV wage offer W_1, the firm will set w_2 so as to maximize the joint income of the worker and the firm. If this implies a high value of w_2, the firm will be compensated by a lower w_1 so that W_1 remains constant. Let $\overline{V} \equiv \max q^u(y - w^u)$. The joint period-2 income of the worker and the firm reads (since we assume rational expectations)

$$\begin{aligned} S_2 &= (1 - \widehat{p}^I(w_2))y + \widehat{p}^I(w_2)(w^I(w_2) + \overline{V}) \\ &= w_2 + \widehat{p}^I(w_2)(w_2^I - w_2) + y - w_2 + \widehat{p}^I(w_2)(\overline{V} - (y - w_2)) \end{aligned}$$

where $\widehat{p}^I(w_2)$ and $w^I(w_2)$ solve (7). Taking the derivative with respect to w_2, and utilizing that p^I and w^I solve (7) gives

$$\begin{aligned} \frac{dS_2}{dw_2} &= \frac{dp^I}{dw_2}(\overline{V} - (y - w_2) \\ &= 0 \text{ if } w_2 = y - \overline{V} \end{aligned}$$

It follows that the optimal period-2 wage for innovating firm is to set the wage equal to the shadow value of the worker, i.e., his output less the expected profit if loosing him. At this wage, the workers' search decision has no external effect on the firm as the worker is residual claimant, and as the competitive search equilibrium maximizes the utility of the searching worker it also maximizes joint income.

To solve the optimal contracting problem of the innovating firm we will also determine the derivative $\frac{d\hat{p}^I}{dw_2}$ Using $p^I \equiv \theta^I q^I$ we get from the zero profit condition:

$$\hat{p}^I = \frac{\hat{\theta}^I K^I}{(1-\epsilon^I)(y-w_2)}$$

Totally differentiating yields:

$$d\hat{p}^I = \frac{K^I}{(1-\epsilon^I)(y-w_2)}\left(\frac{\hat{\theta}^I}{y-w_2}dw_2 + d\hat{\theta}^I\right) = \frac{\hat{p}^I}{y-w_2}dw_2 + q^I d\hat{\theta}^I,$$

where the second equality uses equation (9) once again. Using the definition of the job finding probability we have that $dp^I = d(\theta^I q^I) = q^I(1+\frac{dq^I}{d\theta^I}\frac{\theta^I}{q^I})d\theta^I = q^I(1-\epsilon^I)d\theta^I$. Therefore we can reformulate the previous expression to:

$$d\hat{p}^I = \frac{\hat{p}^I}{y-w_2}dw_2 + \frac{1}{(1-\epsilon^I)}d\hat{p}^I,$$

which finally yields:

$$\frac{d\hat{p}^I}{dw_2} = -\frac{1-\varepsilon^I}{\varepsilon^I}\frac{\hat{p}^I}{y-w_2} < 0. \tag{10}$$

Now we are ready to solve first for the optimal contract and then the optimal recruiting in period one. Substituting out W_1 from the constraint in (5) and adding and subtracting w_2 we can write:

$$\max_{w_2} 2y - W_1 + \hat{p}^I\{q^U(y-w_2^U) - (y-w_2)\} + \hat{p}^I(w^I - w_2) \tag{11}$$

The third term represents the net payoff to the firm from replacing a worker. The last term is the net payoff to the worker from leaving to the imitating firm. Note that this last term is identical to the objective of a worker in the imitating firm's problem (see problem (7) above). Takin the FOC we get:

$$\frac{d\hat{p}^I}{dw_2}\{q^U(y-w_2^U)-(y-w_2)\}+\hat{p}^I+\frac{d}{dw_2}[\hat{p}^I(w^I-w_2)]=0$$

Since the imitating firms already maximize the expression $\hat{p}^I(w^I - w_2)$ (w.r.t. w^I), by the envelope theorem, only the the direct effect of a change in w_2 matters, i.e.

$$\frac{d}{dw_2}[\hat{p}^I(w^I-w_2)]=-\hat{p}^I.$$

This direct effect is the change in the wagė w_2, which is just a redistribution between the worker and the firm, so that we are left with the first term in the FOC. We can also achieve this result by plugging in w^I from 8

$$\frac{d\hat{p}^I}{dw_2}\{q^U(y-w_2^U)-(1-\epsilon^P)(y-w_2)\}+\hat{p}^I(1-\epsilon^P)=0, \tag{12}$$

and then substitue out the derivative from (10) to finally get

$$w_2=y-q^U(y-w^U). \tag{13}$$

This expression says that the worker in period 2 gets all the surplus from period 2 net of the payoff the firm would get from replacing the worker. The expression does not depend on $\frac{d\hat{p}^I}{dw_2}=\frac{dw^I}{dw_2}\frac{d\hat{p}^I}{dw^I}$ since the indirect effect of changing w^I is already maximized out by the optimization problem of the imitating firms. W_1(or relatedly w_1) is determined as a residual given the solution for w_2. Intuitively, the optimal contracting problem with commitment as given in (11) amounts to maximizing the joint net surplus of a worker and a firm in period 2. The optimal decision is to give the full surplus to the worker in period 2 (equation (13) implies zero net surplus for the firm in period 2), and extract surplus only in period 1 through w_1.

The following lemma confirms the validity of omitting the participation constraint in the proceeding optimization problem:

Lemma 1 *The period 2 participation constraint in problem (5) does not bind, i.e.* $U_2 < W_2$.

Proof. Inserting for equilibrium wages, $U_2 < W_2$ can be written:

$$p^U\epsilon^U-q^U(1-\epsilon^U)(1-p^I\epsilon^I)<1,$$

which clearly holds since the lower bound of p^I is zero and (p^U, q^U) cannot simultaneously be at the upper bound of 1. ∎

Next, we solve the optimal recruiting problem by substituting out the constraint in problem in (6) and using the definition of $J(W_1)$ in (5). Taking the first order condition with respect to θ and solving for U_1 this gives

$$U_1 = q'[2y + p^I\{q^U(y - w^U) - (y - w^I)\}] + (1 - q')U_2.$$

Substituting this back into the constraint and solving for W_1 we have

$$W_1 = \epsilon[2y + p^I\{q^U(y - w^U) - (y - w^I)\}] + (1 - \epsilon)U_2.$$

Using the definition for U_2 in (2), the FOC in (12), as well as the results for wages w^U, w^I and w_2, we get :

$$W_1 = \epsilon[2y + p^I(w^I - w_2)] + (1 - \epsilon)p^U w^U.$$

Solving the constraint in (5) for w_1 and using the result for w_2, w^I, and W_1 we obtain for the first period wage

$$w_1 = y\{2\epsilon - 1 + (1 - \epsilon)\epsilon^U p^U + (1 - \epsilon^U)q^U[1 - \epsilon^I(1 - \epsilon)p^I]\}.$$

A priori it is difficult to determine whether w_1 will be positive in equilibrium. A sufficient condition for this is that $\epsilon \geq 1/2$. Furthermore, it is not immediately clear whether the wage profile $\{w_1, w_2\}$ is increasing or decreasing:

$$w_2 - w_1 = y\{2(1 - \epsilon) - (1 - \epsilon)\epsilon^U p^U - (1 - \epsilon^U)q^U[2 - \epsilon^I(1 - \epsilon)p^I]\}.$$

By substituting out all the wages, the 0-profit conditions can now be written only in terms of the parameters. We summarize the characterization of equilibrium in the following proposition:

Lemma 2 *1. An interior equilibrium with full commitment is characterized by a pair of $\{\theta, \theta^I\}$ that solves the following two equations:*

$$V^*(\theta, \theta^I) = q(1-\epsilon)y[2+\epsilon^I(1-\epsilon^U)p^I q^U - \epsilon^U p^U] - K - k = 0, \quad (14)$$

$$V^{I*}(\theta^I, \theta) = q^I q^U y(1 - \epsilon^U)(1 - \epsilon^I)^2 - K^I = 0, \quad (15)$$

and where θ^U is defined as above.

2. Wages are given by

$$w^U = \epsilon^U y,$$

$$w^I = y[1 - q^U(1 - \epsilon^U)(1 - \epsilon^I)],$$

$$w_1 = y\{2\epsilon - 1 + (1-\epsilon)\epsilon^U p^U + (1-\epsilon^U)q^U[1-\epsilon^I(1-\epsilon)p^I]\},$$

and

$$w_2 = y[1 - q^U(1-\epsilon^U)].$$

The following lemma describes properties of equations (14) and (15) that are useful to establish existence of equilibrium.

Lemma 3 *1. $V^*(\theta, \theta^I)$ is strictly decreasing in θ and strictly positive for $K + k$ and θ small enough.*

2. $V^{I*}(\theta^I, \theta)$ is strictly decreasing in both θ^I and θ ,and strictly positive for K^I and θ^I small enough.

Proof. The results follow from the partial derivatives and from inspection of the sign of $V^*(\theta, \theta^I) + K + k$ $(V^{I*}(\theta^I, \theta) + K^I)$ for small θ (θ^I). ■

Lemma 4 *A unique equilibrium exists for $K + k$ and K^I small enough.*

Proof. Using the proceeding lemma, an equilibrium can easily be constructed in the following way: Fix a value of θ and a value of K^I and solve for θ^I, such that $V^{I*}(\theta^I, \theta) = 0$. If no solution exists lower K^I (and/or θ). Given the resulting $\{\theta^*, \theta^{I*}\}$, we can find a $K + k$ such that $V^*(\theta^*, \theta^{I*}) = 0$ (note that this is possible for any θ^I since $V^*(\theta, \theta^I) + K + k \geq 0$ for any θ^I). To show uniqueness we first write the equilibrium value of an innovating firm as: $V^*(\theta, \theta^I(\theta))$, where $\theta^I(\theta)$ is the solution of $V^{I*}(\theta^I, \theta) = 0$. The lemma above established that $\theta^I(\theta)$ is strictly decreasing. It is enough to show that $\frac{dV^*(\theta,\theta^I(\theta))}{d\theta} < 0$. θ enters directly into $q(\theta)$, which is decreasing in θ. Thus we can focus on the ex-post value of the firm:

$$2y + \hat{p}^I\{V^U - (y - w_2)\} + \hat{p}^I(w^I - w_2) - U_2,$$

where $V^U = (y - w^U)q^U$ and $U_2 = w^U p^U$ are the firm's and the worker's value from the replacement market, respectively. Note that the term $\hat{p}^I(w^I - w_2)$ is identical to the (dual) objective of the imitating firm. By the envelope theorem only the direct effect of θ on $\hat{p}^I(w^I - w_2)$ (and not the effect via θ^I) matters. The direct effect, however is only the effect through w_2 which is just a transfer between the worker and the firm that cancels out. Thus, it is enough to show that

$$\frac{d}{d\theta}[p^I V^U - U_2] < 0.$$

Since higher θ decreases θ^I, the probability of being poached p^I decreases with θ. It remains to argue that the replacement value V^U also has to decrease (and correspondingly U_2 has to increase). Note, that V^U decreases directly in θ, and increases indirectly through lowered θ^I. However, θ^I decreases in θ only because a higher θ decreases the value of replacement which implies a higher wage w_2 and lower profits for the imitating firms. In other words, if lower θ^I would outweigh the effect of higher θ on V^U, profits for imitating firms should be higher, and θ^I would not decrease in the first place. Similarly, U_2 goes up for the same reason. Taking everything together the result obtains. ■

4 Efficiency

In this section we determine the constrained efficient allocation and compare that with the equilibrium allocation of the benchmark case.

4.1 Efficient allocation

As it is usual in settings like this with risk-neutral agents, we measure welfare in our economy by total net output, where search frictions and vacancy costs are taken as given. The social planners chooses $\{e_1^*, e_2^*\}$ to maximize

$$\mathcal{F} = e_1[qy - k - K] + e_1 qy[1 - p^I + p^I(q^U - \frac{K^U}{y})] + e_2[q^I y - K^I]. \quad (16)$$

subject to the resource constraint of total unemployed workers in period 1 of $u_1 \equiv 1$ and the law of motion $\theta^U = \frac{p(\theta)p^I(\theta^I)}{1-p(\theta)}$.

Since our agents are risk neutral this is equivalent to a Pareto-optimal allocation where the ex-ante utility of all workers is maximized and all workers have the same welfare weight.

It is more convenient to reformulate the social planner's problem (SPP) in terms of choosing θ and θ^I. Then the SPP can be written as

$$\max_{\theta,\theta^I} F(\theta, \theta^I) = \max_{\theta,\theta^I} y\{p(\theta)[2 + p^I(\theta^I)q^U(\frac{p^I p}{1-p}) - \overline{K}^I\theta^I] - \overline{K}\theta\},$$

subject to $u_1 \equiv 1$, and $\theta^U = \frac{p^I p}{1-p}$. We have used the fact that $\theta = \frac{e_1}{u_1} = e_1$, $\theta^I = \frac{e_2}{p}$, and have defined $\overline{K}^I \equiv \frac{K^I}{y}$ and $\overline{K} \equiv \frac{k+K}{y}$.

The first order condition with respect to θ^I is

$$F_{\theta^I} = yp[p^{I\prime} q^U + p^I \frac{dq^U}{d\theta^I} - \overline{K}^I] = 0,$$

where $p^{I\prime} = q^I(1-\epsilon^I)$, and $\frac{dq^U}{d\theta^I} = -\frac{q^{U\prime}}{(\theta^U)^2}\frac{d\theta^U}{d\theta^I} = -\frac{\epsilon^U q^U p q^I(1-\epsilon^I)}{\theta^U(1-p)} = -\frac{\epsilon^U q^U q^I(1-\epsilon^I)}{p^I}$. The first order condition can be rearranged to:

$$\begin{gathered}[q^I(1-\epsilon^P)q^U - \epsilon^U q^U q^I(1-\epsilon^I) - \overline{K}^I] = 0 \\ \Longleftrightarrow \\ q^I = \frac{K^I}{yq^U(1-\epsilon^I)(1-\epsilon^U)}.\end{gathered} \tag{17}$$

The first order condition with respect to θ is

$$F_\theta = y\{p'[2 + p^I q^U - \overline{K}^I \theta^I] + pp^I \frac{dq^U}{d\theta} - \overline{K}\} = 0,$$

where $p' = q(1-\epsilon)$ and $\frac{dq^U}{d\theta} = -\frac{q'^U}{(\theta^U)^2}\frac{d\theta^U}{d\theta} = -\frac{\epsilon^U q^U p^I(1-\epsilon)q}{\theta^U(1-p)^2} = -\frac{\epsilon^U q^U}{p}\frac{(1-\epsilon)q}{(1-p)}$. The first order condition can be rearranged to give:

$$\begin{gathered}q(1-\epsilon)[2 + p^I q^U - \overline{K}^I \theta^I] - p[\frac{p^I \epsilon^U(1-\epsilon)q^U q}{p(1-p)}] - \bar{K} = 0 \\ \Longleftrightarrow \\ q = \frac{(K+k)}{y(1-\epsilon)[2+p^I q^U \epsilon^I(1-\epsilon^U) - \epsilon^U p^U]},\end{gathered} \tag{18}$$

where we have used $K^I = yq^I q^U(1-\epsilon^I)(1-\epsilon^U)$ from (17).

Sufficient conditions for a maximum of the SPP are established in subsection 7.1 of the appendix.

4.2 Efficiency in the Benchmark Model

Comparing the conditions of the benchmark equilibrium with the conditions for the efficient allocation, we immediately get the following proposition:

Proposition 1 *The equilibrium with commitment is efficient.*

Proof. This follows immediately from that the equilibrium 0-profit conditions, (14) and (15), are the same as the first order conditions of the social planner problem, (17) and (18). ■

Efficiency in the commitment case can be explained by contracting under full commitment and competitive search. The innovating firm designs the wage contract so as to maximize joint surplus over both periods. Given the optimal wage contracts, there are no externalities from the workers' on-the-job search on their employers. Hence, competitive search equilibrium, that maximizes the income of the worker, also maximizes the joint income of the worker and the firm. Furthermore, the social and the private value in period 2 of a firm that innovated in period 1 coincide. Since the search frictions in period 1 does not create distortions, it follows that the social and private value of creating an innovating firm in period one coincides, and efficiency prevails. Although the firm pays for the innovation, and the worker gains from learning the innovation in period 2 because of on-the-job search, the firm is able to extract this gain by paying a low period 1 wage.

In the first period the firm sets W_1 to allocate surplus between the firm and the worker. Firms can freely set the period one wage to optimally trade off the hiring probability and the wage costs. The worker accepts a low wage in the first period, as the firm can credibly promise a higher wage in the second period. Hence, optimal contracting implies separation between surplus maximization and setting the surplus share between worker and firm.

The competitive search mechanism ensures efficient hiring in the first period. In competitive search firms maximize profits given the market unemployment value. The firm rationally anticipates the trade-off between wage and hiring probability. In equilibrium all workers get the same value in all firms. If a small measure of firms would deviate from any given equilibrium wage to a higher wage, they will attract more workers. However, the higher probability will be completely offset by the higher wage since the worker's value in equilibrium is given for the firm.

Due to zero profits, in effect all the surplus (including the surplus coming from the continuation possibilities in the imitating firm and the firms that have replaced a worker) goes to the worker. The problem is therefore equivalent to maximizing the worker's ex-ante utility, which in turn is equivalent to the planner's objective.

To sum up, the optimal decision for the firm is to give the full surplus to the worker in period 2, and extract surplus only in period 1 through w_1. Joint surplus maximization implies that also the worker's surplus is maximized, i.e. the worker will search optimally, which is efficient from the social planner's point of view.

5 Model with No Commitment

The fact that the equilibrium in the benchmark case is efficient case rests on the firms' commitment to future wages. Empirically however, such wage commitments seem to be a strong assumption, since firms are often allowed to fire workers (at low costs) [reference].

This section analyzes the model where firms can only commit to the current wage. We call this the no-commitment case. The model is identical to the benchmark case except that firms set w_2 only once period 2 has arrived. Workers learn about w_2 at the beginning of period 2. As before, workers choose to stay with the firm or quit to apply for jobs in the replacement market.

5.1 Innovating firms

The only difference to the benchmark case is that the firms instead of offering a contract $\{w_1, w_2\}$ in period one, firms offer a total value W_1, which anticipates the optimal wage w_2 that will be set in period 2. In contrast to the benchmark case the two problems, recruiting and retention, are no longer independent. Instead, the problem has to be solved backwards. Starting in the second period, the firm chooses w_2 in order to balance the costs and benefits of retaining the worker:

$$\begin{aligned} V_2 &= \max_{w_2} [y - w_2 + \hat{p}^I(w_2)\{q^U(y - w^U) - (y - w_2)\}] \\ s.to \quad & U_2 \leq W_2, \end{aligned} \tag{19}$$

where the participation constraint says that the value of being employed at the beginning of period 2 needs to be at least equal to the value of being unemployed. If not, the worker quits.[3]

Turning to period 1, the firm offers a total value $W_1$to the worker, that includes the anticipated wage w_2.[4] The firm's recruitment problem in period 1 is then similar to the corresponding problem in the benchmark case:

$$\begin{aligned} V &= \max_{W_1, \theta} q(\theta)[2y - W_1 + \hat{p}^I(w_2)\{q^U(y - w^U) - (y - w^I)\}] - K - k \\ s.to \quad & U_1 \leq p(\theta)W_1 + (1 - p(\theta))U_2. \end{aligned} \tag{20}$$

5.2 Equilibrium

We can now define equilibrium in a similar way as before.

Definition 2 *An equilibrium is a vector of market tightness* $\{\tilde{\theta}, \tilde{\theta}^I, \tilde{\theta}^U\}$,*values for workers* $\{\tilde{W}_1, \tilde{W}_2, \tilde{U}_1, \tilde{U}_2\}$ *and values for firms* $\{\tilde{V}, \tilde{V}_2, \tilde{V}^I\}$*, functions* $\widehat{p}^I(w_2)$ *and* $\widehat{p}^I(w_2)$*, and wages* $\{\tilde{w}_2, \tilde{w}^I, \tilde{w}^U\}$ *such that:*

[3] Though we do not explicitly consider the option of firing a worker, one can interpret a low or zero value of w_2 as a lay-off.

[4] Here we solve out w_1 from the problem, since it is redundant. Instead we only focus on W_1 to describe the innovating firm's problem in period 1.

1. *Optimal Contract, Profit Maximization and Zero Profit Conditions:*

   (a) *Given* $\tilde{W}_2$, $\tilde{U}_2$, *and functions* $\widehat{p}^I(w_2)$ *and* $\widehat{p}^I(w_2)$, $\tilde{w}_2$ *solves problem (19).*

   (b) *Given* $\tilde{W}_2$, $\{\tilde{\theta}^I, \tilde{w}^I\}$ *solve problem (3).*

   (c) *Given* $\tilde{U}_2$, $\{\tilde{\theta}^U, \tilde{w}^U\}$ *solve problem (4).*

   (d) *Given* $\tilde{U}_1$ *and* $\tilde{U}_2$, $\{\tilde{W}_1, \tilde{\theta}\}$ *solve problem (20).*

   (e) *At the equilibrium values,* $\tilde{V} = \tilde{V}^I = 0$, *where* V *and* V^I *are defined by (20) and (3), respectively.*

2. *Optimal Application by Workers: Given* $\tilde{U}_1$, $\tilde{U}_2$, $\tilde{W}_2$, *for any* W_1, w^I, w^U *the following conditions hold with complementary slackness:*

   (a) $\tilde{U}_1 \geq p(\theta)[W_1 - \tilde{U}_2] + \tilde{U}_2$ *and* $\theta \geq 0$,

   (b) $\tilde{W}_2 \geq \tilde{w}_2 + p^I(\theta^I)[w^I - \tilde{w}_2]$ *and* $\theta^I \geq 0$,

   (c) $\tilde{U}_2 \geq \ p^U(\theta^U)w^U$ *and* $\theta^U \geq 0$.

3. *Rational Expectations about imitating firm's behavior: Given* W_2 *and* w_2, $\widehat{w}^I(w_2) = w^{I*}$ *and is* $\widehat{p}^I(w_2) = p^I(\theta^{I*})$, *where* w^{I*} *and* θ^{I*} *is the solution to problem (3).*

4. *Market tightness in the replacement market:* $\tilde{\theta}^U = \frac{p(\tilde{\theta})p^I(\tilde{\theta}^I)}{1-p(\tilde{\theta})}$.

Note that here we eliminated w_1 from the problem. It can computed from the constraint in (5) given W^I.

5.3 Characterization of Equilibrium

We start by solving the period 2 wage setting of the innovating firms. First, we maximize ignoring the participation constraint in (19):

$$\max_{w_2}[y - w_2 + \hat{p}^I(w_2)\{q^U(y - w^U) - (y - w_2)\}],$$

which gives the FOC

$$p^I - 1 + \frac{d\hat{p}^I}{dw_2}\{q^U(y - w^U) - (y - w_2)\} = 0. \tag{21}$$

Substituting in the expression for $\frac{d\hat{p}^I}{dw_2}$ from (10) and solving for w_2 gives

$$w_2 = y(1 - \frac{p^I(1-\epsilon^I)}{p^I - \epsilon^I}q^U(1-\epsilon^U)), \tag{22}$$

where $\frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I} > 1$ if $p^I - \epsilon^I > 0$.

Note that if in equilibrium $p^I \leq \epsilon^I$, then the marginal benefits of lowering p^I are exceeded by the marginal cost of increasing w_2. The firm would then set w_2 equal to the lower bound implied by the participation constraint in problem (19). To have an interior solution with w_2 above the lower bound, it is required that p^I (and thus θ^I) is high enough so that $p^I > \epsilon^I$. Further, since $-\frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I}$ is increasing in p^I (and thus in θ^I), we have from (22) that w_2 is increasing in θ^I for $p^I - \epsilon^I > 0$. Therefore we need θ^I to be high enough in equilibrium to satisfy both the participation constraint in (19) and $p^I > \epsilon^I$. Substituting out all interior equilibrium variables from $U_2 \leq W_2$, an interior equilibrium requires:

$$\epsilon^U p^U + (1-\epsilon^U)q^U(1-\epsilon^I)\frac{p^I(1-\epsilon^I p^I)}{p^I-\epsilon^I} \leq 1, \tag{23}$$

where by interiority of w_2 it is required that $p^I > \epsilon^I$. Further below in Lemma (7) we establish a sufficient condition for (23) to hold. We focus on interior equilibria where this participation condition (23) is satisfied.

Except for the wage setting of the period 2 wage w_2, all other problems are solved the same way as in the commitment case. We summarize the equilibrium characterization in the following proposition:

Proposition 2 *1. An interior equilibrium with no commitment is characterized by a pair of $\{\theta, \theta^I\}$ that solves the following two equations*

$$q^I(\theta^I) = \frac{K^I}{q^U \frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I}(1-\epsilon^I)(1-\epsilon^U)y}, \tag{24}$$

$$q(\theta) = \frac{K+k}{(1-\epsilon)y[2+(1-\epsilon^U)p^I q^U\{1-\frac{p^I(1-\epsilon^I)^2}{p^I-\epsilon^I}\} - p^U\epsilon^U]}, \tag{25}$$

and satisfies the equilibrium participation and interiority constraints

$$\epsilon^U p^U + (1-\epsilon^U)q^U(1-\epsilon^I)\frac{p^I(1-\epsilon^I p^I)}{p^I-\epsilon^I} \leq 1,$$
$$\epsilon^I < p^I.$$

2. *Wages and value W^I are given by:*

$$
\begin{aligned}
w^U &= \epsilon^U y, \\
w^I &= y[1 - \frac{p^I(1-\epsilon^I)^2}{p^I - \epsilon^I} q^U (1-\epsilon^U)], \\
W^I &= y\{2\epsilon + (1-\epsilon)\epsilon^U p^U + \epsilon\epsilon^I(1-\epsilon^U)\frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I} p^I q^U\}, \\
w_2 &= y(1 - \frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I} q^U (1-\epsilon^U)).
\end{aligned}
$$

The following lemma establishes properties of the 0-profit condition for imitating firms.

Lemma 5 *Given any θ, and K^I low enough there is a unique interior solution for θ^I such that $p^I > \epsilon^I$. For given θ, a higher K^I implies a lower θ^I. Moreover, θ^I is decreasing in θ.*

Proof. The LHS of (24) is strictly decreasing in θ^I, going from 1 to 0 as θ goes from 0 to ∞. With our Cobb-Douglas specification, the terms in the denominator of the RHS involving θ^I can be written as:

$$
q^U \frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I} = A\left(\frac{p^I p}{(1-p)}\right)^{-\epsilon} \frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I} = A\left(\frac{p}{(1-p)}\right)^{-\epsilon} \frac{(p^I)^{1-\epsilon}(1-\epsilon^I)}{p^I-\epsilon^I}.
$$

In the relevant range, we have $\frac{d\left[\frac{(p^I)^{1-\epsilon}(1-\epsilon^I)}{p^I-\epsilon^I}\right]}{dp^I} < 0$. Thus, for given θ, the RHS is strictly increasing in θ^I.The RHS ranges from 0 to $\bar{c} \equiv K^I/[q^U \frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I}(1-\epsilon^I)(1-\epsilon^U)y] > 0$ as θ^I goes from $(p^I)^{-1}(\epsilon^I)$ (where $(p^I)^{-1}$ is the inverse function of p^I) to ∞, which ensures a unique intersection point if K^I is small enough.

The comparative statics with respect to K^I follow from the fact that the LHS is decreasing and the RHS is increasing. Moving up the RHS moves the intersection point to the left, i.e. to a lower θ^I. The last statement follows from the fact that the RHS is strictly increasing in θ, which implies that the RHS schedule intersects with the LHS at a lower θ^I if θ is higher. ■

The intuition for the θ^I being decreasing in θ is the following. First, remember that $\theta^I \equiv \frac{e_2}{e_1 q}$. There are two effects. First, more entry in period 1 gives more opportunities for imitating firms to poach a worker. However, when more firms enter in period 1 the replacement market is tighter. This induces innovating firms to set a higher period 2 wage to retain their worker. This second effect of a higher w_2 reduces the profits of entering to imitate since the firms need to offer workers a higher wage. The outcome of the two effects is that the poaching tightness decreases in period 1 entry.

The following Lemma analyses properties of the 0-profit condition for innovators in equilibrium.

Lemma 6 *For given θ^I there exists at least one θ such that the 0-profit condition for innovators (25) holds. Moreover, at a stable equilibrium solution π must cut the x-axis from above, i.e. π is locally decreasing around the equilibrium.*

Proof. Rewrite (25) as:

$$\pi(\theta) = q(1-\epsilon)y[2 + (1-\epsilon^U)p^I q^U\{1 - \frac{p^I(1-\epsilon^I)^2}{p^I - \epsilon^I}\} - p^U\epsilon^U] - K - k = 0$$

First note that $1 - \frac{p^I(1-\epsilon^I)^2}{p^I-\epsilon^I} \gtreqless 0$, and that $\frac{d\left(\frac{p^I(1-\epsilon^I)^2}{p^I-\epsilon^I}\right)}{dp^I} < 0$. As $\theta \to 0$, $q(\theta) \to 1$, whereas the expression in parentheses goes to a positive constant $\bar{c} = (1-\epsilon)y[2 + (1-\epsilon^U)p^I q^U\{1 - \frac{p^I(1-\epsilon^I)^2}{p^I-\epsilon^I}\} - p^U\epsilon^U]$. Thus $\lim_{\theta\to 0}\pi > 0$ given that $k + K$ is small enough (i.e. the economy is productive). Similarly, for $\theta \to \infty$, $q(\theta) \to 0$, whereas the expression in parentheses goes to a positive constant $\underline{c} = (1-\epsilon)y[2-\epsilon] < \bar{c}$. Thus $\pi(\theta)$ will be negative at high a θ. Thus by continuity of the profit function $\pi(\theta)$ a θ with 0 profits, where π cuts the 0 line from above exists. Note, that an intersection where π is increasing cannot be a stable equilibrium since then more entry would increase profits, violating the free entry condition. Therefore, at any stable equilibrium, the profit function is locally decreasing. ■

The previous lemma immediately implies the following relationship between vacancy costs and firm entry:

Corollary 1 *A marginal decrease in K marginally increases θ.*

Finally, we can prove existence of a no-commitment equilibrium given that vacancy and innovation costs are small enough. In contrast to the commitment equilibrium here it is non-trivial to ensure the participation constraint for employed workers in period 2 holds.

Lemma 7 *Given some $K + k$ and K^I small enough there exists an interior no-commitment equilibrium.*

Proof. As in the proof of existence in the commitment case, it is clear that we can find $K+k$ and K^I small enough such that there is a solution to the system of 0-profit conditions. Here, we have to ensure that also the participation constraint (23) is satisfied. If we set K^I so low that $p^I = 1$, then the participation constraint becomes equivalent to the commitment case:

$$\epsilon^U p^U + (1-\epsilon^U) q^U (1-\epsilon^I) \leq 1,$$

and since either p^U or q^U has to be strictly less than 1, so the inequality has to hold strictly. By continuity of (23) in p^I for $p^I > \epsilon^I$, there is a neighborhood to the right of K^I such that the inequality is still strict even if $p^I < 1$. ■

5.4 Comparison of Equilibrium with and without Commitment

We show in the following that commitment implies higher entry to innovation and relatively less poaching (lower θ^I) than an equilibrium without commitment on the firm side.

The following lemma establishes the difference in period 2 wages and poaching tightness between the two cases, for a given level of period 1 entry.

Lemma 8 *For any given θ, the θ^I is higher and w_2 is lower in the no-commitment equilibrium than in the commitment equilibrium.*

Proof. To see the difference in θ^I in the two cases, compare the 0-profit conditions of the poaching firms for the commitment case and no-commitment case, respectively:

$$\begin{aligned} q^I &= \frac{K^I}{q^U(1-\epsilon^I)(1-\epsilon^U)y} \\ q^I &= \frac{K^I}{q^U \frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I}(1-\epsilon^I)(1-\epsilon^U)y}. \end{aligned}$$

The LHS of both equations are a decreasing function, whereas the RHS of both functions are an increasing function in θ^I for given θ (both q^U and $\frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I}$ are decreasing as long as $p^I > \epsilon^I$, where the latter is an equilibrium condition). From before we have that $\frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I} > 1$ for any θ^I that satisfies $p^I > \epsilon^I$. Therefore the RHS of the commitment case has to be above the RHS of no-commitment case, leading to a higher θ^I in the no-commitment case.

Further, from $q^I = \frac{K^I}{(1-\epsilon^P)(y-w_2)}$ it directly follows that higher θ^I implies lower w_2. ■

Next we establish the difference in equilibrium outcomes between the two cases. We superscript commitment variables with C and no-commitment variables with NC when necessary.

Proposition 3 *The no-commitment equilibrium has higher θ^I and lower θ than the commitment equilibrium.*

Proof. Consider an innovating firm. For any given θ we know from the Lemma above that firms under no-commitment will set lower w_2. Since w_2 was the unique profit maximizer under commitment, we have that for any given θ profits are lower under no-commitment. From Lemma 4 we know that the profit line $\pi(\theta) \equiv V^*(\theta, \cdot)$ is downward sloping under commitment. Hence, the fact that profits are lower under no-commitment for any given θ, i.e. $\pi^{NC}(\theta) < \pi^C(\theta)$, together with Lemma 4 , implies that $\theta^{NC} < \theta^C$. Further, we need to show that in fact $\theta^{IC} < \theta^{INC}$. The direct effect coming from a lower w_2 in the no-commitment case is that θ^I is higher. It remains to show that any indirect effect through lower θ doesn't countervail the direct effect. This follow immediately for Lemma 5 which states that lower θ implies higher θ^I. Thus θ^{INC} must be higher than θ^{IC}. ■

Note that we do not know whether more firms enter to poach in no-commitment case. We know that the poaching market is tighter. However, we do not know whether this comes from that more firms enter to poach or from that fewer firms enter to innovate in period 1 as $\theta^I \equiv \frac{e_2}{e_1 \rho q}$.

5.5 Inefficiency of the No-Commitment Case

Since the conditions characterizing the no-commitment equilibrium differ from the ones for the commitment case, it is clear that the equilibrium without commitment cannot be efficient.

Corollary 2 *The no-commitment allocation is not constrained efficient.*

Proof. This follows from that the no-commitment allocation is different than the equilibrium allocation, Proposition 3, and that the commitment allocation is efficient, Proposition 1**,** together with that the efficient allocation is unique due to the global properties of the welfare function in the relevant parameter ranges, Lemma 12**.** ■

The intuition for the proposition is the following. Under commitment the joint surplus of the innovating firm and the matched worker is maximized: firms set w_2^C to perfectly align worker incentives with the firm's, and the firms extract rents by setting a low w_1.Under no-commitment, firms have incentives to lower w_2 from w_2^C. On-the-job search by the worker creates a negative externality of the firm. Competitive search equilibrium maximizes the workers' expected income without taking the externality on the employers into account. Hence w^I is too low and θ^I is too high compared with the values that maximize the joint surplus of the employer and the employee. Lower joint surplus implies lower ex-ante profits. Thus, entry to innovation is lower under no-commitment. Last, the uniqueness of the efficient allocation implies that any deviation must lead to lower welfare.

5.6 Policies in the No-commitment Case

In this section we analyze policies that may increase welfare. It is not likely that we can get the efficient result with only one policy instrument; policy maker needs instruments that can influence e_1 and e_2 independently. We could get efficiency by setting the wage directly, though we do not allow for direct price setting instruments. We first solve for two instruments, then we analyze policy if the set of available policy instruments is limited.

The policies are financed through a lump sum transfer to/from all workers (also unemployed). It is a 0-1 decision to work, which is not distorted by the transfer.

5.6.1 Two Policy Instruments

Define z as subsidy to vacancy cost in period 1 period 1 and τ as a tax vacancy cost in period 2.

Corollary 3 *Two policy instruments, a tax on imitating firms together with a subsidy on innovating firms, can restore efficiency.*

Proof. Follows immediately from Corollary 4. ■

A subsidy induces entry in period 1. The new entries directly reduce tightness in the poaching market. The poaching tightness also changes from increased entry to imitate in period 2. We know that the total effect is that we know that for any θ the θ^I is higher in the no-commitment case than in the commitment case. At the θ^* it is then easy to see that there is too much entry to imitation since $\theta^I = \frac{e_2}{p}$. A tax can then reduce entry to imitation Though note that w_2 is still lower than w_2^C. This is possible due to that the FOC of the poaching firms is 'shifted' by the tax; $q^I = \frac{K^I+\tau}{(1-\epsilon^I)(y-w_2)}$.

5.6.2 Tax on Imitation

Here we analyze the case where the policy maker only can tax imitation, i.e. subsidy to innovation is not available.

First, we establish the effects of a tax on entry to innovate and entry to imitate.

Proposition 4 *A tax on imitating firms will reduce the number of poaching firms and will decrease the number of innovating firms given that scaling factor of the matching function is $A^U \geq \epsilon^{\epsilon-1}$, and $\epsilon = \epsilon^I = \epsilon^U$.*
Proof. *The proof consists of three steps. First we show that a tax on poaching will reduce poaching. Then we establish that profits of an innovating firm increases with θ^I given θ. Finally, we show that entry to innovating has to go down if θ^I decreases. The first two steps are given in the following two lemmata:*

Lemma 9 *A tax on vacancy costs K^I lowers θ^I.*
Proof. *From the LEMMA [0-profit poaching] we know that the LHS of the expression*

$$q^I = \frac{K^I}{q^U \frac{p^I(1-\epsilon^I)}{p^I-\epsilon^I}(1-\epsilon^I)(1-\epsilon^U)y}$$

is strictly decreasing, whereas the RHS is strictly increasing in θ^I. Thus, an increase in K^I will shift the RHS upwards, thereby decreasing θ^I for a given θ. ■

■

Lemma 10 *Let the scaling factor of the matching function $A^U \geq \epsilon^{\epsilon-1}$, and $\epsilon = \epsilon^I = \epsilon^U$. For given θ, an increase in θ^I increases the innovating firm's profits.*

Proof. First, denote $B \equiv \frac{p}{1-p}$.Gross profits can be written as:

$$\pi = q(1-\epsilon)y[2 + (1-\epsilon)p^I q^U(1 - \frac{p^I(1-\epsilon)^2}{p^I-\epsilon}) - p^U\epsilon]$$

Since $\frac{dp^I}{d\theta^I} > 0$, it is enough to show:

$$\frac{\partial}{\partial p^I}\left((1-\epsilon)p^I q^U(1 - \frac{p^I(1-\epsilon)^2}{p^I-\epsilon}) - p^U\epsilon\right) > 0,$$

which can be written

$$\frac{\partial}{\partial p^I}\left(p^I q^U(1-\epsilon - \frac{p^I(1-\epsilon)^3}{p^I-\epsilon} - B\epsilon)\right) > 0.$$

The derivation yields that we need to show

$$(1-\epsilon)(p^I)^{-\epsilon}A^U B^{-\epsilon}[1-\epsilon-\frac{p^I(1-\epsilon)^3}{p^I-\epsilon}-B\epsilon]+(p^I)^{1-\epsilon}B^{-\epsilon}A^U(1-\epsilon)^3\frac{\epsilon}{(p^I-\epsilon)^2} > 0,$$

rearranged to

$$(1-\epsilon)(p^I-\epsilon)^2-(p^I-\epsilon)p^I(1-\epsilon)^3-B\epsilon(p^I-\epsilon)^2+(1-\epsilon)^2\epsilon p^I>0.$$

Next, note that with the assumption on A^U, we must have $B<1$, since the equilibrium condition $p^I>\epsilon$ and the constraint on technology $Bp^Iq^U=p^U\leq 1$ implies $B\leq (A^U)^{1/(\epsilon-1)}/p^I<(A^U)^{1/(\epsilon-1)}/\epsilon$. Inserting the bound for A^U gives the result $B<1$. Also note that the restriction on A^U is only a sufficient condition. Numerical results indicate that it may not be needed. We will assume the upper limit $B=1$. With this inserted in the inequality above we have that is sufficient to show

$$(1-\epsilon)(p^I-\epsilon)^2-(p^I-\epsilon)p^I(1-\epsilon)^3-\epsilon(p^I-\epsilon)^2+(1-\epsilon)^2\epsilon p^I>0.$$

Next, we first rearrange the first two terms, then rearrange the last two terms, and last combine this to show our result. The first two terms can be written as:

$$\begin{aligned}&(1-\epsilon)(p^I-\epsilon)^2-(p^I-\epsilon)p^I(1-\epsilon)^3\\&=(1-\epsilon)(p^I-\epsilon)[(p^I-\epsilon)-p^I(1-\epsilon)^2]\\&=(1-\epsilon)(p^I-\epsilon)[p^I-\epsilon-p^I+2\epsilon p^I-\epsilon^2p^I]\\&=(1-\epsilon)(p^I-\epsilon)\epsilon[p^I-1+p^I(1-\epsilon)].\end{aligned}$$

Since $p^I>\epsilon$, we have

$$p^I-1+p^I(1-\epsilon)>\epsilon-1+p^I(1-\epsilon)=(1-\epsilon)(p^I-1).$$

Thus for the first two terms we have:

$$(1-\epsilon)(p^I-\epsilon)^2-(p^I-\epsilon)p^I(1-\epsilon)^3>(1-\epsilon)^2\epsilon(p^I-\epsilon)(p^I-1),$$

where the bounding term is negative.

The last two terms can be written as:

$$\begin{aligned}&-\epsilon(p^I-\epsilon)^2+(1-\epsilon)^2\epsilon p^I\\&=\epsilon[(1-\epsilon)^2p^I-(p^I-\epsilon)^2]\\&=\epsilon[p^I-2\epsilon p^I+\epsilon^2p^I-(p^I)^2+2\epsilon p^I-\epsilon^2]\\&=\epsilon[p^I(1-p^I)+\epsilon^2(p^I-1)]\\&=\epsilon(p^I-\epsilon^2)(1-p^I).\end{aligned}$$

Combining all terms we have:

$$\begin{gathered}(1-\epsilon)(p^I-\epsilon)^2-(p^I-\epsilon)p^I(1-\epsilon)^3-\epsilon(p^I-\epsilon)^2+(1-\epsilon)^2\epsilon p^I\\>(1-\epsilon)^2\epsilon(p^I-\epsilon)(p^I-1)+\epsilon(p^I-\epsilon^2)(1-p^I)\\=(1-p^I)\epsilon[(p^I-\epsilon^2)-(p^I-\epsilon)(1-\epsilon)^2]>0,\end{gathered}$$

since $(p^I - \epsilon^2) - (p^I - \epsilon)(1-\epsilon)^2 > (p^I - \epsilon^2) - (p^I - \epsilon) > 0$.

Finally, to conclude that θ goes down if profits of the innovating firm go down, we use LEMMA [Properties of the 0-profit condition for innovators] . If the profit curve of an innovating firm is shifted downwards due to the decrease in poaching, the intersection point with the 0-profit line must move to the left since the profit function is locally decreasing. ■

Next we establish the welfare effects of the tax.

Proposition 5 *The welfare effect of a tax on imitating firms is ambiguous, given that scaling factor of the matching function is $A^U \geq \epsilon^{\epsilon-1}$, and $\epsilon = \epsilon^I = \epsilon^U$.*

Proof. Follows immediately from Proposition 4 above and the concavity of the welfare function stated in Lemma 12 in the Appendix. ■

To summarize, a tax on imitating firms will reduce the number of imitating firms and will decrease the number of innovating firms. This gives a positive welfare effect of reduced imitation that is countervailed by a negative welfare effect of lower innovation. Intuitively, a tax reduces imitation. For given wages, joint surplus of innovation firms and workers increase as workers leaves too often in the no-commitment case. However, the innovating firms' trade-off changes due to the tax and w_2 decreases. This lowers the workers' part of the surplus, more than the gain for the firm, and joint surplus is reduced. Thus period 1 entry is lower.

5.6.3 Subsidy to Innovation

Here we analyze the case where the policy maker only can subsidize innovation, i.e. tax on imitation is not available.

Proposition 6 *A subsidy $z > 0$ to the innovating firms will increase the number of innovating firms (as well as θ), and decrease the tightness in the poaching market θ^I, and thus will improve welfare.*

Proof. That a subsidy leads to increased θ is stated in Corollary 1. That this gives lower θ^I is stated in Lemma . Finally, the concavity of the welfare function stated in Lemma 12 gives the result. ■

Intuitively, a subsidy will directly increase the number of innovating firms, and thereby θ. This will increase the cost of replacement and thereby increase the optimal wage w_2, and hence acts as a commitment device. Therefore the poaching probability will be lower and hence θ^I decreases. Both effects contribute to move the no-commitment equilibrium closer to the efficient equilibrium.

5.6.4 Restrictions on labor mobility

A much discussed policy tool is to restrict worker movements between jobs. Suppose that a new match is allowed to develop into an employment relationship with a probability lower than 1. Technically, this is equivalent to a reduction in the efficiency parameter A^I in the matching function in the search market for imitating firms.

Proposition 7 *Restrictions on labor mobility, interpreted as a reduction in the matching efficiency parameter A^I, reduces welfare*

A reduction in A^I has the same effect on the equilibrium outcome as a tax on innovating firms. This reduces θ and θ^I, and hence reduces the *ex ante* income of workers. It follows that welfare is reduced.

6 Conclusion

We analyze whether firms have the right incentives to innovate in the presence of spillovers. Spillovers between firms take place through labor flows within a framework of competitive search. Firms choose to innovate or to imitate by hiring a worker from a firm that has already innovated. We show that if firms can commit to long-term wage contracts, the spillovers are efficiently internalized. In the absence of such contracts, there is too little innovation and too much imitation, and hence a scope for policy. We show that it is important to have the right mix of policy instruments to improve efficiency. First, a subsidy to innovators together with a fee on imitation can implement the efficient allocation. Second, a stand-alone subsidy to innovating firms is always welfare improving. Third, a fee on imitation by itself has countervailing effects as it reduces imitation but also innovation. The net effect is inconclusive. Finally, a restriction on worker mobility is welfare deteriorating.

References

Acemoglu, D. and J.S. Pischke (1999). The Structure of Wages and Investment in General Training. *Journal of Political Economy* 107, 539-572.

Almeida, P. and B. Kogut (1999). Localization of Knowledge and the Mobility of Engineers in Regional Networks, *Management Science* 45, 905-917.

Balsvik, R. (2011). Is labor Mobility a Channel for Spillovers from Multinationals? Evidence from Norwegian Manufacturing. *Review of Economics and Statistics* 93, 285-297.

Breschi, S. and F. Lissoni. (2009). Mobility of skilled workers and co-invention networks: an anatomy of localized knowledge flows. *Journal of Economic Geography* 9, 439-468.

Combes, P.P. and G. Duranton (2006): Labour pooling, labour poaching, and spatial clustering, *Regional Science and Urban Economics* 36, 1-28.

Cooper, D.P. (2001). Innovation and reciprocal externalities: information transmission via job mobility, *Journal of Economic Behavior* 45, 402-425.

Fosfuri, A. and A. Rønde (2004). High-tech clusters, technology spillovers, and trade secret laws, *International Journal of Industrial Organization* 22, 45-65.

Greenstone, M., R. Hornbeck, and E. Moretti (2010). Identifying Agglomeration Spillovers: Evidence from Winners and Losers of Large Plant Openings. *Journal of Political Economy* 118, 536-598.

Görg, H. and E. Strobl (2005). Spillovers from Foreign Firms through Worker Mobility: An Empirical Investigation. *Scandinavian Journal of Economics* 107, 693-709.

Jaffe, A.B., M. Trajtenberg, and R. Henderson (1993). Geographic localization of knowledge spillovers as evidenced by patent citations. *Quarterly Journal of Economics* 108, 577-598.

Kim, J. and G. Marschke (2005). Labor Mobility of Scientists, Technological Diffusion, and the Firm's Patenting Decision, *RAND Journal of Economics* 36, 298-317.

Marimon, R., and Quadrini, V. (2011). Competition, human capital and income inequality with limited commitment. *Journal of Economic Theory* 146, 976-1008.

Moen, E.R. (1997). Competitive Search Equilibrium, *Journal of Political Economy* 105, 385-411.

Moen, E.R. and Å. Rosén (2004). Does Poaching Distort Training, *Review of Economic Studies* 71, 1143-1162.

Møen, J. (2005). Is Mobility of Technical Personnel a Source of R&D Spillovers? *Journal of Labor Economics* 23, 81-114.

Pakes, A. and S. Nitzan (1983). Optimum Contracts for Research Personnel, Research Employment, and the Establishment of "Rival" Enterprises, *Journal of Labor Economics* 1, 345-365.

7 Appendix

7.1 Concavity of the Welfare Function

The welfare function is concave in each argument but not jointly concave in both arguments for all of the domain. In what follows, we state two lemmata that establish parameter bounds such that within these bounds: i) the welfare function is strictly concave on a restricted domain, ii) a commitment equilibrium exists, and iii) a no-commitment equilibrium exists. To simplify the exposition we set the elasticity parameters equal across markets: $\epsilon = \epsilon^I = \epsilon^U$.

The first lemma establishes parameter bounds such that the welfare function is strictly concave:

Lemma 11 *There exist bounds on $\bar{K}^I$ and a number $m(\epsilon)$ such that if $\bar{K}^I$ are within these bounds then the welfare function F is strictly concave on the set $[m(\epsilon), \infty] \times [0, \infty]$.*

Proof. First, simplify notation by $y \equiv 1$. We have to first show that $F_{\theta^I\theta^I} = (1-\epsilon)A^U p^{I(-\epsilon)} p^{1-\epsilon}(1-p)^{\epsilon}[p^{I\prime\prime} - \epsilon p^{^{I-1}}(p^{I\prime})^2] < 0$ which

is clearly the case for any $(\theta, \theta^I) \geq 0$, since $p^{I\prime} = (1-\epsilon)A^I\theta^{I(-\epsilon)} > 0$ and $p^{I\prime\prime} = -\epsilon(1-\epsilon)A^I\theta^{I(-1-\epsilon)} < 0$ (Lemma 13 below shows that $F_{\theta\theta} < 0$) . Secondly, we have to show that the determinant of the Hessian of F is strictly positive on a restricted set, that is $D = F_{\theta\theta}F_{\theta^I\theta^I} - (F_{\theta\theta^I})^2 > 0$. D can be written as

$$\begin{aligned}
&\left[p''[2-\theta^I\bar{K}^I] - A^U p^{I(1-\epsilon)}p^{-\epsilon}(1-p)^{\epsilon-1}[(p')^2\epsilon(1-\epsilon)p^{-1}(1-p)^{-1} - p''(1-\epsilon-p)]\right] \\
&* (1-\epsilon)A^U p^{I(-\epsilon)}p^{1-\epsilon}(1-p)^{\epsilon}[p^{I\prime\prime} - \epsilon p^{p^{-1}}(p^{I\prime})^2] \\
&- \left[(1-\epsilon)p^{I\prime}p'A^U p^{I(-\epsilon)}p^{-\epsilon}(1-p)^{\epsilon-1}[1-\epsilon-p] - p'\bar{K}^I\right]^2 .
\end{aligned}$$

Next using the facts $p' = \frac{p}{\theta}(1-\epsilon)$, $p'' = \frac{-\epsilon p'}{\theta}$, $p^{I\prime} = \frac{p^I}{\theta^I}(1-\epsilon)$, and $p^{I\prime\prime} = \frac{-\epsilon p^{I\prime}}{\theta^I}$, we can write $D \gtreqless 0$ as

$$\begin{aligned}
&\left[\begin{array}{c} -\frac{\epsilon(1-\varepsilon)p}{\theta^2}[2-\theta^I\bar{K}^I] \\ -A^U p^{I(1-\epsilon)}p^{-\epsilon}(1-p)^{\epsilon-1}[(\frac{p}{\theta}(1-\epsilon))^2\epsilon(1-\epsilon)p^{-1}(1-p)^{-1} + \frac{\epsilon(1-\epsilon)p}{\theta^2}(1-\epsilon-p)] \end{array}\right] \\
&* \left[(-\epsilon)(1-\epsilon)^2\frac{A^U}{(\theta^I)^2}p^{I(1-\epsilon)}p^{1-\epsilon}(1-p)^{\epsilon}[2-\epsilon]\right] \\
&- \left[\frac{p^I}{\theta^I}\frac{p}{\theta}(1-\epsilon)^3A^U p^{I(-\epsilon)}p^{-\epsilon}(1-p)^{\epsilon-1}[1-\epsilon-p] - \frac{p}{\theta}(1-\epsilon)\bar{K}^I\right]^2 \gtreqless 0.
\end{aligned}$$

Rearranging and factoring out $p^2 \frac{1}{\theta^2} \frac{1}{(\theta^I)^2}(1-\epsilon)^3 > 0$ we get

$$\begin{aligned}
&\left[-\epsilon[2-\theta^I \bar{K}^I] - \epsilon A^U \beta[(1-\epsilon)^2(1-p)^{-1} + (1-\epsilon-p)]\right] \\
&*(-\epsilon)A^U \beta(1-p)[2-\epsilon] \\
&-(1-\epsilon)\left[(1-\epsilon)A^U \beta[1-\epsilon-p] - \frac{\theta^I \bar{K}^I}{(1-\epsilon)}\right]^2 \gtreqless 0,
\end{aligned}$$

where $\beta = p^{I(1-\epsilon)} p^{-\epsilon}(1-p)^{\epsilon-1} > 0$.

In the following we will establish bounds on $\bar{K}^I$ so that $D > 0$ on a restricted set. First consider the case $\bar{K}^I = 0$. Then the inequality can be simplified to

$$2\epsilon^2(1-p)[2-\epsilon] + \beta A^U Z(p,\epsilon) \gtreqless 0,$$

where $Z(p,\epsilon) = \epsilon^2[2-\epsilon](1-\epsilon)^2 - \epsilon^2(1-p)[2-\epsilon](1-\epsilon-p) - (1-\epsilon)^3[1-\epsilon-p]^2$. Note that the LHS is strictly positive if $Z \geq 0$. It is easy to show that $Z > 0$ if $p \geq 1-\epsilon$. Next we implicitly define $m(\epsilon)$ by $p(m(\epsilon)) = 1-\epsilon$. Then for any $\theta \geq m(\epsilon)$, D is strictly positive, i.e. F is strictly concave on the set $[m(\epsilon),\infty] \times [0,\infty]$.[5] Now consider the case $\bar{K}^I < 0$. Since D is strictly positive for $\theta \geq m(\epsilon)$, by continuity there exists a $\gamma \geq 0$ such that if $\bar{K}^I < \gamma$, D will still be strictly positive.

To sum up, we can find a $\bar{K}^I$ small enough and a number $m(\epsilon)$ such that the determinant is strictly positive on a restricted set. ■

These bounds on the parameters are clearly a sufficient and not a necessary condition. Simulations show that the welfare function is strictly concave for a much wider set of parameters.

The following lemma establishes that parameters can be chosen within bounds such that: i) the welfare function is strictly concave, ii) the commitment equilibrium exists, iii) the no-commitment equilibrium exists.

Lemma 12 *There exist bounds on $\bar{K}$, $\bar{K}^I$, and a number $m(\epsilon)$. If $\bar{K}$, $\bar{K}^I$ are within these bounds, then $(\theta^C, \theta^{CI}), (\theta^{NC}, \theta^{NCI}) \in [m(\epsilon),\infty] \times [0,\infty]$ and the welfare function F is strictly concave on the set $[m(\epsilon),\infty] \times [0,\infty]$.*

Proof. Define $m(\epsilon)$ as in the Lemma above. Using the 0-profit condition for the innovators of the non-commitment case (eq???), we can choose a $\bar{K}$ small enough such that $\theta^{NC} \geq m(\epsilon)$. Further, if $\bar{K}^I$ is close enough to 0, we can ensure that $p^I > \epsilon$, so that an interior non-commitment allocation exists for $\bar{K}$, $\bar{K}^I$. Then $(\theta^{NC}, \theta^{NCI}) \in [m(\epsilon),\infty] \times [0,\infty]$ and by the previous Lemma, F is strictly concave on $[m(\epsilon),\infty] \times [0,\infty]$.

[5] From the FOC for the welfare function, $F_\theta = 0$ (see equation XX), it can be seen that we can always get p big enough, equivalently θ big enough, if we set $\bar{K}$ small enough.

Since we only put upper bounds on $\bar{K}$, $\bar{K}^I$, it is clear that we can always find $\bar{K}$, $\bar{K}^I$ such that the previous conditions hold and also an interior commitment equilibrium exists. We know from Proposition 3 that $\theta^C > \theta^{NC}$. Therefore (θ^C, θ^{CI}) is also contained in $[m(\epsilon), \infty] \times [0, \infty]$. ■

To summarize, we constructed a set of parameters, within which the welfare function is strictly concave and which contains both equilibria. This allows us to determine the direction of welfare changes coming from policies aimed at the non-commitment allocation.

In the following lemma we show that $F_{\theta\theta} < 0$.

Lemma 13 $F_{\theta\theta} < 0$.

Proof. First, simplify notation by $y \equiv 1$. Next define $G(\theta, p(\theta)) \equiv p[2 + \hat{A}p^{(1-\epsilon)}(1-p)^{\epsilon} - \overline{K}^I\theta^I] - \overline{K}\theta$, where $\hat{A} = A^U p^{I(1-\epsilon)}$. To show that $F_{\theta\theta} = G\left(p(\theta), \theta\right)_{\theta\theta} < 0$, we have to prove that:

$$(p')^2 G_{pp} + p'' G_p < 0,$$

since $G_{p\theta} = G_{\theta\theta} = 0$. Further, we have $p'' = -\epsilon\frac{p'}{\theta} < 0$ and $p' > 0$, so we have to show:

$$\theta p' G_{pp} - \epsilon G_p < 0.$$

We can write the first term as:

$$\theta p' G_{pp} = -\epsilon(1-\epsilon)\widehat{A}\theta p' p^{-\epsilon}(1-p)^{\epsilon}[p^{-1} + 2(1-p)^{-1} + p(1-p)^{-2}],$$

which can be rearranged to

$$\theta p' G_{pp} = -\epsilon(1-\epsilon)\widehat{A}\theta p' p^{-\epsilon-1}(1-p)^{\epsilon-2}\theta < 0.$$

It remains to show that the second term

$$-\epsilon G_p = \epsilon\widehat{A}p^{1-\epsilon}(1-p)^{\epsilon}[\epsilon(1-p)^{-1} - (1-\epsilon)p^{-1}] - \epsilon(2 - \theta^P\overline{K}^I),$$

is negative. Note, in order for the poaching market to be productive, we need to have $p^I q^U - \overline{K}^I\theta^I > 0$. Further, since $p^I q^U \leq 1$, we must have $\theta^I\overline{K}^I < 1$. Moreover, for solutions in the range that do not involve kink points of the matching functions, we must have $\widehat{A}p^{1-\epsilon}(1-p)^{\epsilon}(1-p)^{-1} = p^U \leq 1$. Thus, within this range, we must have:

$$\epsilon\widehat{A}p^{1-\epsilon}(1-p)^{\epsilon}\epsilon(1-p)^{-1} - \epsilon[1 + (1 - \theta^I\overline{K}^I)] < 0.$$

which gives the result, since the only remaining term, $-\epsilon\widehat{A}p^{1-\epsilon}(1-p)^{\epsilon}(1-\epsilon)p^{-1}$, is also negative. ■

7.2 Bounds on CD-Matching Functions

Throughout the paper we assume that the matching function is of Cobb-Douglas type. This simplifies the algebra since it implies constant elasticities (ϵ^i's). In order to be able to use derivatives of matching probability functions we have to restrict the parameters, in particular the matching function parameters (ϵ^i and A^i), to ensure that the functions p^i and q^i are neither at a kink point nor on a flat part of the range. That is, in each matching market i of the model, we have to ensure that the market tightness is such that

$$\overline{B}^i \equiv (A^i)^{-\frac{1}{1-\epsilon^i}} \geq \theta \geq (A^i)^{\frac{1}{\epsilon^i}} \equiv \underline{B}^i,$$

where the upper bound $\overline{B}^i$ is the highest θ^i that satisfies the inequality $p^i(\theta^i) \leq 1$, and the lower bound $\underline{B}^i$ is the lowest θ^i that satisfies the inequality $q^i(\theta^i) \leq 1$.

We can establish the following two results:

Lemma 14 *If $A \geq 2^{-\epsilon}$ then if $(\theta, \theta^I) \geq \underline{B}$, then $\theta^U \geq \underline{B}$.*
Proof. *This follows from plugging in the lower bound for θ and θ^I into $\theta^U = \frac{A\theta^{1-\epsilon}A(\theta^I)^{1-\epsilon}}{1-A\theta^{1-\epsilon}}$.* ■

For the upper bound we cannot get a counterpart result.

The following establishes the weak result that given a restriction on A we can find θ and θ^I so that all inequalities (for θ, θ^I, and θ^U) hold:

Lemma 15 *If $A^{\cdot-\frac{1}{1-\epsilon}} - A^{\cdot\frac{1-2\epsilon}{\epsilon(1-\epsilon)}} - A^{\cdot\frac{2}{\epsilon}} \geq 0$, then there exists θ, θ^I such that all upper bounds for θ, θ^I, and θ^U hold simultaneously.*
Proof. *The result is obtained by plugging in the lower bounds for θ and θ^I into the expression for θ^U and compare it to the upper bound for θ^U.* ■

These results are not sufficient to ensure that all allocations are within the bounds.[6] Such bounds are not easy to describe explicitly. However, since have not assumed that the matching function parameters are the same across markets i, we have a lot of flexibility to pick parameters. Indeed, we can show by numerical simulation (in the next subsection of the appendix) that there is some range of parameters such that both the parameter restrictions for the policy analysis as well as those restrictions for the "interiority" of the matching functions are satisfied.

[6] In particular, the policy analysis in section (5.6) requires that we have parameters for which both equilbria exists and for which the welfare function is strictly convex.