

# Looking for jobs with good pay and good working conditions

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**Work in Progress**

## **Abstract**

In perfectly competitive labour markets, there is a market for non-material job amenities in which workers' willingness to pay for these goods implies that workers accept compensating wage differentials, such that jobs with better working conditions should have lower wages. In labour market characterised by frictions, workers' wages typically depend also on firm productivity. However many job characteristics with consumptive value also influence productivity, such that compensating differentials may not be apparent. Job characteristics that may influence productivity are considered here in a labour market with search frictions and different levels of worker ability. In this framework, compensating differentials may not be evident despite the fact that workers are willing to substitute better working conditions for lower pay.

Two methods of empirical estimation are proposed based on subjective evaluation of working conditions, productivity, wage and labour market transition data. Results for an approach imputing productivity are presented, estimation of the model including productivity data is work in progress.

**JEL:** J28, J31, J64, J81, M52

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## 1 Working conditions and Job market frictions

Working conditions are at the heart of the classical exposition of the labour market, since labour supply is equated to the disutility of work. As a result, differences in job amenities, by influencing labour supply, generate corresponding (compensating) wage differentials. The classic work by [Rosen \(1986\)](#) reviews the implications: a negative correlation between wages and working conditions. Empirical work testing this correlation requires good controls since effectively a shadow “hedonic price” must be calculated. Whilst there have been various attempts to test for specific types of workplace characteristics, cross-sectional results have often been disappointing<sup>1</sup>. Two types of problems have been stressed in this literature: on the one hand, the difficulties involved in identifying relevant working conditions; on the other hand, the role of unobservable heterogeneity in worker ability and firm productivity. An early work questioned along these lines is [Masters \(1969\)](#), who interpretes wage variance across industries as compensation for differences in working conditions.

First, regarding the measurement of working conditions, we here test different measures of working conditions, including subjective job satisfaction as an indicator for good working conditions. By specifying (and estimating) the role that these job characteristics may play for firm productivity in a dynamic model we make explicit a linkage that troubles many analyses of compensating differentials.

Second, productive heterogeneity on both the firm and worker side are allowed for in the current set-up. Important work has focused on disentangling worker and firm effects ([Abowd et al. \(1999\)](#)). The role of job quality has rarely been considered in this framework however<sup>2</sup>. Previous work testing for the existence of compensating differentials has often focused on characteristics that may be thought to also influence productivity (for example industries, occupations, job amenities etc.<sup>3</sup>). We allow for hedonic job characteristics to be correlated with firm productivity differentials. In a first step, we use productivity data to identify the effect that job satisfaction may have in explaining differences in productivity. As a compari-

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<sup>1</sup>See [Benz and Frey \(2003\)](#) for an interesting exception and [Bonhomme and Jolivet \(2008\)](#) for a recent review of the inconclusive evidence.

<sup>2</sup>For example, contrasting to the work by [Masters \(1969\)](#) cited above, cross-sectional industry wage differentials have been broken down into firm productivity (fixed) effects and average worker productivity (fixed) effects (see [Abowd et al. \(2003\)](#))

<sup>3</sup>[Clark and Senik \(2006\)](#) contrast job satisfaction across industries and occupations using subjective data. They argue that the findings indicate that whilst compensating differentials may be important between industries, differences in rents explain the wage variation across occupations. The current paper provides a novel rationale for these findings.

son, we estimate a closely related model that makes a functional form assumption about the relationship between productivity and job satisfaction and treat productivity as an unobservable. Since we do not have firm-level data on productivity, comparing these alternative routes of dealing with missing data appears promising.

In a search framework allowing for on-the-job search we can consider the dynamic effect that hedonic job characteristics may have when choosing a job and the trade-offs that individuals may make with respect to future career choices. As [Hwang et al. \(1998\)](#) have shown, the classical prediction of compensating differentials will typically be much affected by the assumptions of limited information that search models of the labour market make. Recently, [Bonhomme and Jolivet \(2008\)](#) have shown that introducing relatively modest labour market frictions can considerably reduce the expected negative correlation between wages and working conditions. Whilst [Bonhomme and Jolivet \(2008\)](#) use exogenous mobility costs, the model here assumes search frictions in line with the job search literature. In particular, workers are subject to an exogenous job arrival rate determining potential matches. Furthermore, offers arrive at a given rate (both to unemployed and employed workers, since there is on-the-job search) and employers can respond to rival firms' offers by raising their own wage, following the model of [Postel-Vinay and Robin \(2002\)](#). By allowing for firms to counter offers by other firms we allow for firm competition in a natural way.

Taking into account the impact of good working conditions may be particularly important to the extent that working conditions impact on productivity. This is an assumption that the efficiency wage literature has made (an early example specifically using subjective data is [Freeman \(1978\)](#)). This is in line with considerable evidence from the Human Resource management literature. The simplest motivation underlying the assumption refers to the fact that efficiency wage models are typically based on a certain amount of utility generating behavioural changes in the population. It should be noted that the model here is also consistent with the basic assumption of a different class of efficiency wage models based on adverse selection, namely that workers' productivity is positively correlated with workers reservation wage.<sup>4</sup>

Focusing on health insurance in the US, [Dey and Flinn \(2005\)](#) combine the assumption that health insurance raises worker productivity in a job search frame-

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<sup>4</sup>This justification requires relaxing the assumption that firms can perfectly observe workers productivity. It may for example be assumed that there is a discretionary element in productivity which gives rise to the productive impact of working conditions.

work. They find a demand for health insurance despite a lack of cross-sectional wage differentials. Their model differs from that presented here, since they only have two levels of job quality (health insurance status) and they do not allow for on-the-job search. Furthermore, whereas they model job quality as an endogenous firm choice variable, differences in job quality are here assumed to be exogenous, determined for example by the arduousness of a given occupation.

Workers with different levels of ability may have preferences for jobs which pay well and have good working conditions. However, in a non-competitive labour market, this may not lead to compensating differentials for job amenities. Additionally, a good working environment (subjectively evaluated by workers) may increase worker productivity.

This paper uses an equilibrium search framework in which workers are engaged in job search on the job, generating job offers from competing firms. In the spirit of [Postel-Vinay and Robin \(2002\)](#), whilst firms are assumed to have market power over workers, but job offers generate Bertrand competition for workers by allowing current firms to match offers. As a result, workers with no market power capture some of the match rent, depending on their labour market history. Differences in productivity related to working conditions may be passed on to the worker in terms of higher wages, thus attenuating or reversing the expectation of a negative correlation between working conditions and wages.

The particular way of including working conditions in a search framework allows for a broad applicability: Instead of using objective differences in working conditions (e.g. health insurance, workplace mortality), the current framework can make use of data on subjective evaluation of working conditions.

## 2 The Model

Workers care about wages  $w$  and non-material job satisfaction (which we will interchangeably call working conditions)  $s$ . Functional form assumptions for the utility function are required to generate an explicit expression for the equilibrium wage. When necessary, the utility function will be assumed to be log utility<sup>5</sup>. Job characteristics may influence workers' utility either positively or negatively depending

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<sup>5</sup>For an argument why logarithmic utility may be a good assumption, see [Layard et al. \(2008\)](#), who find a coefficient of risk aversion  $\rho$  close to 1 such that marginal utilities are inversely proportional to income

on  $\gamma$  and we do not assume constant returns to scale in this Cobb-Douglas type utility function.

$$u(w, s) = \log w + \gamma \log s \quad (1)$$

Match productivity depends on worker's individual ability  $x$ , workers' job satisfaction  $s$  and firm productive heterogeneity unrelated to job satisfaction. It is assumed that constant returns to scale technology allows us to focus on productivity per worker,  $p$ .

$$\ln(p) = \ln(x) + \ln(z) + \alpha \ln(s) \quad (2)$$

It is an appealing simplification to assume that the reservation wage of the unemployed is a function of their labour market productivity ( $x$ ). Home (or self-employed) productivity can then be given as follows, for some  $z_0, s_0$  constant across the population:

$$p_0 = x z_0 s_0^\alpha \quad (3)$$

In order to purge our measure of firm productive heterogeneity  $p$  of the potential reverse causality of job satisfaction  $s$ , we decompose firm heterogeneity in a component  $z$  which is unrelated to  $s$ . We observe sector-level productivity  $p^*$  and write match productivity of workers and firms  $(x, z, s)$  as producing  $p(x, z, s) = x z s^\alpha$ . Given separability (no sorting by individual characteristics) we have  $E(\ln(x)|z, s) = 0$  and can identify the impact of job satisfaction on sector-level productivity by regressing  $s$  on  $p$ :

$$\ln(p^*) = p_0 + \alpha \ln(s^*) + u \quad (4)$$

$$z \equiv \exp(p_0 + u). \quad (5)$$

The labour market is characterised by search frictions: individual workers receive job offers at an exogenous rate  $\lambda$  and choose to accept or reject these. Firms can match wage offers, such that there is a Bertrand-type competition for workers between current and potential employers. Workers discount the future at rate  $\rho > 0$ . Given we are not modelling planned exit from the labour market (retirement), this can be taken to include the risk of an exogenous exit from the labour market.

We assume that workers have bargaining power and rents are shared as in [Cahuc et al. \(2006\)](#). Worker bargaining power is  $\beta$ .

### 3 Equilibrium Wage contracts

The value of a labour market state,  $V(\cdot)$ , depends on three variables: the worker's type  $x$ , the worker's wage  $w$  and the firm heterogeneity  $z, s$ . Notice that working conditions  $s$  impact on utility both directly through their hedonic value and as a determinant of productivity. Both factors will later be found to determine to what extent workers may accept lower wages to move to a firm with better working conditions.

An employer can pay a worker at most  $w = p(x, z, s)$  yielding utility

$$\begin{aligned} u(p, s) &= \log x + \log z + (\alpha + \gamma) \log s \\ &\equiv v(x, y) \end{aligned}$$

where the firm characteristics - both productive and hedonic - relevant to an individual can be given as  $y(z, s) = z s^{\alpha+\gamma}$ . For now on, let us index firm by  $(y, s)$  instead of  $(z, s)$ . The index  $y$  determines firm competitiveness to workers.

Let  $V_0(x)$  denote the value of unemployment and  $V_1(x, y)$  the employees' reservation values. Let  $V(w, x, y)$  be the value of a wage contract  $w$ <sup>6</sup>. Unemployed workers are paid a wage  $\phi_0(x, y, s)$  such that

$$V(\phi_0, x, y) = V_0(x) + \beta [V(x, y) - V_0(x)]. \quad (6)$$

The wage resulting from two firms competing for the same worker is  $\phi_1(x, y, s, y')$ , for  $y < y'$ , such that

$$V(\phi_1, x, y) = V(x, y) + \beta [V(x, y') - V(x, y)]. \quad (7)$$

The value of unemployment  $V_0(x)$  solves the following Bellman equation:

$$r V_0(x) = v(x, b) + \lambda_0 \beta \int_{y_{\inf}}^{y_{\max}} [V(x, y') - V_0(x)] dF(y'), \quad (8)$$

where  $y_{\inf}$  is such that  $V_0(x) = V(x, y_{\inf})$ , where  $V(x, y)$  is the firm's reservation value. The employees's reservation value is equal to the maximum value the reference firm could pay a worker, such that  $V(x, y) \equiv V(x, y_R)$ :

<sup>6</sup>We will show later that it depends on  $w$  and  $s$  only via  $y$ , i.e. not on the composition of  $y$  - see (13)

$$\begin{aligned}
 V(x, y) &= \frac{v(x, y)}{1+r} + \delta \frac{V_0(x)}{1+r} \\
 &+ \frac{\lambda_1}{1+r} \int_{V(x, y') > V(x, y)}^{V(x, y_{max})} [(1-\beta)V(x, y) + \beta V(x, y')] dF_y(y') \\
 &+ (1-\delta - \lambda_1 \bar{F}(y)) \frac{V(x, y)}{1+r}
 \end{aligned}$$

$$[\delta + r + \lambda_1 \beta \bar{F}(y)] V(x, y) = v(x, y) + \delta V_0(x) + \lambda_1 \beta \int_{V(x, y') > V(x, y)}^{V(x, y_{max})} V(x, y') dF_y(y') \quad (9)$$

$$(\delta + r) V(x, y) = v(x, y) + \delta V_0(x) + \lambda_1 \beta \int_{V(x, y') > V(x, y)}^{V(x, y_{max})} V'(x, y') \bar{F}(y') d(y') \quad (10)$$

$$r V(x, y) = v(x, y) + \delta [V_0(x) - V(x, y)] + \lambda_1 \beta \int_y^{y_{max}} [V(x, y') - V(x, y)] dF(y'). \quad (11)$$

where integration by parts allows us to move from (9) to (10).

Workers will move to the firm that can offer the higher value. Intuitively there is only one state variable (the current firm's job characteristics which condition job offer quality by helping bargaining), potentially increasing period utility in a future period (by influencing bargaining) as well as influencing current utility. Taking the derivative of equation (10) we find that:

$$\frac{\partial V(x, y)}{\partial y} = \frac{\partial v(x, y)}{\partial y} \frac{1}{r + \delta + \lambda_1 \beta \bar{F}(y)}. \quad (12)$$

Given that the sign of the denominator on the right-hand side will always be positive, we see that the preference ordering of firm characteristics depends solely on the sign of  $v'(x, y)$ , i.e. that we need only consider the rôle of job characteristics in the (instantaneous) utility function in order to assess their ordering in the value function. This implies that a worker will move to a firm with productivity  $z'$  and hedonic characteristics  $s'$  where potential instantaneous utility is higher<sup>7</sup> iff:

<sup>7</sup>The qualifying "potential" is required because firms will not give workers full productivity unless workers either have full bargaining power or have an offer from a firm with equally high levels of productivity. However, as long as workers have positive bargaining power, workers will be better off in the firm with greater instantaneous utility.

$$\begin{aligned}
 V(x, z', s') &> V(x, z, s) \\
 u(x, y', z', s') &> u(x, y, z, s) \\
 \ln(x, y') + \theta \log(s') &> \ln(x, y) + \theta \log(s) \\
 \alpha \log(z') + (\beta + \theta) \log(s') &> \alpha \log(z) + (\beta + \theta) \log(s) \quad (13)
 \end{aligned}$$

The criterion for mobility can thus be given directly in terms of our linear combination of productive and hedonic characteristics defined by  $y \equiv z^\alpha + s^{\beta+\theta}$ .

And it follows from integration by part that

$$\begin{aligned}
 rV_0(x) &= v(x, b) + \lambda_0 \beta \int_{y_{\text{inf}}}^{y_{\text{max}}} \frac{\partial v(x, y')}{\partial y'} \frac{\bar{F}(y')}{r + \delta + \lambda_1 \beta \bar{F}(y')} dy', \\
 (r + \delta)V(x, y) &= v(x, y) + \delta V_0(x) + \lambda_1 \beta \int_y^{y_{\text{max}}} \frac{\partial v(x, y')}{\partial y'} \frac{\bar{F}(y')}{r + \delta + \lambda_1 \beta \bar{F}(y')} dy',
 \end{aligned}$$

and

$$v(x, y_{\text{inf}}) = v(x, b) + (\lambda_0 - \lambda_1) \beta \int_{y_{\text{inf}}}^{y_{\text{max}}} \frac{\partial v(x, y')}{\partial y'} \frac{\bar{F}(y')}{r + \delta + \lambda_1 \beta \bar{F}(y')} dy'. \quad (14)$$

The value of a wage contract  $w$  is given by

$$\begin{aligned}
 [r + \delta + \lambda_1 \bar{F}(y_R)] V(w, x, y, s) &= v(x, y) + \delta V_0(x) \\
 &+ \lambda_1 \int_y^{y_{\text{max}}} [\beta V(x, y') + (1 - \beta)V(x, y)] dF(y') \\
 &+ \lambda_1 \int_{y_R}^y [\beta V(x, y) + (1 - \beta)V(x, y')] dF(y') \\
 &= v(x, y) + \delta V_0(x) \\
 &+ \lambda_1 \bar{F}(y_R) [\beta V(x, y) + \lambda_1 (1 - \beta) V(x, y_R)] \\
 &+ \lambda_1 \beta \int_y^{y_{\text{max}}} \frac{\partial v(x, y')}{\partial y'} \frac{\bar{F}(y')}{r + \delta + \lambda_1 \beta \bar{F}(y')} dy' \\
 &+ \lambda_1 (1 - \beta) \int_{y_R}^y \frac{\partial v(x, y')}{\partial y'} \frac{\bar{F}(y')}{r + \delta + \lambda_1 \beta \bar{F}(y')} dy', \quad (15)
 \end{aligned}$$

where  $y_R(w, x, y, s)$  is such that  $\phi_1(x, y, s, y_R) = w$ .

Consider the value of a job with less than maximum wage, where  $y_R$  is the threshold for jobs workers can use to bargain for higher wages (where  $y_R \leq y$



- workers move jobs for offers with  $y' > y$ .  $y_R$  can be seen as a state variable encapsulating past labour market experience (or simply, the best past job offer received). So for  $y > y_R$  using the definition for wage contract  $\phi(x, y, s, y')$ ,

$$V(w, x, y, s) = V(x, y_R) + \beta[V(x, y) - V(x, y_R)], \quad (16)$$

Substituting the expression on the left-hand side using (11) and again (7) we obtain

$$\begin{aligned} u(w, s) = & v(x, y_R) + \beta[v(x, y) - v(x, y_R)] \\ & - (1 - \beta)^2 \lambda_1 \int_{y_R}^y \frac{\partial v(x, y')}{\partial y'} \frac{\bar{F}(y')}{r + \delta + \lambda_1 \beta \bar{F}(y')} dy'. \end{aligned} \quad (17)$$

The wage contract  $\phi_0(x, y, s)$  is obtained as

$$\phi_0(x, y, s) = \phi_1(x, y, s, y_{\text{inf}}). \quad (18)$$

Using our utility function, the equilibrium wage follows as

$$\begin{aligned} \ln(w) = & \ln(x) + (1 - \beta) \ln(y_R) + \beta \ln(y) - \gamma \ln(s) \\ & - (1 - \beta)^2 \lambda_1 \int_{y_R}^y \frac{1}{y'} \frac{\bar{F}(y')}{\delta + r + \lambda_1 \chi \bar{F}(y')} dy'. \end{aligned} \quad (19)$$

The different elements of the wage are easily visible in (19): individual productivity, bargained firm rent, compensating differentials and deduction of expected future benefits from counteroffers.

## 4 Equilibrium distributions

We observe wages  $w$ , reported job satisfaction  $s$  and the other components of  $y$ . Transition parameters are estimated in a first stage using duration data (see (??)).

We have noted that the distribution of  $y$  can be derived from the joint population distribution of  $z$  and  $s$  for which we assume bivariate normality,  $(\log(z), \log(s)) \xrightarrow{D} N(\mu, \Sigma)$ <sup>8</sup>.

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<sup>8</sup>Note that by construction of  $z$  we have some knowledge about the covariance-matrix.

The observed sample distribution  $G(y)$  is different from the underlying population distribution as a specific subsample of offers are accepted by workers. Equating the outflow out of employment to the inflow from employment in a job with no better characteristics than  $y$  we have:

$$\begin{aligned} [\delta + \lambda_1 \bar{F}(y)] G(y) (1 - u) &= [\lambda_0] F(y) \\ \delta F(y) &= [\delta + \lambda_1 \bar{F}(y)] G(y) \\ G(y) &= \frac{F(y)}{\delta + \lambda_1 \bar{F}(y)} \end{aligned} \quad (20)$$

where we use the fact that  $u = \frac{\delta}{\delta + \lambda_0}$  by a similar argument of flows into and out of unemployment and where in standard notation  $k_1 \equiv \frac{\lambda_1}{\delta}$ .

Normality of  $z, s$  also imply normality of  $y$  given that the latter is a linear combination of the former:

$$\begin{aligned} y \xrightarrow{D} N \left( \beta_0 + \beta_1 \mu_{\log(z)} + (\beta_2 + \theta) \mu_{\log(s)}, \right. \\ \left. \beta_1^2 \sigma_{\log(z)}^2 + (\beta_2 + \theta)^2 \sigma_{\log(s)}^2 + 2 \beta_1 (\beta_2 + \theta) \sigma_{\log(z), \log(s)} \right) \end{aligned} \quad (21)$$

Finally, we have the latent terms  $x$  and  $y_R$ , denoting individual productivity and best past offer realisation respectively. The expected value of the best past job offer will depend on the transition rates and the (observed) value of  $y$ . To derive the conditional distribution of  $y_R|y$  consider the flows into and out of jobs with wage less than or equal to  $w$  and job quality  $y$ :

$$\begin{aligned} [\delta + \bar{F}(y_R)] (1 - u) J(w|x, y) g(x, y) &= \left[ \lambda_0 u n(x) + \lambda_1 (1 - u) \int_{y_{min}}^{y_R} g(x, y') dy' \right] f(y) \\ [\delta + \bar{F}(y_R)] \frac{\lambda_0}{\lambda_0 + \delta} J(w|y) g(y) n(x) &= \left[ \frac{\lambda_0 \delta}{\lambda_0 + \delta} n(x) + \frac{\lambda_1 \lambda_0}{\lambda_0 + \delta} \int_{y_{min}}^{y_R} g(x, y') dy' \right] f(y) \\ [\delta + \bar{F}(y_R)] J(w|y) g(y) n(x) &= \delta f(y) n(x) + \lambda_1 n(x) \int_{y_{min}}^{y_R} g(x, y') dy' \\ J(w|y) \frac{1 + k_1}{[1 + k_1 \bar{F}(y_R)]^2} &= \delta + \frac{\lambda_1 F(y_R)}{1 + k_1 \bar{F}(y_R)} \\ J(w|y) &= \left[ \frac{1 + k_1 \bar{F}(y)}{1 + k_1 \bar{F}(y_R)} \right]^2 \end{aligned} \quad (22)$$

Where moving from the first to the second line uses separability of  $x$  and  $z, s$ , i.e. that there is no matching of individual ability to job characteristics. This is an implicit assumption given our framework of constant returns to scale firms and no directed search on workers' side, which can be seen by considering in- and outflows for the top of the wage-distribution (conditional on job characteristics), i.e. workers receiving their full productivity ( $w = x y$ ) such that  $J(w|x, z, s) = 1$  and  $t_R = t$ .

$$[\delta + \lambda_1 \bar{F}(t)] g(x, z, s) \frac{\lambda_0}{\lambda_0 + \delta} = \left[ \lambda_0 u n(x) + \lambda_1 (1 - u) \int_{t_{min}}^t g(x, z', s') dz', s' \right] f(z, s)$$

$$g(x, z, s) = \frac{\left[ \delta n(x) + \lambda_1 \int_{t_{min}}^t g(x, z', s') dz', s' \right] f(z, s)}{\delta + \lambda_1 \bar{F}(t)} \quad (23)$$

$$g(x, z, s) = \frac{1 + k_1}{[1 + k_1 \bar{F}(z, s)]^2} n(x) f(z, s) \quad (24)$$

where (24) is the solution to the preceding differential equation and demonstrates separability.

## 5 Estimation

### 5.1 Partial likelihood for mobility

We use the information on the first unemployment or employment spell and do not consider workers who transit to or from labour market states other than unemployment and employment.

The likelihood contribution of an unemployed worker is thus composed of the probability of being unemployed, given by  $\frac{1}{1+k_0}$  (where  $k_0 \equiv \frac{\lambda_0}{\delta}$ ) and the density of unemployment duration given as two independent processes: the observed duration of unemployment up to the interview date ( $t_{0b}$ ) and the remaining duration of unemployment if the end of the unemployment spell is observed from later interview information, i.e.  $d_{0f} = 0$ .

$$\frac{1}{1 + k_0} (\lambda_0 \tilde{f}(z, s))^{1-d_{0f}} \exp \left[ -\lambda_0 (t_{0b} + t_{0f}^{1-d_{0f}}) \right] \quad (25)$$

where  $\tilde{f}(z, s)$  is the sampling distribution of  $(z, s) = (y s^{-(\alpha+\gamma)}, s)$ :

$$\tilde{f}(z, s) = s^{\alpha+\gamma} f_z(z s^{\alpha+\gamma}) f_s(s|z s^{\alpha+\gamma}) \quad (26)$$

The likelihood contribution of an employed worker consists first of the probability of employment  $\left(\frac{k_0}{1+k_0}\right)$ , second of the probability of observing the combination of individual and job characteristics in the employment state - the population distribution  $g(x, z, s)$  - as given by equation (24). For individuals for whom we observe a job-to-job transition (i.e. non-right-censored individuals ( $d_{1f} = 0$ )), we can also use the information about job characteristics after the transition.

Employed workers either transit to unemployment ( $v = 1$ ) or to another job, the latter case occurring at rate  $\lambda_1 \bar{F}(z, s)$ , the arrival rate of jobs with better working conditions. Furthermore, for non-censored workers transiting from job to job we observe job characteristics before and after the transition. Their likelihood contribution then includes the likelihood of sampling a particular combination of job characteristics from the population  $\tilde{f}(z', s')$  (as defined in equation (26)) given the characteristics of the job to which employees transits ( $z', s'$ ). With random timing of the interview across total duration, tenure and remaining duration are again independent random processes.

$$\frac{k_0}{1+k_0} g(x, z, s) \exp\left[-(\delta + \lambda_1 \bar{F}(z, s)) (t_{1b} + t_{1f}^{1-d_{1f}})\right] \left[\left[\lambda_1 \bar{F}(z, s) \tilde{f}(z', s')\right]^{1-v} \delta^v\right]^{1-d_{1f}} \quad (27)$$

## 5.2 Conditional wage likelihood

In the second step we use the likelihood of individual wage observations. The conditional pdf of wages  $J(w|x, z, s)$  has been shown to be a function of the distribution of job characteristics, the transition parameters estimated in the first step and the level of working conditions (see 22).

The likelihood of a worker's wage  $w$  conditional on the job type  $(y, s)$  is

$$\mathcal{L}(w|y, s) = \int \frac{\partial L(w|x, y, s)}{\partial w} h(x) dx = \int \frac{\partial y_r}{\partial w} \frac{\partial L(w|x, y, s)}{\partial y_r} h(x) dx \quad (28)$$

where

$$\frac{\partial y_R(w, x, y, s)}{\partial w} = \left[ \frac{\partial \phi_1(x, y, s, y')}{\partial y'} \Big|_{y'=y_R(w, x, y, s)} \right]^{-1} \quad (29)$$

with

$$\frac{\partial \ln \phi_1(x, y, s, y')}{\partial y'} = (1 - \beta) \frac{\partial v(x, y')}{\partial y} \frac{r + \delta + \lambda_1 \bar{F}(y')}{r + \delta + \lambda_1 \beta \bar{F}(y')}. \quad (30)$$

The marginal distribution of wages for workers who have received job offers since their last unemployment spell (such that  $y_R \in [y_{min}, y_{max}]$ ) can be given as:

$$\frac{\partial L(w|x, y, s)}{\partial y_R} = \frac{\left[1 + \frac{\lambda_1}{\delta} \bar{F}(y)\right]^2}{\left[1 + \frac{\lambda_1}{\delta} \bar{F}(y_R)\right]^3} 2 k_1 f(y_R) \quad (31)$$

Furthermore there is the possibility of a point mass in the distribution of reference job characteristics  $y_R$  corresponding to workers who have not received any job offer ( $y_R = y_0$ ) previously. Such that<sup>9</sup>

$$L(w|x, y, s) = P(y_R = y_0) \quad \text{for } t_R == t_0 \quad (32)$$

$$= \frac{1}{[1 + k_1 G(y)]^2}. \quad (33)$$

The overall likelihood contribution of a wage observation is then given by the sum of the likelihood contribution of wage observations with reference firm characteristics in the range  $y_R \in [y_{min}, y_{max}]$ ,

$$\mathcal{L}(w|y, s) = \frac{1}{1 - \beta} \int \int \frac{y'_R}{w} \frac{r + \delta + \lambda_1 \beta \bar{F}(y'_R)}{r + \delta + \lambda_1 \bar{F}(y'_R)} \frac{f(y'_R) [\delta + \lambda_1 \bar{F}(y)]^2}{[\delta + \lambda_1 \bar{F}(y'_R)]^3} h(x') dx' dy'_R, \quad (34)$$

and the likelihood contribution of wage observations for individuals who have not received any offers (apart from their current job) since unemployment,

$$\mathcal{L}(w|y, s) = \frac{1}{1 - \beta} \int \frac{y_0}{w} \frac{r + \delta + \lambda_1 \beta}{r + \delta + \lambda_1} \frac{h(x')}{[1 + k_1 G(y)]^2} dx'. \quad (35)$$

Now all we need to deal with are the unobservable  $x$  and  $y_R$ .

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<sup>9</sup>This can be calculated using (22) and noting that  $F(y_0) = 0$  and that the value of the cdf  $J(w|x, t)$  at  $y_R = y_0$ .

The distribution of  $y_R$  has already been given - conditional on  $y$ , the distribution of  $y_R$  is random and we can integrate over values of  $y_R$ :

$$\mathcal{L}(w|y, s) = \int \int \frac{\partial y'_R}{\partial w} \frac{\partial L(w|x', y, s)}{\partial y'_R} dH(x') dJ(y'_R) \quad (36)$$

$$= \int \int \frac{\partial y'_R}{\partial w} \frac{\partial L(w|x', y, s)}{\partial y'_R} h(x') j(y'_R|y) dx dy_R \quad (37)$$

with

$$j(y_R|y) = \frac{\left[1 + \frac{\lambda_1}{\delta} \bar{F}(y)\right]^2}{\left[1 + \frac{\lambda_1}{\delta} \bar{F}(y_R)\right]^3} 2 k_1 f(y_R). \quad (38)$$

Given the separability result with respect to individual unobserved heterogeneity  $x$ , we can similarly integrate out values of  $x$ , whereby we use the wage equation to replace  $x$  by the observables  $w, z, s$ . Using the equilibrium wage function (19),

$$\begin{aligned} \ln(x) = & \ln(w) - (1 - \beta) y_R - \beta y + \gamma \ln(s) \\ & + (1 - \beta)^2 \lambda_1 \int_{y_R}^y \frac{\bar{F}(y')}{\delta + r + \lambda_1 \chi \bar{F}(y')} dy' \end{aligned}$$

the density of individual heterogeneity can be expressed as

$$\begin{aligned} h_x = & h_x \left( \ln(w) - (1 - \chi) t_R - \chi t + \theta \ln(s) \right. \\ & \left. + (1 - \chi)^2 \lambda_1 \int_{t_R}^t \frac{\bar{F}(t')}{\delta + r + \lambda_1 \chi \bar{F}(t')} dt' \right). \quad (39) \end{aligned}$$

Using a suitably flexible distribution for the unobservable individual heterogeneity  $h(\cdot)$  then completes this step of the estimation procedure. Note that  $y$  is a function of observed firm characteristics weighted by  $\alpha + \gamma$  and that  $F(\cdot)$  can be non-parametrically estimated from the observed distribution  $G(\cdot)$ .

The maximisation of the likelihood of a cross-section of wages yields a consistent estimate of  $\beta$  and  $\gamma$ .

## 6 Alternative Specification

Per-worker productivity is now given directly as a function of workers' ability ( $x$ ) and the quality of working conditions ( $s$ ). Working conditions may increase or decrease firm productivity (for  $\beta > 0$  and  $\beta < 0$  respectively), but firms differ in productivity proportionally to working conditions.

$$p = x y^\beta \quad (40)$$

The advantage of this approach is that for the estimation of all parameters we use firm-level information. The disadvantage is that we must make functional form assumptions about the relation between job satisfaction and productivity, the latter being unobservable.

In a first version we assume furthermore that firms have full market power (allowing for worker bargaining power has been empirically implemented and follows the same steps as above, following in the footsteps of [Cahuc et al. \(2006\)](#)). In this case firms offer unemployed workers the equivalent of their reservation utility and the unemployed are indifferent between working at the wage offer  $\mu_0(x, y)$  or remaining unemployed.

We know write working conditions  $y$  as having both hedonic and productive value, such that

productivity is given by  $p = x y^\beta$  and utility is written  $u(w, y) = \alpha \log(y) + \gamma \log(w)$ . We then find:

$$V(x, \mu_0(x, y), y) = V_0(x) \quad (41)$$

All firms will do this if they can - this requires an assumption on the support of  $y$  with respect to  $y_0$  but which also depends on the technology and preference parameters as we require the instantaneous utility of taking up a job to be positive<sup>10</sup>:

$$U(x y^\beta, y) > U(x y_0^\beta, y_0) \quad (42)$$

Similarly, for employed workers, the offer  $\mu(x, y, y')$  is defined by (43). This equilibrium wage offer may be lower than productivity (and thus maximal earn-

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<sup>10</sup>Section (6.1) goes through the relationship between relative sizes of current and offered  $y$  to entice an employed worker to switch firms as a function of the parameter values of  $\alpha, \beta$  and  $\gamma$ . The same arguments apply here.

ings) at the current firm since it takes into account expected future earnings trajectories. Thus for some firm with job characteristics  $y'$  the equilibrium offer  $\mu(\cdot)$  would have to satisfy:

$$V(x, \mu(x, y, y'), y') = V(x, x y^\beta, y) \quad (43)$$

Assuming that there is an exogenous Poisson arrival rate of job offers for unemployed of  $\lambda_0$ , the probability that an unemployed receives an offer in a small period is  $\lambda_0 \Delta$ . With probability  $1 - \lambda_0 \Delta$  no job offer is forthcoming and workers remain unemployed. The instantaneous (flow) utility of unemployment is given by  $u(x, y_0)$ . The expected discounted utility of being unemployed is therefore:

$$V_0(x) = \Delta U(x y_0^\beta, y_0) + e^{-\rho \Delta} [\lambda_0 \Delta E_F [V(x, \mu_0(x, Y), Y)] + (1 - \lambda_0 \Delta) V_0(x)] \quad (44)$$

where  $Y$  is a random variable distributed according to a function  $F(\cdot)$ . Subtracting  $e^{-\rho \Delta} V_0(x)$  from both sides gives:

$$(1 - e^{-\rho \Delta}) V_0(x) = \Delta U(x y_0^\beta, y_0) + e^{-\rho \Delta} [\lambda_0 \Delta E_F [V(x, \mu_0(x, Y), Y)] - V_0(x)] \quad (45)$$

Using (41) and since in equilibrium,  $E_F [V(\cdot)] = V(\cdot)$ , the expression for the expected capital gains resulting from a first job are zero. Dividing by  $\Delta$  and using l'Hôpital's rule to show that  $\lim_{\Delta \rightarrow 0} \left[ \frac{1 - e^{-\rho \Delta}}{\Delta} \right] = \rho$ :

$$V_0(x) = \frac{U(x y_0^\beta, y_0)}{\rho} \quad (46)$$

this shows that the value of the unemployed state is simply the discounted sum of the value of the flow utility, since gaining a job is not associated with an increase in *expected* value -a minimum wage changes this by setting a wage floor. <sup>11</sup>

<sup>11</sup> A minimum wage creates capital gains of finding a job. As noted in the supplement to Cahuc et al. (2006), a binding minimum wage in this context leads to assortative matching, as worker-firm pairs with productivity below the minimum wage will not match, whereas both agents may match (and produce productivity above the minimum wage) with higher-productivity partners. Thus offers for which workers are paid the minimum wage cannot be offered indiscriminately to workers of different types. Lack of separability in the wage equation of the component of individual heterogeneity makes the model much less tractable. Thus Cahuc et al. (2006) abandon individual heterogeneity in their analysis of the impact of including a minimum wage, which they find to be small.



For an employed worker receiving a wage offer, three scenarios are of interest:

First, the offering firm may not be able to offer the worker a more attractive package of wage and working condition than she is currently receiving. We assume that firms always benefit from employing workers if they can afford to pay a wage that will attract them. Thus firms' highest wage offer in the bargaining process will grant workers their full productivity. If this offer is below the value of the current wage-working conditions package, workers will not report the offer. This is the case if:

$$V(x, x y'^{\beta}, y') \leq V(x, w, y) \quad (47)$$

Second, it may be the case that although the utility of the offered wage-working condition bundle exceeds the value of the current wage and working conditions package, the current firm can offer a more appealing counter-offer (match the offer).

$$V(x, x y'^{\beta}, y') \geq V(x, w, y) \quad (48)$$

and

$$V(x, x y'^{\beta}, y') \leq V(x, x y^{\beta}, y) \quad (49)$$

For productive job characteristics ( $\beta > 0$ ) that are also appreciated by workers ( $\alpha > 0$ ) this will only be the case if  $y' > y$ . For productive working conditions that are disliked by workers ( $\alpha < 0$ ) productive firms may not be able to sufficiently compensate workers to induce them to move firms. We now sketch the strategy to ensure that we achieve a monotonic ordering of firms' values as a function of  $y$ .

## 6.1 Preference ordering of the value of firms

In order to determine the outcome of the bargaining game between firms with different job characteristics we need to consider the impact job characteristics have on workers' value functions. These depend on the parameter values of  $\alpha$  and  $\beta$ :

- If job characteristic  $y$  is productive ( $\beta > 0$ ) and liked ( $\alpha > 0$ ), job values increase monotonically with the prevalence of this job characteristic for two reasons: because higher productivity-levels will allow the firm to counter potential job offers and raise individuals' wage and for the hedonic value.

- If job characteristic  $y$  is counter-productive ( $\beta < 0$ ) and disliked ( $\alpha < 0$ ), job values decrease monotonically with this job characteristic. In this case, we redefine  $y^* = \frac{1}{y}$  and we can analyse the situation as in the first case.
- If job characteristic  $y$  is productive but disliked ( $\beta > 0, \alpha < 0$ ), will workers prefer firms with higher  $y$ ? Let us first consider under what circumstances the instantaneous utility of a job in firm  $y' > y$  exceeds that in firm  $y$ :

$$u(w', y') > u(w, y) \quad (50)$$

$$\gamma \log(w') + \alpha \log(y') > \gamma \log(w) + \alpha \log(y) \quad (51)$$

$$\gamma \log(w') - |\alpha| \log(y') > \gamma \log(w) - |\alpha| \log(y) \quad (52)$$

The maximum utility that the current firm (with characteristics  $y$ ) can offer is given by the worker's productivity  $x y^\beta$ . We can see under what conditions firm ( $y'$ ) can offer the worker higher utility by assuming that it too provides the worker with maximum utility  $x y'^\beta$ .

$$\gamma \log(x y') - |\alpha| \log(y') > \gamma \log(x y) - |\alpha| \log(y) \quad (53)$$

$$\gamma \log\left(\frac{x y'^\beta}{x y^\beta}\right) > |\alpha| \log\left(\frac{y'}{y}\right) \quad (54)$$

$$\beta > \frac{|\alpha|}{\gamma} \quad (55)$$

This intuitively says that the marginal material benefits given by  $\gamma \beta$  must exceed the hedonic cost  $\alpha$ .

The firm with higher instantaneous utility has bargaining power and will reduce its wage offer to that level as to make individuals marginally indifferent between the two firms. Furthermore, a dynamic effect is taken into account: Individuals will prefer to move to the higher-utility firm because future job offers from firms with utilities above the level implied by the offer will lead to wage increases (wages being the only strategic variable). Dynamic considerations cannot reverse a preference for the firm with higher instantaneous utility.<sup>12</sup>

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<sup>12</sup>This becomes obvious when we consider the wage-setting condition (67) which indicates that the wage offer made by the firm - which includes a dynamic element - is decreasing in the difference in instantaneous utilities between the bargaining firms.

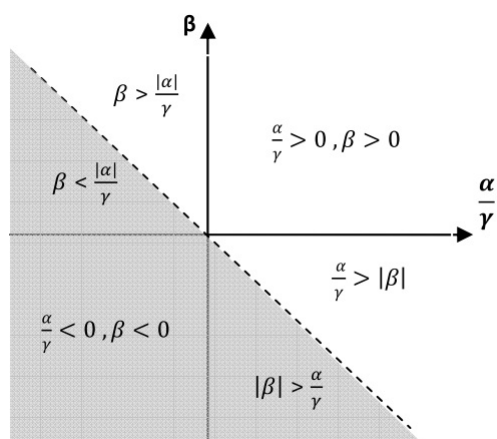


Figure 1: The condition for preference of firms with higher  $y$ -values is  $\frac{\alpha}{\gamma} + \beta > 0$ . This is the case in the non-shaded area. In the shaded area, the parameter space satisfies  $\frac{\alpha}{\gamma} + \beta < 0$  and the  $y$ -variable needs to be redefined to be monotonically increasing. Using  $y^* \equiv y_{max} + y_{min} - y$ , we thus move from the South-West to the North-East quadrant and from the shaded part in the North-West to the non-shaded part in the South-East and vice-versa for the shaded part in the South-East quadrant.

Thus we can analyse this situation analogously to the first case if the condition  $\beta > \frac{|\alpha|}{\gamma}$  holds.

What if, by contrast,  $\frac{|\alpha|}{\gamma} > \beta$ ? In that case we can redefine  $y^* = \frac{1}{y}$  and the analysis just considered goes through analogously.

- Finally, we have the case of non-productive job characteristics that are liked by workers ( $\beta < 0, \alpha > 0$ ). If  $|\beta| < \frac{\alpha}{\gamma}$ , this corresponds precisely to the previous situation with the redefined  $y^*$ , however, where we found a monotonic ordering of firms by  $y^*$ . If, by contrast,  $|\beta| > \frac{\alpha}{\gamma}$ , we can again use the inverse of  $y$  and we are in the situation of a productive but moderately disliked good as above.

In summary, we thus find that for all parameter values we have a monotonically increasing preference relation if we adequately define our measure of job characteristics. Figure (1) summarises the different possible configurations of the parameter space.

## 6.2 Bargaining and equilibrium wage determination

In bargaining between firms' offers it should be noted that since firms' working conditions are assumed exogenous, the only variable of adjustment is the wage. In what follows we will for simplicity of exposition consider the case of productive and liked job characteristics. The analysis does not rest on the specific values of  $\alpha, \beta$  and  $\gamma$  as long as instantaneous utility is monotonic with respect to firms' job characteristics the analysis goes through for all real-valued  $\alpha, \beta, \gamma$ . Given our assumptions on the production function the previous paragraph has argued that this relationship exists throughout the parameter space. An offer  $y' > y$  then implies a new "matched" wage defined by  $w''$  such that  $V(w'', y) = V(x, y, y)$ .

The new "matched" wage may be below the offered wage if working conditions in the current firm are better, but workers' wage must increase if the firm wants to avoid a worker leaving the firm.<sup>13</sup>

Third, it might be the case that the offering firm's wage-working condition bundle cannot be matched by the current firm even if the current firm pays the worker her productivity. In this case  $y' \geq y$  and for some  $\mu(x, y, y') \leq x y'$

$$V(x, \mu(x, y, y'), y') \geq V(x, x y, y) \quad (56)$$

In this case, the second-price auction structure of the model implies that the worker will move firms and gain a wage determined by the job amenity-adjusted equivalent of her productivity at the old firm  $w'''$  such that  $V(w''', y') = V(x, y, y)$ .

What distinguishes these three cases? It appears that the worker's equilibrium strategy only requires information on the two firms' working conditions. There is a threshold value of working conditions above which workers *report* all offers.

The monotonic relationship between working conditions and firms' value to workers that we have shown to hold for different values of  $\alpha, \beta$  and  $\gamma$  implies a threshold level of working conditions defined by<sup>14</sup>

$$\mu(x, z, y) = w \quad (57)$$

<sup>13</sup> For a precise expression of the matched wage, see (76)

<sup>14</sup> It can be noted that the relationship between working conditions and offered wages is monotonically increasing in the range in which the current firm matches the offer (for all values of  $z$  up to  $y$ ) and workers use outside wage offers to raise their wages. For offers above this level (which incite workers to move), differences in working conditions and future earnings paths may imply that workers accept lower wages.

where  $z = z(x, w, y)$

and where  $\mu(\cdot)$  is the wage level needed - for some  $y'$  - to give a worker an equivalent level of value as if she were to receive her full productivity  $x y$  elsewhere.

The function  $z(\cdot)$  is a function mapping wages and current working conditions to “reference values of job amenities” which an offer  $\mu$  equal to the current wage would need to provide.

This is the minimum level of working conditions which will induce the worker to report a job offer given her current wage and working conditions. The argument above implies that this level is below that which induces workers to actually accept the offer. Thus not all job offers result in a change in the capital gains in the Bellman equations - only those with a value of working conditions above  $z(\cdot)$ . Furthermore, only those offers with working conditions above  $y$  can induce a worker to move firms.

$$V(x, w, y) = \Delta U(w, y) + e^{-\rho\Delta} \left[ \lambda_1 \Delta E_Y \max \left[ V(x, x Y^\beta, Y) - V(x, w, y), 0 \right] + \delta \Delta [V_0(x) - V(x, w, y)] + V(x, w, y) \right] \quad (58)$$

For workers we can thus distinguish the impact of job arrivals in a value function like (58) into different components using the distribution of firm working conditions relative to the threshold level.

As noted above, the value of a job offer  $Y$  in this model depends crucially on the productivity of the offering firm - only offers above  $z(x, w, y)$  are reported. For lower values of  $z(x, w, y)$  the worker will not move firms. Thus defining the cdf of working conditions  $F(y)$  and  $\bar{F} \equiv 1 - F$  the value of employment for a short time period  $\Delta$  (58) can be rewritten as:

$$V(x, w, y) = \Delta U(w, y) + e^{-\rho\Delta} \left\{ \lambda_1 \Delta E_Y \left[ \left( V(x, x Y^\beta, Y) | z < Y < y \right) - V(x, w, y) \right] + \lambda_1 \Delta \bar{F}(y) \left[ V(x, x y^\beta, y) - V(x, w, y) \right] + \delta \Delta [V_0(x) - V(x, w, y)] + V(x, w, y) \right\} \quad (59)$$

where  $Y$  is a random draw from the offer distribution. The interpretation of the expression for the value of job mobility on the right-hand-side of this equation relies on the argument that wages are set to equate the value of employment to the outside option available to the worker.<sup>15</sup>

Collecting terms, dividing by  $\Delta$  and evaluating the equation at  $\lim_{\Delta \rightarrow 0}$  using l'Hôpital's rule:

$$\begin{aligned} [\rho + \delta + \lambda_1 \bar{F}(z(x, w, y))] V(x, w, y) &= U(w, y) + \\ \lambda_1 [F(y) - F(z(x, w, y))] E_Y [V(x, x Y^\beta, Y) | z(x, w, y) \leq Y \leq y] &+ \\ \lambda_1 \bar{F}(y) V(x, x y^\beta, y) + \delta V_0(x) & \end{aligned} \quad (60)$$

Evaluating the expectation term:

$$\begin{aligned} [\rho + \delta + \lambda_1 \bar{F}(z(x, w, y))] V(x, w, y) &= U(w, y) + \\ \lambda_1 \int_z^y V(x, x r^\beta, r) f(r) dr + \lambda_1 \bar{F}(y) V(x, x y^\beta, y) &+ \delta V_0(x) \end{aligned} \quad (61)$$

In the special case that workers are paid their productivity ( $w = x y^\beta$ ), the expected option value of encountering other firms is zero:

First, the option value of encountering firms with *worse* job amenities is zero, since no threat can be made as workers are already paid their marginal productivity, so  $z = y$ .

Second, we know that firms with *better* working conditions will lower their wage to the extent of gains resulting from the higher option value of better working conditions. Thus there is no expected gain from encountering firms with better working conditions either.

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<sup>15</sup>Note that workers do not receive the wages and working conditions listed in the value functions when they meet a firm with working conditions of value  $z(\cdot)$  or higher: They do not receive the bundle of working conditions and wages  $x Y^\beta, Y$  with probability  $\lambda_1 \Delta \Pr[z < Y < y]$ , nor a wage of  $x y^\beta$  and working conditions  $y$  with probability  $\lambda_1 \Delta \bar{F}(y)$ . Rather, when these events occur, the value of the offer they receive is equivalent to  $V(x, x Y^\beta, Y)$  and  $V(x, x y^\beta, y)$ , respectively. This is because in both cases, the value function expresses the value of the outside option that individuals hold in these cases.

Thus (60) becomes <sup>16</sup>:

$$V(x, x y^\beta, y) = \frac{U(x y^\beta, y) + \delta V_0(x)}{\rho + \delta} \quad (62)$$

Integrating by parts and using the fact that  $\int_x^{x'} f(x)dx = -\int_x^{x'} \bar{F}'(x)dx$  and that  $\frac{dV}{dy} = \frac{d}{dy} \left( \frac{U(x y^\beta, y)}{\rho + \delta} \right) = \frac{U'_Y(x y^\beta, y)}{\rho + \delta}$ , equation (61) can be rewritten using (62):

$$(\rho + \delta) V(x, w, y) = U(w, y) + \delta V_0(x) + \frac{\lambda_1}{\rho + \delta} \int_z^y U'_Y(x r^\beta, r) \bar{F}(r) dr \quad (63)$$

This equation gives the discounted value of a current job. This expression can now be used to back out the equilibrium wage offer that makes the individual indifferent between firms with different working conditions. In (63), wage and working conditions can be replaced by  $(x z(x, w, y)^\beta)$  since by definition of  $z$  <sup>17</sup>:

$$V(x, w, y) = V(x, x z(x, w, y)^\beta, z(x, w, y)) \quad (64)$$

Inserting this in (62):

$$V(x, x z(x, w, y)^\beta, z(x, w, y)) = \frac{U(x z(x, w, y)^\beta, z(x, w, y)) + \delta V_0(x)}{\rho + \delta} \quad (65)$$

So:

$$V(x, w, y) = \frac{U(x z(x, w, y)^\beta, z(x, w, y)) + \delta V_0(x)}{\rho + \delta} \quad (66)$$

The value of any particular job can be expressed as the discounted sum of wages by a firm of threshold level productivity  $z(\cdot)$  and paying workers full productivity. Replacing this in (64) yields:

$$U(x z(x, w, y)^\beta, z(x, w, y)) = U(w, y) + \frac{\lambda_1}{\rho + \delta} \int_z^y U'_Y(x r^\beta, r) \bar{F}(r) dr \quad (67)$$

Now considering the situation in which a firm with better working conditions  $y' > y$  encounters a worker. Equation (68) then provides a condition on the wage

<sup>16</sup>The case is analogous to the case of the unemployed who also receive their full home productivity (see (46))

<sup>17</sup>Note that the wage may be larger or smaller than  $x z^\beta$  depending on working conditions and individuals' taste parameter  $\alpha$  which determine how large  $z$  needs to be to make the individual indifferent.

offer  $\mu(\cdot)$  required by combining the instantaneous utilities of the bundles of wage and working conditions in the two firms with the option value of working in a firm with better job amenities (and thus higher productivity).

$$U(x y^\beta, y) = U(\mu(x, y, y'), y') + \frac{\lambda_1}{\rho + \delta} \int_y^{y'} U'_y(x r^\beta, r) \bar{F}(r) dr \quad (68)$$

The equilibrium wage offer  $\mu(\cdot)$  can be backed out for employed workers using (68) - relating the utility of an offer by a firm with better working conditions. This can only be done using a specific functional form for the utility function. Using the log-utility form such that  $U(x y^\beta, y) = \gamma \log(x y^\beta) + \alpha \log(y)$  and  $U'_y(x y^\beta, y) = \frac{\beta \gamma + \alpha}{y}$ . Then (68) implies:<sup>18</sup>

$$\log \mu(x, y, y') = \log x + \left( \beta + \frac{\alpha}{\gamma} \right) \left( \log y - \frac{\lambda_1}{\rho + \delta} \int_y^{y'} \frac{\bar{F}(r)}{r} dr \right) - \frac{\alpha}{\gamma} \log y' \quad (69)$$

This is the equilibrium wage, whereby the working conditions in the highest past wage offer can be designated  $z$  and current working conditions as  $y$ :

$$\log w(x, z, y) = \log x + \left( \beta + \frac{\alpha}{\gamma} \right) \left( \log z - \frac{\lambda_1}{\rho + \delta} \int_y^z \frac{\bar{F}(r)}{r} dr \right) - \frac{\alpha}{\gamma} \log y \quad (70)$$

There are two terms that reduce the wage paid to the individual: on the one hand, the option value of encountering future jobs given by the integral term, on the other hand a term relating to the working conditions in the current job, weighted by the workers taste for working conditions.

### 6.3 Equilibrium wage and Compensating Differentials

The expression for the equilibrium wage allows estimation of the bivariate relation between working conditions and wages. By tying better working conditions to productivity, the effect of working conditions on wages is no longer obvious.

<sup>18</sup> This expression is also valid in the case where the current firm matches the offer of a worker who receives an offer from a firm with better working conditions (here  $y'$ ), following equations (48) and (49). In any competition for the worker, the firm with the higher  $y(\equiv y')$  will win and pay according to (69).



Table 1: Simulated  $\beta_{sim}$  from Pooled OLS Regression of Wages on Working conditions

$\ln \bar{\mu} = \beta_0 + \beta_{sim} \bar{Y}$	$E(Y) = 2$	$E(Y) = 4$	$E(Y) = 6$
$\alpha$			
0.1	0.5546	0.1658	0.0260
0.2	0.5156	0.1553	0.0246
0.3	0.5737	0.1489	0.0209
0.4	0.5663	0.1443	0.0215
0.5	0.4123	0.1528	0.0284
0.6	0.4329	0.0809	0.0267
0.7	0.4204	0.1214	0.0027
0.8	0.2717	-0.0621	-0.0213
0.9	0.3124	-0.1661	-0.0615

On the one hand, jobs with better working conditions may have higher wages as well as greater expected wage increases, since more productive firms are more able to match future potential job offers. On the other hand, the argument of compensating differentials - suggesting a negative correlation between wages and working conditions also applies: workers are equally attracted to a post with better working conditions and a lower wage.

It can easily be shown by differentiation that wages decrease with better working conditions  $y$  - for a given level of reference working conditions  $z$ .

#### 6.4 Equilibrium wage and Separability

Equation (69) indicates that the equilibrium wage is separable into a component resulting from workers' individual productivity, a compensating differential effect and a firm productivity and option value effect. The separability into individual and firm and friction effects directly replicates a result from [Postel-Vinay and Robin \(2002\)](#). This result offers a framework for estimating the impact of compensating differentials in a job search framework. The separability result greatly simplifies the estimation procedure. In particular, the estimation of the fixed worker effect (individual ability) can be separated from the effect of search frictions and working conditions. However, the standard econometric technique of controlling for individual productive heterogeneity using fixed effects appears warranted.

The separability result shows that in the current model individual heterogeneity does not influence observed compensating differentials. It might be thought that the absence of controlling for individual fixed effects explains the lack of compensating differentials: The argument would be that individuals with better characteristics sort into jobs with better working conditions. The theoretical model here is not consistent with this story. The reason is related to another result; the “No Sorting” condition (see below): Even in the absence of perfect competition, introducing individual productive heterogeneity on both sides of the market and a production technology based on complementarity does not imply sorting if preferences are ordinally the same and directed search is not allowed.

In the current model, the ultimate cause of the lack of compensating differentials is the fact that job characteristics are productive, and that despite imperfect competition, workers are rewarded some of the additional productivity.

To argue that differentials in individual ability do explain a lack of compensating differentials requires an alteration of the model structure in a way that will generate sorting by individual effects. One way of generating this result may be by assuming that job arrival rates vary as a function of individual productivity. This amounts to weakening the labour market frictions, since the exogenous job arrival rate is a constraint for the agents of the model. Differential job arrival rates may help coordinate workers and firms, generating sorting and increasing the overall match surplus.

$$\frac{\partial \log w}{\partial y} \Big|_z = -\frac{\alpha}{\gamma} \frac{1}{y} + \left( \beta + \frac{\alpha}{\gamma} \right) \frac{\lambda_1}{\delta + \rho} \frac{\bar{F}(y)}{y} \quad (71)$$

The first term in (71) is the compensating differentials effect: In particular, as the preference for better working conditions (given by the parameter  $\frac{\alpha}{\gamma}$ ) tends towards zero, this term vanishes. Also, by nature of the utility function, workers have a taste for diversity (i.e. there is complementarity in working conditions and money) and are thus willing to forego a larger amount of wages for better working conditions if working conditions are bad (i.e.,  $y$  is low). Thus the compensating differentials term is decreasing in  $y$ .

The second term in (71) indicates the option value impact of better working conditions increasing utility<sup>19</sup>. For a given level of reference alternative options,

<sup>19</sup>Note that by definition of  $y$ ,  $\beta + \frac{\alpha}{\gamma} > 0$

workers in firms with better working conditions are paid less even if workers do not care about working conditions *per se*.

Conversely, wages unambiguously increase with higher levels of reference working conditions  $z$  - for a given level of current working conditions  $y$ :

$$\frac{\partial \log w}{\partial z} | y = \left( \beta + \frac{\alpha}{\gamma} \right) \left( \frac{1}{z} + \frac{\lambda}{\delta + \rho} \frac{\bar{F}(z)}{z} \right) > 0 \quad (72)$$

Since  $z$  and  $y$  are positively correlated, there is no clear bivariate relationship between wages and working conditions, i.e. no prediction of compensating differentials: wages and working conditions may be positively or negatively correlated.

Figure (2) simulates the impact of different values of average working conditions  $E(Y)$ , and  $\alpha$  on average wages<sup>20</sup>. The simulation shows that for a low value of  $\alpha$ , wages do increase, but that at higher values the effect is negative. Increasing the level of job amenities leads to a greater impact of the preference parameter  $\alpha$  on wages. Preferences over job amenities matter more if these are relatively more important for the determination of wages. Furthermore, the scale reveals that for the values chosen here the wage becomes negative for values of  $\alpha$  above  $\approx 0.6$ , even for very different average levels of working conditions in the economy. As noted below, the differences in  $\alpha$  can be interpreted as varying across the population, such that we might interpret the negative values of wages as being associated with individuals engaged in work for charities, clubs or other activities which are valued by individuals for non-monetary reasons and which may carry opportunity cost<sup>21</sup>.

The corresponding values for the coefficient of wages on working conditions - according to the theory of compensating differentials argues this is negative - are simulated for different values of  $\alpha$  and average level of working conditions in table (1).

#### *Equilibrium wage and taste for job amenities*

For a given level of working conditions, as people care relatively less about the wage (i.e. for lower absolute levels of  $\alpha$ ), the impact of hedonic preferences on wages decreases, as (73) shows. The extent to which firms can reduce workers'

<sup>20</sup>The underlying distributional assumptions and the simulation procedure are detailed in chapter 3.

<sup>21</sup>The model has some interesting effects when a lower wage-floor is imposed, since this somewhat weakens the competitive pressure for firms in this segment of the labour market (since wages are the only strategic variable). This may be an interesting extension to the model.

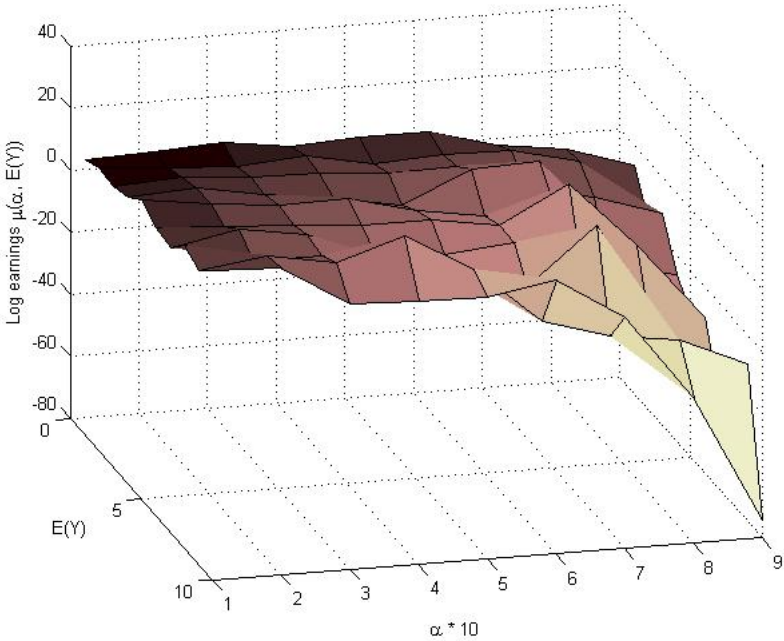


Figure 2: Average log earnings  $\mu$ , Preferences over job amenities  $\alpha$  and their expected level  $E(Y)$

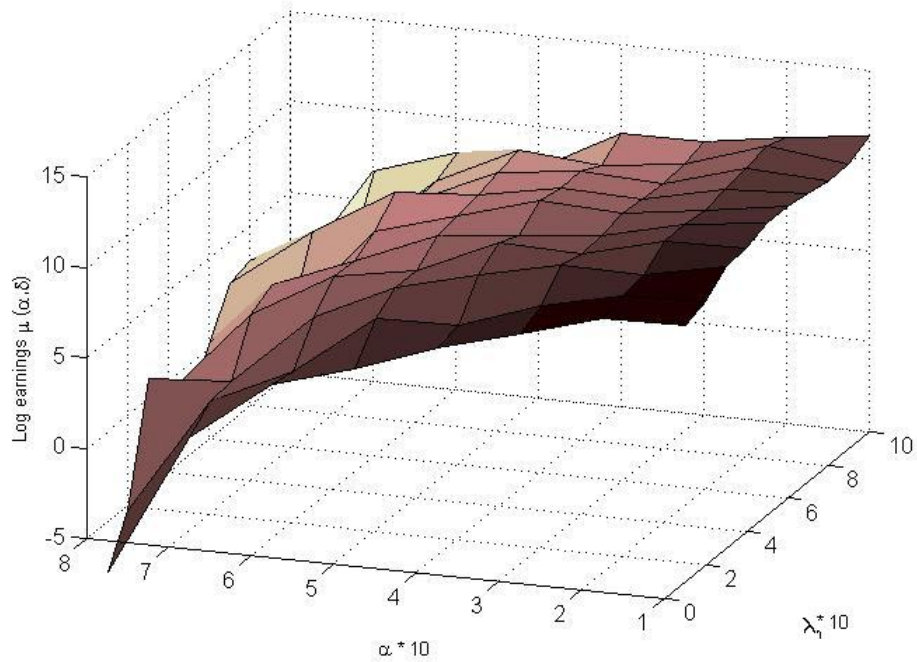


Figure 3: Average log earnings  $\mu$ , Preferences for job amenities ( $\alpha$ ) and Job arrival rate ( $\lambda_1$ )

wages according to their hedonic preferences is related to workers' expectations of the future opportunities of increasing their utility by encountering firms with higher  $y$ . Since the expected future increase in utility will be greater for individuals with a higher positive value of  $\alpha$ , employers can reduce their wage by more ex ante. Another factor affecting the probable returns to job arrivals is the frequency of such offers, as is visible by the negative impact of  $\lambda_1$  on the equilibrium wage. For  $\alpha > 0$  we have:

$$\frac{\partial \log w|y, z}{\partial \alpha} = -\frac{1}{\gamma} \left[ \left( \log \frac{y}{z} \right) + \frac{\lambda}{\delta + \rho} \int_z^y \frac{\bar{F}(r)}{r} dr \right] < 0 \quad (73)$$

Using the simulation technique outlined above, the impact of changing the value of  $\alpha$  on simulated earnings is shown in figure (3).

*Efficiency, Separability and "No sorting"*

Given the multiplicity in productivity, firms with better working conditions can pay workers with higher productivity over-proportionally high wages. Assortative matching of high productivity firms and high productivity workers would maximise overall match rents. However, as in other job search models, the well-known result of "No Sorting" (e.g. [Burdett and Mortensen \(1998\)](#)) of productive workers to productive firms is reproduced here.<sup>22</sup>

That is, it is not the case that more able workers are over-represented in jobs with better working conditions.<sup>23</sup> The intuition for this is simple: The overall production in the economy would be maximised by allocating the most productive workers to the most productive firms (given the particular form of the production function). However, workers' preference over working conditions is independent of  $x$  and firms have no interest in rejecting workers with lower ability.

Although higher productivity workers earn higher wage premiums than lower productivity workers in firms with higher productivity, thus accentuating or mitigating workers' preference over working conditions (depending on the sign of  $\alpha$ ), they have no way of expressing this preference.

One way of interpreting the result is that the exogenous job arrival rate creates a missing market: If there was a market for jobs, higher-productivity individuals would be willing to buy jobs off lower-productivity individuals. If this possibility were given, sorting would result. In the absence of this possibility only the ordinal preferences over firms are relevant - but as long as the conditions outlined in section (6.1) are satisfied, the order is the same for all workers.

### *Heterogeneous tastes and No Sorting*

Equation (76) indicates that equilibrium wages are lower if workers enjoy a particular job characteristic  $y$  (i.e.  $\alpha > 0$ ). It might be thought that allowing for heterogeneity in the tastes for working conditions would lead to a situation in which those workers who care most about working conditions move to jobs with the best working conditions. The argument in the previous section indicates that this will only be the case if we have a reversal of ordinal preferences.

These considerations underline the inefficiency resulting from the search fric-

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<sup>22</sup>[Abowd et al. \(2005\)](#) report that the correlation of person and firm effects is fairly low across industries. Though not necessary, a lack of correlation across industries may be taken as suggestive evidence in favour of No Sorting.

<sup>23</sup>For a formal statement, see (78).

tions as well as the bundling of the markets for labour services and working conditions. Whilst the intensity of workers' preference to move to jobs with good working conditions is a positive function of the relative weight of working conditions in the utility function  $\frac{\alpha}{\gamma}$ , the coordination problems in the labour market make it impossible to "switch jobs" in the same way as agents can trade goods. Given the differences in the intensity of preferences, a labour market that does not sort workers with different marginal willingness to pay is inefficient.

The standard efficiency wage scenario in which good working conditions generate higher productivity can explain why compensating differentials are not more frequently observed under certain assumptions of the preference parameter  $\alpha$ . In fact, one of the contributions of the current paper is to show that sorting does not occur in the equilibrium if good working conditions are productive and individuals vary by ability and relative preferences  $\frac{\alpha}{\gamma}$ . By contrast, sorting by tastes may result if the difference in workers' preferences is large enough to generate different orderings. Furthermore, this may also occur if working conditions are costly to the employer or if the likelihood of encountering a job is a function of workers' preference for working conditions (directed search).

## 6.5 Equilibrium in the alternative specification

Equilibrium in this model is characterised by a stationary distribution of workers types, firm types and wages, as a function of the structural parameters of the model.

## 6.6 Equilibrium flows

The direct relationship between firms' job amenities and worker mobility allows us to write a flow equation describing the in- and outflows of workers (74).<sup>24</sup>

$J(\cdot)$  gives the cumulative distribution of wages, such that a group  $J(w|x, y)$   $s(x, y)$   $(1-u)$   $M$  of workers of type  $x$  earn wages up to  $w$  in firms of type  $y$ , where  $M$  is the measure of the sample population, and  $u$  is the rate of unemployment in the sample population. It consists of the joint density of type- $x$  workers in type- $y$  firms  $s(x, y)$ , the employment rate  $(1 - u)$  (shown to be independent of worker types in (75) and the unconditional density of job amenities  $f(y)$ .

There are two sources of outflow: Workers are laid off at rate  $\delta$  and find jobs they prefer at rate  $\lambda_1 \bar{F}(z(x, w, y))$ .

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<sup>24</sup>This section follows ??

Similarly,  $\lambda_0 u M n(x)$  people are hired out of unemployment into this pool<sup>25</sup>. Furthermore,  $\lambda_1 (1 - u) M \int_{y_{min}}^z s(x, r) dr$  workers from firms with worse job amenities are hired into the pool.

Thus the flow into and out of a pool of workers type  $x$  in firms  $y$  earning up to  $w$  can be given as:

$$\begin{aligned} & [\delta + \lambda_1 \bar{F}(z(x, w, y))] J(w|x, y) s(x, y) (1 - u) M \\ &= \left[ \lambda_0 u M n(x) + \lambda_1 (1 - u) M \int_{y_{min}}^{z(x, w, y)} s(x, r) dr \right] f(y) \end{aligned} \quad (74)$$

The standard relationship for the unemployment rate can be derived by equating inflow  $\delta(1 - u)$  to outflow  $\lambda_0 u$  (supposing that all job offers are accepted) from the unemployment pool, since  $(M - U)\delta = \lambda_0 U$ :

$$u = \frac{\delta}{\lambda_0 + \delta} \quad (75)$$

This expression can then be used to simplify (74):

$$\begin{aligned} & [\delta + \lambda_1 \bar{F}(z(x, w, y))] J(w|x, y) s(x, y) = \\ & \left[ \delta n(x) + \lambda_1 \int_{y_{min}}^{z(x, w, y)} s(x, r) dr \right] f(y) \end{aligned} \quad (76)$$

Applying this steady state condition for those workers who gain their full labour productivity  $w = x y^\beta$  we know that  $J(w|x, y) = J(x y^\beta|x, y) = 1$ , and that the threshold level for reporting is at the actual level of working conditions  $z(x, w, y) = y$ .

$$[\delta + \lambda_1 \bar{F}(y)] s(x, y) = \left[ \delta n(x) + \lambda_1 \int_{y_{min}}^y s(x, r) dr \right] f(y) \quad (77)$$

Solving this differential equation yields:

$$s(x, y) = \frac{1 + \frac{\lambda_1}{\delta}}{\left[1 + \frac{\lambda_1}{\delta} \bar{F}(y)\right]^2} n(x) f(y) \quad (78)$$

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<sup>25</sup>Recall that all job offers are accepted by the unemployed since by assumption,  $U(x, y, y) > U(x, y_0, y_0) \forall y$ . On why this may be reasonable, see discussion relating to (98)



It is found that this equation is separable into  $n(x)$  and a term only dependent upon  $y$ . The joint density of workers of type  $x$  working in jobs with working conditions  $y$ , is given by a separable equation:

$$s(x, y) = s(y)n(x) \quad (79)$$

Better workers do not get more productive jobs on average. See above (section (6.4)) for a discussion of this important feature of the model. The unconditional density of workers in firms with working conditions  $y$  can then be found by integrating (76) over all  $x$ :

$$s(y) = \frac{1 + \frac{\lambda_1}{\delta}}{\left[1 + \frac{\lambda_1}{\delta} \bar{F}(y)\right]^2} f(y) \quad (80)$$

Integrating over  $y$  between  $y_{min}$  and  $y$  then gives the relationship between the observed density of workers in firms with different job amenities as a function of the underlying offer distribution of working conditions  $F(y)$ <sup>26</sup>.

$$S(y) = \frac{F(y)}{1 + \frac{\lambda_1}{\delta} \bar{F}(y)} \quad (81)$$

We can thus deduce the distribution of  $F(y)$  from the observed distribution  $S(y)$ :

$$F(y) = \frac{\left(1 + \frac{\lambda_1}{\delta}\right) S(y)}{1 + \frac{\lambda_1}{\delta} S(y)} \quad (82)$$

$$f(y) = \frac{\left(1 + \frac{\lambda_1}{\delta}\right) s(y)}{\left(1 + \frac{\lambda_1}{\delta} S(y)\right)^2} \quad (83)$$

Substituting (81), (78) and (79) in (74) and noting that  $S(y_{min}) = 0$  then yields:

$$\begin{aligned} J(w|x, y) &= \left[ \frac{1 + \frac{\lambda_1}{\delta} \bar{F}(y)}{1 + \frac{\lambda_1}{\delta} \bar{F}(z(x, w, y))} \right]^2 \\ &= \left[ \frac{1 + \frac{\lambda_1}{\delta} S(z(x, w, y))}{1 + \frac{\lambda_1}{\delta} S(y)} \right]^2 \end{aligned} \quad (84)$$

<sup>26</sup>These relationships were first clearly stated in [Burdett and Mortensen \(1998\)](#)

The number of people employed up to the wage level  $w$  is the same as the number of people employed up to the level of threshold working conditions  $z(\cdot) \in \{y_0 \cup [y_{min}, y]\}$  for some  $y$  (see (57)).

$$J(w|x, y) = J(\mu(x, y', y|x, y)) \quad (85)$$

$$J(z|x, y) = \left[ \frac{1 + \frac{\lambda_1}{\delta} \bar{F}(y)}{1 + \frac{\lambda_1}{\delta} \bar{F}(z(\cdot))} \right]^2 \quad (86)$$

Note that the point mass at  $z = y_0$  implies that the cumulative density function at the lowest possible value of  $z$  is strictly positive as long as  $\frac{\lambda_1}{\delta} < \infty$ , i.e. as long as there is a positive probability that the worker has received no job offer. Given  $z \leq y$  we have  $J(w|\cdot) = 1$  for  $z = y$ .

These results finally enable us to characterise the steady state as a function of three distributions: It has been found that the distribution of  $x$  is independent of both the distribution of  $y$  and  $z$  (though not independent of the distribution of wages - more productive workers earn more everywhere). The distribution of  $z$  is related to the distribution of  $y$  by equation (86), and can be calculated conditional on  $y$ .

It can be noted that the decisions over job mobility here do not depend on individuals' preferences over wages and working conditions, since the distributions of  $x, y, z$  do not depend on  $\alpha$ . The reason for this is that employers with full market power will make individuals weakly indifferent between their wage offer and workers' outside alternative. The outside alternative here is fully characterised by the level of working conditions. In a model with heterogeneous  $\alpha$ , workers are paid differentially, but the critical value of the offer required to increase their wage is the same unless the ordinal preferences over  $y$  are reversed - the specific form of the utility function merely mediates the impact of higher quality working conditions.

## 6.7 Duration of Labour Market States

Unemployment is independent of individual productivity and the hazard rate of finding a job when unemployed is given by the Poisson process with arrival rate  $\lambda_0$ . Mean unemployment duration is then given by  $E(T_0) = \frac{1}{\lambda_0}$ .

Similarly, the rate of becoming unemployed is an exogenous hazard occurring at Poisson rate  $\delta$ . Since it does not vary across different jobs, and is independent of how long a person has been employed for, it is independent of job offers and worker mobility. The expected duration of employment is thus given by  $E(T_1) = \frac{1}{\delta}$ . Jobs can finish due to two events in the model: Worker mobility to another firm; or unemployment. As noted above, the latter event occurs at exogenous rate  $\delta$ . For the former we have a Poisson arrival rate of job offers given by  $\lambda_1$ . Workers will only move from one firm to another if the offering firm's working conditions exceed the current firm's, with probability  $\overline{F}(y)$ . Since the offer arrival rate and the density function of offering firms working conditions are independent, the joint hazard rate of this event occurring is  $\lambda_1 \overline{F}(y)$ . Since this event is independent of unemployment hazard, we have a merged conditional poisson rate of  $\lambda_1 \overline{F}(y) + \delta$ . The conditional expected job duration is then:

$$E(T_1|y) = \frac{1}{\lambda_1 \overline{F}(y) + \delta} \quad (87)$$

The unconditional expected job duration is then achieved by integrating over the whole support of  $y$ :

$$E(T_1) = \int_{y_{min}}^{y_{max}} \frac{f(r)}{\lambda_1 \overline{F}(r) + \delta} dr \quad (88)$$

The analysis could attempt to take into account the frequency of promotions, that is, taking into account the frequency with which individuals report wage offers to their employers without moving employer. The structure of data with respect to promotions (recall data at infrequent intervals) is slightly different to that of employer changes and not used in the current analysis.

## 6.8 Empirical estimation of the alternative specification

It is proposed to estimate the current model using a two-step Maximum Likelihood estimation procedure. In the first step the information and theoretical predictions on durations in different labour market states are used to estimate the transition parameters conditional on observed working conditions which are used to infer an underlying offer distribution of working conditions  $F(y)$ <sup>27</sup>. In the second step, the equilibrium wage distribution is used to express the likelihood of individual wage observations conditional on the transition parameters estimated in the first

<sup>27</sup>For the relation between sampling and underlying offer distribution, see equations (81-83).

step. Results using the current procedure can then be compared to a Full Information Maximum Likelihood strategy, the latter being potentially more sensitive to misspecification. A further advantage is that the sources of identification of the parameter estimates are also used for estimation and the steps of estimation are more easily comprehensible.

## 6.9 Step 1 - Transition Parameters

We use the information on the first unemployment or employment spell and do not consider workers who transit to or from labour market states other than unemployment and employment.

The likelihood contribution of an unemployed worker is thus composed of the probability of being unemployed, given by  $\frac{1}{1+k_0}$  (where  $k_0 \equiv \frac{\lambda_0}{\delta}$  - see (75)) and the density of unemployment duration given as two independent processes: the observed duration of unemployment up to the interview date ( $t_{0b}$ ) and the remaining duration of unemployment if the end of the unemployment spell is observed from later interview information, i.e.  $d_{0f} = 0$ .

$$\frac{1}{1+k_0} \lambda_0^{2-d_{0f}} \exp \left[ -\lambda_0 (t_{0b} + t_{0f}^{1-d_{0f}}) \right] \quad (89)$$

The likelihood contribution of an employed worker consists first of the probability of observing an employed worker  $\left( \frac{k_0}{1+k_0} \right)$  and the density of working conditions in the employed population  $s(y)$ . If workers are not right-censored ( $d_{1f} = 0$ ), we can also use the information about workers' status after the employment spell:

Employed workers either transit to unemployment ( $v = 1$ ) or to another job, the latter case occurring at rate  $\lambda_1 \bar{F}(y)$ , the arrival rate of jobs with better working conditions. Given that the timing of the interview is random across total duration, tenure and remaining duration are again independent random processes.

$$\frac{k_0}{1+k_0} \left[ \delta + \lambda_1 \bar{F}(y) \right]^{2-d_{1f}} s(y) \exp \left[ - \left[ \delta + \lambda_1 \bar{F}(y) \right] (t_{1b} + t_{1f}^{1-d_{1f}}) \right] \left[ \left[ \lambda_1 \bar{F}(y) \right]^{1-v} \delta^v \right]^{1-d_{1f}} \quad (90)$$

Table 2: Estimated transition parameters using data on job and unemployment durations\*

	$\delta$	$\lambda_0$	$\lambda_1$
	job destruction	job offers (unemp.)	job offers (emp.)
overall	0.1594	0.5311	0.5118
female	0.1652	0.5458	0.6590
male	0.1542	0.5191	0.4340
wholesale/retail	0.1681	1.0931	0.4810
manufacture	0.1365	0.9431	0.3897
health/commun./social	0.1524	1.6298	0.4452
transport/comm.	0.1352	2.6020	0.3296
finance	0.1817	1.4867	0.7559
real est./business	0.1813	1.640	0.8417
public admin/sales	0.1357	1.1319	0.8426
education/training	0.1383	1.0650	0.6745

\* *Industry samples based on workers who worked in the same sector during the observation period and excluding the long-term unemployed*

## 6.10 Step 2 - Likelihood of wage observations

In the second step we use the likelihood of individual wage observations. The conditional pdf of wages  $J(w|x, y, z)$  has been shown to be a function of the empirical cdf of working conditions  $S(\cdot)$ , the transition parameters estimated in the first step, the level of working conditions and  $\alpha$  (see 84). The density of working conditions,  $s(y)$  is observed<sup>28</sup> and the distribution of the reference level of working conditions  $z \in \{y_0 \cup [y_{min}, y_{max}]\}$  (the threshold for reporting offers - see (86)) is given by (86). The only unknowns at this step is the distribution of  $x$  and  $\alpha$ .

The marginal distribution of wages for workers who have received job offers since their last unemployment spell (such that  $z \in [y_{min}, y_{max}]$ ) can be given as:

$$j(w|x, y) = \int_{y_{min}}^y j_w(w|z, \alpha) j_z(z|y) dz \quad (91)$$

where, using the expression for the distribution of  $z$  (see (86)),

<sup>28</sup>Currently it is approximated parametrically, but the use of kernel density or the empirical discrete distribution are work in progress

$$j(z|x, y) = \frac{\left[1 + \frac{\lambda_1}{\delta} \bar{F}(y)\right]^2}{\left[1 + \frac{\lambda_1}{\delta} \bar{F}(z)\right]^3} 2 k_1 f(z) \quad (92)$$

There is a point mass in the distribution of reference working conditions  $z$  corresponding to workers who have not received any job offer ( $z = y_0$ ):

$$j(w|x, y) = P(z = y_0) j_w(w|z = y_0, x) \quad (93)$$

The probability  $P(z = y_0)$  can be calculated using (86)<sup>29</sup>.

Having stated the density of wage observations as the density of  $w$  conditional on  $z$  and the density of  $z$ , we can use a deconvolution argument to state the density of  $w$  as a convolution of  $x$  and  $z|y$ . It has been shown that at equilibrium the distributions of  $y$  and  $z$  are independent of  $x$  (see (79)).

Making use of the independence of  $x$  from  $y$ ,  $z$  and with parametric restrictions on the distribution of  $x$ , we can now use the wage equation to replace  $x$  by the observables  $w, y$ . By (69),

$$\log x = \log w - \left[\beta + \frac{\alpha}{\gamma}\right] \left(\log z - \frac{\lambda_1}{\rho + \delta} \int_y^z \frac{\bar{F}(r)}{r} dr\right) + \frac{\alpha}{\gamma} \ln y$$

It thus follows from(91) that the likelihood of wage observations for workers with  $z \in [y_{min}, y_{max}]$  is given by:

$$\int_{y_{min}}^y f_x \left( \log w - \left[\beta + \frac{\alpha}{\gamma}\right] \left( \log z - \frac{\lambda_1}{\rho + \delta} \int_z^y \frac{\bar{F}(r)}{r} dr \right) + \frac{\alpha}{\gamma} \ln y \right) j_z(z|y) dz \quad (94)$$

Similarly, for workers with  $z = y_0$ , we have:

$$f_x \left( \log w - \left[\beta + \frac{\alpha}{\gamma}\right] \left( \log y_0 - \frac{\lambda_1}{\rho + \delta} \int_{y_0}^y \frac{\bar{F}(r)}{r} dr \right) + \frac{\alpha}{\gamma} \ln y \right) P(z = y_0) \quad (95)$$

The overall likelihood of wage observations is then the sum of (94) and (95).

???

<sup>29</sup>Simplifying (86) noting that  $F(z_0) = 0$  and that the value of the cdf  $J(w|x, y)$  at  $z = y_0$  we find:  
 $P(z = y_0) = \frac{1}{[1+k_1 S(y)]^2}$ .

<b>1st spell</b>	employed	unemployed
right-censored	12,369	2,198
uncensored	5,401	795
<i>job-to-job</i>	<i>5,263</i>	-
<i>to unemp</i>	<i>138</i>	-
total	17,770	2,993

The cross-section of wages is thus sufficient to identify the density of the individual heterogeneity component  $x$ , given  $z$  and  $y$ .

We allow the distribution of  $x$  to be a Gaussian mixture (estimated using an EM-algorithm), thus allowing for considerable flexibility in the distribution of the individual heterogeneity term.

The likelihood of wage observations can finally be maximised over  $\frac{\alpha}{\gamma}$  and  $\beta$ .

## 6.11 Data

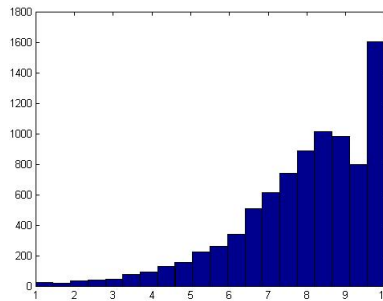
We have set up the British Household Panel for the purposes of this estimation. It includes data on labour earnings, labour market status, duration in employment and unemployment, job characteristics (in particular, subjective evaluation of working conditions).<sup>30</sup>

The data includes several sources of information on working conditions. In particular, individuals are asked to state their job satisfaction in a number of sub-categories: with "work itself", "promotion prospects", "total pay", "relations with boss", "use of initiative", "hours worked" and "use of initiative". A great advantage of these data is that a clear distinction is made between "satisfaction with pay" and satisfaction with other aspects of employment. For our purposes it is required that the evaluation of working conditions  $y$  is made independently of monetary remuneration.

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<sup>30</sup>Since currently no matched employer-employee data with information on working conditions exists, the key assumption that the failure of previous work to find compensating differentials is related to the productivity of working conditions must be tested indirectly. The equilibrium search framework outlined here provides a consistent method to do this.

Figure 4: Distribution of  $y$ : synthetic measure of job satisfaction using three job satisfaction components: satisfaction with relations with boss, work in itself, job security - weighted using PCA; scale standardized to 1-10



In order to work with a tractable measure of job quality,  $y$ , we use a factor analysis of these different dimensions to group these variables. There is limited scope for cross-validation of the subjective indicator developed here with objective working conditions as given by workplace characteristics.

## 6.12 Results

Estimation of the model using a two-step maximum likelihood procedure is work in progress.

What can we say about our key preference and technology parameters  $\alpha$ ,  $\gamma$  and  $\beta$ ?

Preliminary results are reported in table (3) and show positive productivity and hedonic impacts of our measure of job characteristics  $y$ . We created scalar working conditions using the principal component of three measures of job satisfaction other than satisfaction with pay and which were available throughout the data period: satisfaction with relations with a worker's superior, satisfaction with job work in itself and satisfaction with job security<sup>31</sup>. Figure (4) gives the distribution of the values of working conditions according to the new measure which have been scaled to vary from 1 to 10.

<sup>31</sup>These preliminary instrumentation may be challenged - for example, the measure of job satisfaction with job security may be taken as an indication of differing rates of firing probability  $\delta$  which are however assumed homogeneous across the population. The trade-off is between generating enough variation in the instrument and avoiding potential misspecification. In simulations a continuous instrument performed better than discrete instruments, which is one of the advantages of the factor analysis using several instruments.



Table 3: First results using two-step Maximum Likelihood. Standard errors derived using simple (naïve) bootstrap (very small bootstrap: 10 resamples)

transition	estimate	bootstr. s.e.
$\lambda_0$	0.5319	0.0006
$\lambda_1$	0.3884	0.0035
$\delta$	0.1516	0.0046
utility & technology		
$\frac{\alpha}{\gamma}$	0.1521	0.0030
$\beta$	0.1631	0.0025

The fact that  $\alpha > 0$  implies that there are preferences for jobs with better working conditions, but that these do not translate to marginal willingness to pay estimates. The reason this study advances is that productive working conditions may be crucial for this results.

When the population is stratified by sector (see table (5)) and the model is estimated using Indirect Inference (see the appendix), we find interesting - and large - differences across different labour markets. Thus in education we find the highest preferences for good working conditions, almost twice the size as in the health and social sectors. The latter are again significantly larger than in the business and finance sectors. Finally, in the retail and trade as well as the hotel and restaurant sectors we find apparently no significant<sup>32</sup> preferences for better working conditions. Other sectors were not used since sample sizes were too small.

### 6.13 Other Empirical Work

The empirical estimation of the current model should provide some information for example on the importance of working conditions and frictions in determining wages in this framework, as opposed to individual fixed effects. Stratifying the estimation by different subgroups of the population allows for a pragmatic approach to possible heterogeneity in the values of  $\alpha$ , as well as a potential explanation for

<sup>32</sup>Bootstrapped standard errors are work in progress.

differences in earnings profiles as well..

Complementing empirical estimation we can note the consistency of the current model in light of previous empirical evidence. The model appears for example provides an interpretation for the observation that whilst there is no evidence of compensating differentials between occupations, compensating differentials between workers in different industries have been found (see [Clark and Senik \(2006\)](#)). Clark interprets these stylised facts as follows: Workers receive rents within firms, and these rents are essentially distributed at promotion. Workers with different occupations within one industry will be found to have wage differentials not compensated by differences in working conditions. By contrast, since most firms operate only in one industry, industry-differentials will not be driven by within-firm wage variance. As a result, differences between industries may result from compensating differentials. The particular structure of rents resulting from promotions would here be interpreted as a "matching of outside offers".

Empirical estimation of the model can establish how important preferences for good working conditions are. Most previous work focussing on compensating differentials in a competitive framework will be misleading if the labour market is characterised by important search frictions.

## 7 Discussion

This paper introduces productive job amenities into an equilibrium search model with on-the-job search and workers with differing ability. The assumption that good working conditions may be productive inputs is inspired by the efficiency wage literature. In perfect competition, jobs with good working conditions should be associated with lower wages, contrary to empirical findings. Search models have proven to be consistent with labour market data in various areas<sup>33</sup>. The impact of working conditions on wages in a model with on-the-job search and firms competing for workers is presented here.

In the equilibrium, wages are not uniformly lower for better working conditions. Whilst workers who prefer good working conditions receive a lower wage, higher-paid jobs are more likely to be higher-productivity and have better working conditions. This result is consistent with previous empirical findings and contrary to the competitive prediction of a negative relationship between wages and good

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<sup>33</sup>For a discussion, see [Mortensen \(2003\)](#).

working conditions (compensating differentials).

Whilst the model includes heterogeneity in workers, the result is obtained without any prediction of assortative matching between workers and jobs<sup>34</sup>. That is, it is not the case that workers with higher ability are more likely to be found in more productive jobs. Furthermore, adding differential tastes for working conditions does not impact on who gets the jobs with the good working conditions. This result also allows for empirical distinction between the individual effect and a search friction and working conditions effect in the wage equation.

Previous empirical evidence can be interpreted in light of the current model. In particular, the model provides an interpretation for the observation that whilst there is no evidence of compensating differentials between occupations, compensating differentials between workers in different industries have been found (see [Clark and Senik \(2006\)](#)). Clark interprets these stylised facts as follows: Workers receive rents within firms, and these rents are essentially distributed at promotion. Workers with different occupations within one industry will be found to have wage differentials not compensated by differences in working conditions. By contrast, since most firms operate only in one industry, industry-differentials will not be driven by within-firm wage variance. As a result, differences between industries may result from compensating differentials. The particular structure of rents resulting from promotions would here be interpreted as a "matching of outside offers".

Empirical estimation of the model can establish how important preferences for good working conditions are. Most previous work focussing on compensating differentials in a competitive framework will be misleading if the labour market is characterised by important search frictions. Similarly, however, estimations of the importance of search frictions (such as those presented in [Postel-Vinay and Robin \(2002\)](#)) may be reconsidered in light of the importance of differential working conditions as a factor of wage determination. Separate estimation for different sub-groups of the population allows for a pragmatic approach to possible heterogeneity in the values of  $\alpha$  and is consistent under the assumptions outlined here.

Since currently no matched employer-employee data with information on working conditions exists, the key assumption that the failure of previous work to find compensating differentials is related to the productivity of working conditions must be tested indirectly. The equilibrium search framework outlined here provides a

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<sup>34</sup>Here the equilibrium we present remains structurally similar to that of [Postel-Vinay and Robin \(2002\)](#).

consistent method to do this.

The current model allows for a decomposition of the wage into a part resulting from individual ability and an element resulting from search frictions and working conditions. The data to estimate the current model non-parametrically is available. The proposed strategy for obtaining data on working conditions uses subjective evaluation of working conditions (see for example, [Clark and Senik \(2006\)](#)). In particular, the British Household Panel Survey includes data on job duration, subjective evaluation of working conditions and wages in different jobs.

## A Derivation of the equilibrium wage offer for unemployed workers

The equilibrium offer to the unemployed must provide her with at least (and - as a result of the assumption that workers have no bargaining power - no more than) her reservation utility given by  $V_0(\cdot)$ , whereby it is assumed that workers gain their “full home productivity”.

$$V_0(x, x y_0, y_0) = V(x, \mu_0(x, y), y) \quad (96)$$

By definition,  $\mu_0$  this is equivalent to extracting the full marginal productivity from a firm with job amenities of  $y_0$ . As a normalising assumption, workers receive their full productivity when unemployed. The formula (68) can thus be applied directly to a firm  $y' > y_0$ <sup>35</sup> that offers a job with wage  $\mu_0$  to an unemployed.

$$U(x y_0, y_0) = U(\mu(x, y_0, y'), y') + \frac{\lambda_1}{\rho + \delta} \int_{y_0}^{y'} U'(x y, y) \bar{F}(r) dr \quad (97)$$

Following the same steps as for (69) and using the log utility function the equilibrium offer for the unemployed is found to be:

$$\ln w(x, y_0, y) = \log x + \left[ \beta + \frac{\alpha}{\gamma} \right] \left( \ln y_0 - \frac{\lambda_1}{\rho + \delta} \int_{y_0}^y \frac{\bar{F}(r)}{r} dr \right) - \frac{\alpha}{\gamma} \ln y \quad (98)$$

Note that equation (98) indicates that the unemployed are willing to accept a reduction in their monetary compensation in order to move out of unemployment. The intuition is that once in employment the worker will receive job offers and career advancement. This is a result of the assumption that non-monetary conditions in employment are preferred by individuals to unemployment. This is consistent with the fact that unemployment is consistently found to decrease subjective levels of well-being conditional on income - contrary to standard economic models.

## B Workers with bargaining power

Assuming that workers have some bargaining power and

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<sup>35</sup>See below why  $y_{min} > y_0$  may be a good assumption.

## C Estimation using Indirect Inference

### *Simulating the steady state wage distribution*

The wage distribution is characterised by the joint distribution of working conditions, workers' types and the level of working conditions of a previous firm or offer (the worker's reference level of  $y$ ).

- First draw a random  $x \in [x_{min}, x_{max}]$  from distribution  $G(\cdot)$ .
- Independently draw a level of working conditions from distribution  $S(x, y)$  (see (78)).
- Draw a reference level of working conditions  $z(\cdot)$  for each worker according to density  $J(z(\cdot)|y)$  (see (86))<sup>36</sup>

The model has also been estimated using simulated method of moments. The intuition for this method is to simulate data using the structural model developed here to see whether the model could replicate certain moments found in the data. Since moments are matched rather than the actual parameters estimated, this strategy is an application of “indirect inference” (see Gouriéroux et al. (1993)). To this end a simulation procedure has been set up following the procedure outlined in section (6.5).

*Which moments to match?* To make sure that matching the moments is sufficient to estimate the structural parameters of interest, we require one moment for each structural parameter. Using the simulations we then test whether changing a particular moment in the data is associated with the corresponding structural parameter.

Obvious candidates for the moments to use for the transition parameters are labour market durations:

- Unemployment duration for  $\lambda_0$
- Job duration for  $\lambda_1$
- Employment duration (in different jobs) for  $\delta$

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<sup>36</sup>The algorithm used is based on Postel-Vinay and Robin (2002). After drawing a level of current working conditions  $y^*$ , the larger of two independent draws  $z_1, z_2$  determines  $z|y^*$ . Both  $z_1$  and  $z_2$  equal  $y_0$  with probability  $\frac{1}{1 + \frac{\lambda_1}{\delta} S(y)}$  and are otherwise drawn from a conditional distribution of job amenities  $f(y)$  truncated above at  $y^*$ . This is consistent with (86)

As should be intuitively clear, changing these moments should be associated with a change in the estimated structural parameter. For instance, longer observed unemployment duration should lead to a decreased estimate of the job offer arrival rate,  $\lambda_0$ .

For the remaining parameters we first set up an auxiliary model generating moments that describe the conditional correlations between variables of interest, in our case in particular income and job satisfaction. The auxiliary moment is not intended to have any direct theoretical interpretation, but to test the relationship between important variables for which our model predicts the existence of causal relationships (see [Gouriéroux and Monfort \(1995\)](#)). The moments describing their relationships should be approximated well by the set of estimated structural parameters:

$$\ln w_i = a + b \ln y_i + \sigma \epsilon_i \quad (99)$$

From this auxiliary wage regression we can use the constant and error variance  $a, \sigma$  to estimate the parameters of individual heterogeneity (the parameters of the discrete multinomial distribution of individual heterogeneity  $G(x)$ ). Since  $\alpha$  gives the degree of marginal willingness to pay for higher job satisfaction, we can use the auxiliary regression parameter  $b$  to match the simulated moments to the data. This leaves us with  $\rho$ , which we do not attempt to identify but parameterise as 0.05.

The result of the estimation procedure (Simulated Method of Moments) is a binding function linking the simulated and the empirical moments. To test that the binding function is injective, i.e. that the binding function effectively links individual moments to their structural analogues, we use Monte Carlo simulations to test what happens when moving every single variable holding others constant. The vector of structural moments for which this can be done is <sup>37</sup>:

$$\Theta = f(m_x, p_x, \mu_y, \sigma_y, \alpha, \lambda_0, \lambda_1, \delta) \quad (100)$$

Thus we tested the impact of marginal changes of the structural parameters on the value of the estimated (simulated) moments that are matched. In particular, we confirmed that:

- increasing  $m_x$  increases the value of  $a$ ? This is consistent with average individual productivity should increasing the intercept of the wage regression.

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<sup>37</sup>Note that for simplicity I have assumed that heterogeneity in  $x$  can be captured in only two groups such that the distribution of  $x$  consists of two parameters.

Table 4: Parameters of the model

parameter	estimated value <sup>1</sup>
$\alpha$	0.191
$\lambda_1$	0.513
$\lambda_0$	0.702
$\delta$	0.054

<sup>1</sup> Simulated sample size: 10,000  
 Empirical sample size: 10,000

Table 5: Estimated  $\alpha$  stratified by sector

	estimated $\hat{\alpha}^1$	proportion of sample
Education	0.3789	9.3 %
Health and Social	0.1888	11.9 %
Other Social/Local	0.1980	5.7 %
Business / Real Est	0.1131	9.8 %
Finance	0.1200	4.1 %
Retail / Trade	0.0042	15.3 %
Hotels/Rest.	0.0000	5.13 %

<sup>1</sup> Simulated sample size: 10,000; Empirical sample size: 10,000

- increasing  $\lambda_0$  reduced unemployment duration.
- increasing  $\lambda_1$  increased the duration of employment.

Using these moments to match, we find the following parameter estimates

Note that apart from the result for  $\delta$ , these results have similar orders of magnitude to the aggregate results across sectors reported above (figure (4)) using maximum likelihood-based estimation.



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