Unemployment Duration and Worker Turnover *
(very preliminary)

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July 25, 2014

Abstract

Worker turnover is large in the US economy. Using SIPP data, we find that over 43% of employment relationships end within their first year. We also document that longer unemployment durations are associated with a lower job tenure in the subsequent employment spell. To address this evidence, we build a directed search model of the labor market with adverse selection. Workers differ by their ability to form good matches. We assume that match quality is both an inspection and an experience good. Different types of workers are sorted out in different submarkets in equilibrium because of the firms' ability to backload wages. Wages and the probability of a job ending increase with job tenure. While the latter is by construction, wages increase because firms offer risk averse workers insurance to income shocks by raising the hiring chances, and limited commitment pushes wages further down for short than for long tenures. A composition effect explains the declining exit rates from unemployment and the falling job tenure with unemployment duration. We design the optimal unemployment insurance.

Keywords: Unemployment Duration, Worker Turnover, Directed Search, Adverse Selection
JEL Codes:

*We thank Johannes Gierlinger, Joan Llull, and Martí Mestieri for their feedback. We are also grateful to numerous participants at the NBER Summer Institute 2014, REDg-Dynamic General Equilibrium Macroeconomics Workshop 2014, and at various seminars. Sekyu gratefully acknowledges financial support from the Spanish Ministry of Economy and Competitiveness through grant ECO2012-32392 and through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2011-0075), and Javier from the Spanish Ministry of Science and Technology under Grant Nos. ECO 2010-19357.

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1 Introduction

A large number of employment spells are short lived in the US economy. Using data from the Survey of Income and Program Participation (SIPP), we find that above 43% of the newly employed workers return to non-employment within a year, whereas this separation rate drops to 25% in the second year. To the best of our knowledge, the only comparable reference is Farber (1999), that raises the first number to 50%.[1] This high worker turnover rate is driven neither by a set of individuals who frequently enter and leave employment, nor by recalls. Furthermore, we document that these transitions from employment to non-employment within a year are more likely the longer the previous non-employment spell is. For example, the probability of staying employed one year after is 15% lower for an initial non-employment spell of 6 months relative to a 1-week spell. This evidence suggests, first, that the distribution of match qualities is quite right-skewed, and, second, that the quality of the matches formed is rather low, and particularly so the longer the previous non-employment spell is.

We conjecture that an underlying reason for this high turnover rate is the limited unemployment insurance (UI) provision in the US economy. When facing risk-averse workers in an economy with incomplete markets, private markets offer their own insurance to income shocks by lowering the hiring standards.[2] A large number of matches of low quality are formed and, hence, are likely to be dissolved in a short period of time. Because workers are less selective the longer a non-employment spell is, this pattern becomes accentuated for long non-employment durations.

To address the above evidence, we build a directed search model of the labor market with learning about match quality. There are two types of workers (type ℓ and h). Workers differ in their ability to form good matches. An applicant's type is unobserved by recruiting firms. Therefore, an adverse selection problem may arise if type ℓ workers crowd out the type h ones. Firms commit to job offers to attract applicants. Following Pries and Rogerson (2005), we think of match quality both as an inspection and an experience good.[3] We assume that

[1] Farber (1999) consider full time jobs and uses National Longitudinal Survey of Youth (NLSY) data. The discrepancy between SIPP and NLSY estimates is mostly due to that 1) younger workers (oversampled in the NLSY) are more likely to experience job terminations, and 2) full time jobs (which he considers exclusively) are usually of longer duration.

[2] Our paper is consistent with the idea casted in Acemoglu and Shimer (1999): unemployment benefits make firms create better jobs. In their work, better jobs are associated to positions with larger capital investments and, consequently, higher wages. We abstract from this mechanism and also from human capital accumulation, that is the main component of the earnings increase with potential experience according to Altonji et al. (2013). We also abstract away from crowding out effects of unemployment insurance on precautionary savings, which are found to be large by Engen and Gruber (2001).

[3] This characterization was first introduced by Nelson (1970) meaning that the quality of a match can only

the idiosyncratic match quality can only be partially inspected when a worker and a firm meet. Match quality is an inspection good in the sense that an employment relationship is formed if the observed signal is sufficiently good. If a match is formed, its actual quality is learned by experiencing it. Formally, the match quality of worker-firm pairs is observed with some noise, leading to a delay in the learning process. If the match quality is learned to be bad, then it is mutually beneficial to destroy the match. By posting contracts that specify a reservation value for the signal and a two-wage schedule, firms make different workers self-select in different markets in equilibrium. We impose limited commitment in the sense that contracts cannot be renegotiated, but agents can walk away from them.

Workers are risk averse and markets are incomplete in the sense that workers can only insure themselves against income shocks through job search. Risk neutral firms provide insurance by posting low-wage jobs with low hiring standards. Although firms would benefit from offering perfect consumption smoothing to workers while employed, limited commitment requires a lower wage over the stage at which match quality remains unknown. Therefore, equilibrium individual and average wages increase with job tenure. Given that type h workers are more likely to form good matches, job creation is larger for this type of workers in equilibrium. Furthermore, by construction, longer job tenures lead to smaller separation rates. These results are consistent with the empirical evidence. See for example Topel (1991), Dustmann and Meghir (2005), and Altonji et al. (2013) for wage dynamics, and Farber (1999) and Nagypál (2007) for turnover rates.

Exit rates from unemployment fall with unemployment duration because of a composition mechanism, in line with the empirical literature. Job-separation rates are also monotone in the length of the previous unemployment spell, but need not be decreasing as we find in the data. To capture the heterogeneity linked to the distributions of signals of match quality in our calibration exercise, in line with Moscarini (2003) and Menzio and Shi (2011), we target the separation rates within the first year as a function of the unemployment duration and achieve a good fit. The interpretation of the duration functions of exit rates is that type h workers face more job opportunities and higher hiring standards. As a result, they leave unemployment faster than type ℓ workers and their spells also last longer.

We then use our calibrated model to determine the optimal unemployment insurance. Two of the well-known features of the problem are modeled. First, the optimal policy provides workers with resources to smooth consumption while unemployed. Second, more generous unemployment benefits make workers more selective and reduces job creation, leading

be assessed *experiencing* it. In contrast, Hirshleifer (1973) coined the inspection term to point to evaluations carried out prior to the formation of the match.

to an increase in the average unemployment duration. This trade-off is optimized in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). In addition, UI plays, in Burdett (1979) words, a search subsidy role reducing the insurance private markets offer and, hence, the extend of mismatch. Unlike to Marimon and Zilibotti (1999) and Teulings and Gautier (2004), where the separation rate is independent of the match quality, unemployment benefits also lead to longer lasting relationships. Then, the overall effect on the unemployment rate cannot be theoretically asserted. We quantitatively estimate that the optimal unemployment insurance is monotone (?) and increases welfare relative to the status quo by x% (?).

Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) find that unemployment benefits should decline with unemployment duration to provide incentives for workers to search for jobs. When allowing workers to save and borrow, Shimer and Werning (2008) find that benefits should slightly increase with duration, yet the welfare cost of time-invariant benefits are tiny. Our quantitative work may seem at odds with the findings in the empirical literature. For example, Card et al. (2007), for Austria, and van Ours and Vodopivec (2008), for Slovenia, find that a variation in the generosity of unemployment benefits does not significantly lengthen postunemployment spells. However, this needs not be inconsistent with our results because, first, worker turnover in European countries is much smaller than in the US (see Pries and Rogerson (2005)), and, second, the European unemployment insurance systems are much more generous than that of the US along many dimensions. Supporting our conjecture, Centeno (2004) finds that a more generous UI lengthens subsequent job tenure using NLSY data.

We contribute to the theory of worker turnover building on Jovanovic (1979). Specifically, we borrow the learning process from Pries (2004) and Pries and Rogerson (2005). Nagypál (2007) finds that learning about match quality accounts for a large share of worker turnover (relative to learning by doing), particularly for job tenures longer than 6 months. Moscarini (2005) brings Jovanovic (1984) to the Diamond-Mortensen-Pissarides framework with on the job search to set a unified theory of unemployment, wage inequality and turnover. While in the latter, wages increase with job tenure because they reward the worker's marginal productivity, they are the outcome of a bargaining game in the former. Instead, as explained above, the within-firm wage dynamics in our setting result from the interplay of incomplete markets, wage-posting and limited commitment. Although search on the job is not allowed, our model also delivers wage inequality among observationally identical jobs and workers.

Our paper contributes to the competitive (and directed) search literature by analyzing an economy with adverse selection and learning on the match. This branch of the search literature starts with Peters (1991) and Moen (1997), and Guerrieri et al. (2010) investigate the effects of adverse selection in the market economy.

Notice that average productivity increases with a more generous UI in our case because of the effects on the formation of better matches. Other policies are also of interest. In particular, it has been found in the literature significant effects on productivity and welfare from employment protection policies and minimum wages, see e.g. Acemoglu (2001), Pries and Rogerson (2005) and Hopenhayn and Rogerson (1993). As in the two former articles, minimum wages and unemployment benefits both increase the average output per worker. However, in our case, minimum wages reduce the insurance private markets offer to risk-averse workers. Employment protection policies also affect the hiring standards. The latter estimate that a tax on job destruction equal to one year wages reduces consumption and average productivity by over 2 percent. We analyze these two other policies and compare the results with our benchmark (UI) policy.

The paper is organized as follows...

2 Data

In this section we describe our data work. To study unemployment and employment histories at high frequency (weekly level) we focus on two data sets which meet our requirements. First, the Survey of Income and Program Participation (SIPP) which collects retrospective data on individuals for time windows of up to four years. In what follows, we use the 1996 and 2001 panels, containing labor force histories observed between 1996 and 2003.[4] Second, we consider the 1979 National Longitudinal Survey of Youth (NLSY), a panel data focusing on a single cohort of individuals. Survey participants in the NLSY were first observed in 1979 when they were between 14 and 22 years old. Since then, these individuals have been surveyed on a yearly basis until 1994 and biannually from then onwards. We consider information from 1979 and 2006, inclusively. The benefit of the NLSY over the SIPP is that it contains much longer employment and unemployment histories. However, since it focuses on only one cohort, one cannot separately identify time versus cohort effects and its overall representativity changes with time. In both datasets, we restrict our attention to workers

[4]The estimates are larger if only considering the 1996 panel and shorter if adding the 2004 and 2008 panels, which include the last boom and great recession. Therefore, we decided to take a somewhat conservative approach.

aged 16 to 65.[5] See the Data Appendix 8.1 for further details.

2.1 The SIPP

In the SIPP, individuals are interviewed every four months for up to four years. In particular, they are asked to report their employment status for each week of the previous four month period, their wages, working hours if employed and a number of demographic and other relevant characteristics. We restrict our sample to individuals who lose and subsequently find a job within the period spanned by the survey. We do not interpret non-employment spells shorter or equal than 2 weeks as separations if the job id number does not change. Furthermore, we only consider spells with a positive number of working hours reported at re-employment. Since we aim to evaluate the turnover within the fist year, we only consider individuals whose employment histories are observable for at least a year after reemployment.[6] Our sample then consists of 11099 observations. Individuals with a single observation account for approximately 80% of our sample, whereas those with 3 or more observations add up to less than 4%.

We label a worker as employed (E) in a given week if he reports to have a job, regardless of whether he is working, absent or on temporary layoff, worked as a paid employee or in their own business.[7] Everyone else is labeled as non-employed ($\not{E}$). An employment to non-employment ($E\not{E}$) transition is identified when the worker has no job for at least one week within a month. We will refer to the newly employed workers who return to non-employment within a year as non-stayers, and as stayers otherwise. Similarly, a non-employment to employment ($\not{E}E$) transition occurs when the worker has a job during all weeks of the month following a week of non-employment. For clarification, our sample consists of spells that correspond to $E\not{E}E$ transitions and that can end in either E or $\not{E}$ within a year.

We find that 43.88% of the newly employed workers are non-stayers.[8] As a reference, we also estimate this statistic for different time horizons. For example, 29.05% of the new

[5] Given the structure of the NLSY data, the age restriction for older workers doesn't apply: the oldest considered worker is 49 years old by 2006 in our sample.

[6] Arguably, our estimate of the turnover rate is likely to be conservative because of a potential attrition bias. We compare the turnover rates at m¡12 months for our sample with the ones obtained from a sample formed by all spells with a history observable for at least m months after re-employment. We find that the latter rates are consistently a bit higher.

[7] As noted in the Data Appendix 8.1, this classification differs from the 'employment' category in the Current Population Survey.

[8] Although we have reduced our data analysis to the 1996 and 2001 panels, the estimates do not vary significantly across the post-1996 panels. The turnover rate for the 1996, 2001, 2004 and 2008 panels is 45.32, 41.06, 44.76, and 43.27, respectively.

Table 1: Descriptive Statics: Non-stayers vs. Stayers

Variable	**Non-stayers**		**Stayers**	
	Average	St. dev.	Average	St. dev.
Log hourly wage	1.746	0.785	2.032	0 .758
Age	27.498	11.620	32.452	12.065
Female	0.525	.499	0.530	.499
White	.842	.365	.845	.362
Black	.113	.316	.103	.304
Marital status	.294	.455	.449	.497
College	.077	0.267	.150	.367
Post-college	.021	.145	.055	.228
Non-emp. duration	20.291	18.869	17.109	16.949
Working hours	30.919	13.557	34.720	12.932
Firm size	1.792	0 .837	1.890	0.855
Number of employers within first year	1.205	0.446	1.387	0.622

employment spells conditional on the observation being in the sample for at least half a year end in non-employment within the first six months. In contrast, the $E\not{E}$ transition rate within the second year of employment conditional on being a stayer and remaining in the sample for at least two years since re-employment is 24.96%. For comparison, using NLSY data, Farber (1999) reports that one third of new full-time jobs end in the first six months, one half in the first year, and two thirds within the first two years.

This estimate is sensitive to a number of factors. First, it differs across a number of observable characteristics as Table 1 suggests. Second, there are individuals with several spells in our sample. See Table 3 in the Data Appendix 8.1. If we only look at individuals with one observation, 41.73% of them are non-stayers. Third, similar to Fujita and Moscarini (2013), we find 'seam effects' in the sense that the turnover rate is significantly higher for spells that start with a new wave. See Table 4 in the Data Appendix 8.1. Fourth, although we did not consider separations with a length shorter than 2 weeks if the worker returned to the same employer, it can be argued that a number of $E\not{E}E$ observations with a very short non-employment spell mask job-to-job transitions and the turnover rate within the first year may differ from the above estimates. Instead, we find it to be approximately 44% when only considering observations with $\not{E}$ spells longer than 3 or 4 weeks.

Recalls deserve a special consideration.[9] Fujita and Moscarini (2013) find that about slightly over 30% of the $E\not{E}E$ spells in the 1996 and 2001 SIPP panels are accounted for by recalls by a former employer. To identify a recall, we use the two job id numbers the SIPP provides. We find it to be 21.72% in our sample, which is quite close to their number prior to

[9] For example, Katz and Meyer (1990) find that workers with recall expectations exert less search effort, have shorter unemployment spells and face smaller wage cuts.

the implementation of their imputation procedure. That is, we have 8688 observations not involving a recall. The percentage of non-stayers amounts to 36.62 after a recall and 45.89 otherwise. Furthermore, out of the over 20% of individuals with more than one spell in our sample, recall only affects 2.04% of them, and only 0.06% out of those with more than one spell who were recalled.

Therefore, we run a Probit regression to account for all these factors. Specifically, we control for a number of observables characteristics, use the national unemployment rate as a business cycle indicator and monthly dummies to capture seasonality effects, and include seam dummies. We obtain that the predicted turnover rate amounts to 43.35%, and goes up to 45.56% when excluding the spells involving a recall.

Table 1 shows that non-stayers differ from stayers along many dimensions. In particular, stayers are older and more educated, and work more hours on average. They also earn significantly higher wages, even if reducing the sample to college and post-college graduates. We also find that over 68% of the stayers have a single employer within the first year after re-employment and over 93% at most two jobs, whereas the first number goes up to 81% for the non-stayers. The stayers had also had shorter non-employment spells prior to re-employment. If we only consider non-recall spells, the average non-employment duration is 23.20 weeks for non-stayers and 19.85 for stayers. If we further restrict our sample to individuals aged 25 to 65, we find that the turnover rate within a year falls to 33.55%. Moreover, while age and marital status differences vanish, the differences in log wages, education attainment and working hours as well as the gap in the length of the previous non-employment spell reduce, but still remain quite significant.

To investigate the relationship between the probability of transiting back into non-employment within a year and the length of the previous unemployment spell, we run a Probit regression with the EU transition rate as the dependent variable. We use the same regressors as before. In particular, the effect of non-employment duration is estimated using a quartic polynomial.

Figure 1 depicts the dynamics of the turnover rate as a function of the length of the last non-employment spell for $E\not{E}E$ spells not involving a recall. Plotted values are normalized by the value corresponding to the first week. It shows that the one-year turnover rate increases with non-employment duration. In particular, the predicted turnover rate within a year after re-employment increases by 15% (30%) for those workers who became employed after half a year (a year) of non-employment, relative to those who only had stayed one week without a job.[10] The average non-employment duration for our non-recall subsample is 21.38 weeks,

[10]These percentages are 12 and 31% for the whole sample.

and about 72% of non-employed workers find jobs within the first 6 months.

To sum up, our estimates for turnover rates within a year are high and comparable to other numbers found in the literature, being even higher for spells not following a recall. We find a positive relationship between the probability of transiting into non-employment within a year after re-employment and the length of the previous non-employment spell.

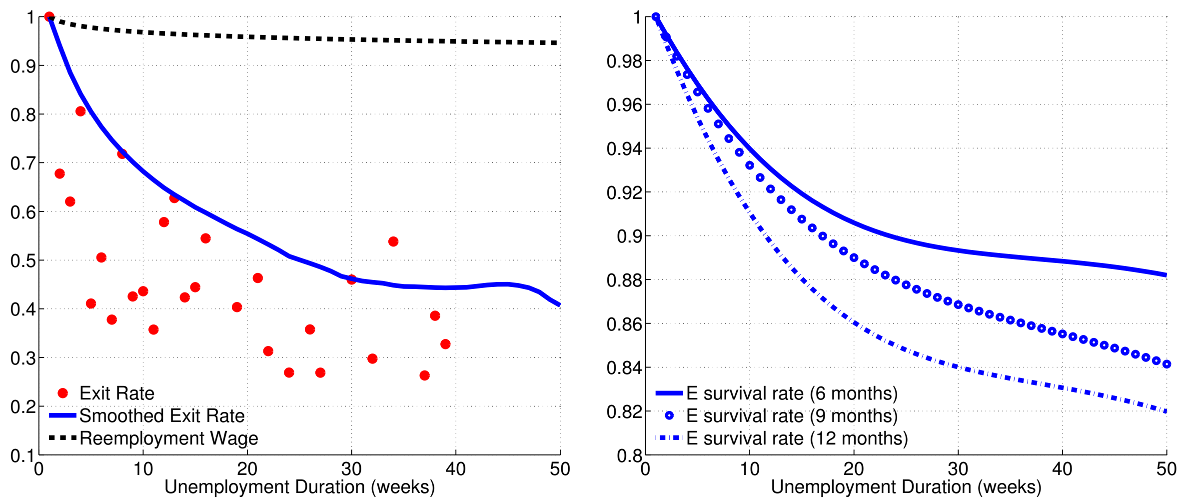

Figure 1: Normalized re-employment wages and predicted exit probabilities from unemployment (Left) and normalized predicted probabilities of remaining employed after 6, 9 and 12 months (Right). Note: The data are from the 1996 and 2001 SIPP panels. All variables are normalized by the corresponding value at the first week. The predicted probabilities are the probit-predicted values with all regressors, except for the unemployment duration, are evaluated at their sample mean. For the exit probabilities from unemployment, we consider the subsample of workers whose unemployment spell is longer than the specified duration, and also plot the values obtained from a Kernel-weighted local polynomial smoothing.

2.2 The NLSY

Information in the NLSY is gathered in an event history format, recording start and end dates of important life events (like jobs and unemployment). The survey contains weekly information on labor force participation, unemployment, jobs and several other measures, as well as demographic information on the household of the individual. Our sample consists of all individuals observed between 1979 and 2006. For each of them, we record unemployment to employment transitions and the information on the subsequent employment spell.

The main benefit of using the NLSY is its length. We have information on weekly labor force statistics for up to 27 years, leading to recorded employment histories which are much longer than those in the SIPP. Thus, right censoring issues and length bias are less of a concern. On the other hand, the focus of the NLSY is on just one cohort which produces a couple of issues: time and cohort effects cannot be separately identified and the sample is not fully representative of the US economy.

We try to use the same demographic controls and observables as with the SIPP data. The most glaring differences are in terms of state and firm size variables, which are not present in the NLSY. After computing employment spells and restricting our sample to those which started when individuals were 20 years of age or older,[11] we are left with 11834 individuals and 73649 individual cross spell observations.

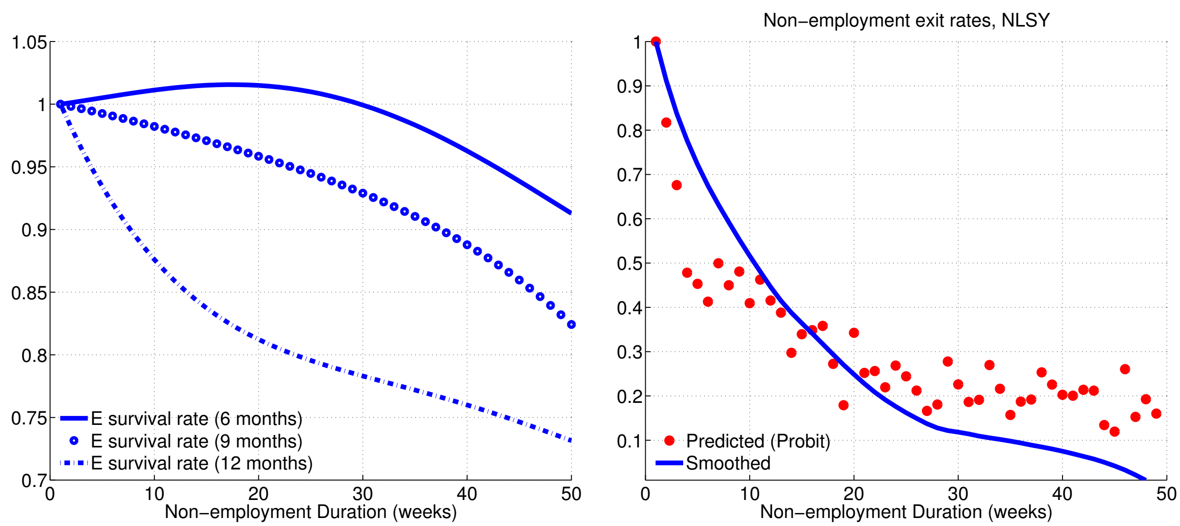

Figure 2: Relative predicted probabilities (smoothed) of remaining employed after 6, 9 and 12 months (Left) and relative predicted exit probabilities from unemployment (Right) from NLSY data.

Figure 2 presents unemployment exit rates and employment tenure statistics by previous unemployment duration for the NLSY. Both panels of the figure show that the conclusions from the SIPP are quite robust. The right panel shows unemployment exit rates by length of the previous UD spell. Given the increased sample size in the NLSY, the noise in the raw data is much less than in the SIPP. The main difference between both datasets is the the decrease in the relative probabilities of finding a job given different unemployment durations: for example, the probability of finding a job after half a year of unemployment is around 45% (Right panel of Figure 1) for the SIPP sample. The number for the NLSY sample falls to around 10%. Given that the latter survey focuses on younger individuals, the difference

[11]By 2006, the oldest individuals in the NLSY were 49 years old, so we impose no sample restrictions on later years.

could be explained in the oversampling of spells where individuals chose to go to school instead of keep searching for a job.

With respect to job tenure after an unemployment spell, the information in the NLSY confirms the findings from the SIPP: longer unemployment durations are associated with lower probabilities of remaining employed. As explained above, these probabilities decrease the longer the considered employment spell: the left panel of Figure 2 shows the probability of remaining employed after 6, 9 and 12 months. The difference between the NLSY and the SIPP lays in the strength of the effect: in the NLSY, the probability of staying in a job for one year, after an unemployment spell of half a year, falls to around 90%, while the same figure is 82% in the SIPP.

In spite of the stark contrasts between the SIPP and the NLSY, in terms of size, length and considered sample, in this section we showed that unemployment durations have important effects on both the probability of finding and retaining jobs.

3 Model

Time is discrete. The economy is populated by a mass one of infinitely-lived workers and a large continuum of risk-neutral firms. Workers are risk averse. Let v denote the utility function of workers. It is assumed twice continuously differentiable, and increasing and concave in its argument. We also assume that $v(0) = 0$ and $\lim_{w \to 0} v'(w) = \infty$. All agents discount future utility at common rate β. Markets are incomplete in the sense that workers do not have access to credit markets and cannot save. That is, they consume their income every period.

Workers can be either employed or unemployed at any period. When seeking a job, they derive utility from home production and unemployment benefits net of taxes, $v(z - T)$. Production at the market requires a firm and a worker. Firms incur cost c when posting vacancies. Each firm holds a single job. When a job-seeker and a recruiting firm get together, they decide whether or not to form a match. Their decision is based on the expected match quality. We assume the extreme case in which the match quality is either good or bad.[12] If the former, the match quality amounts to y. Otherwise, the output produced is 0 as a shortcut for the dissolution of bad matches to be mutually desirable. While firms are ex-ante identical, workers differ by their abilities to form matches of good quality. For simplicity,

[12]We abstract from potential market productivity differences across workers. The primary reason is that, because of our focus on limited commitment equilibrium, firms might screen out low type workers by offering contracts that promise a wage above their productivity.

workers can be of either high or low type. Let $\mu_h = \mu$ denote the mass of type h workers in the economy. The remaining $\mu_\ell = 1 - \mu$ workers are of type ℓ.

We build upon Jovanovic (1979) and Pries and Rogerson (2005) and model match quality as both an inspection and an experience good. We now describe the sense of these two features.

Inspecting. Upon meeting, a type i worker-firm pair draws a probability π of the match being of good quality. A match is formed if such a probability is sufficiently high. Let Π_i denote the continuous random variable of such probabilities for (potential) type i matches. Let F_i be its differentiable cdf with support within the unit interval, with no mass points and the same supremum across types.[13] We make the following assumptions.

Assumption 1 Π_ℓ *is less than* Π_h *in the strict mean residual life order,* $\Pi_\ell \leq_{mrl} \Pi_h$. *That is,*

$$\int_{\pi \geq t} \pi \frac{dF_\ell(\pi)}{1 - F_\ell(t)} < \int_{\pi \geq t} \pi \frac{dF_h(\pi)}{1 - F_h(t)}, \ \forall t \geq \underline{\pi}_\ell$$

Assumption 2 Π_h *first-order stochastically dominates* Π_ℓ. *That is,*

$$F_\ell(t) \geq F_h(t), \ \forall t, \ \textit{and strict inequality for some } t$$

We refer to the first assumption as *strict* mean residual life order because it requires a strict inequality instead of weak inequality. This stochastic order implies that the truncated expected probability of the match quality being good is always larger for type h workers. While Assumption 1 is a sort of single-crossing condition, it does not suffice to ensure that firms benefit more from matching with type h workers. To this aim, we also assume first order stochastic dominance. It is worth noticing that, under some additional condition, the mean residual life order implies first order stochastic dominance.[14] To fix ideas, consider a uniform distribution for low type workers, $F_\ell \sim U[0, 1]$, and a perturbation of it for high types, $F_h(\pi) = \epsilon\pi^2/2 + (1 - \epsilon/2)\pi$. One particular case of such an F_h would be a triangular distribution with mode at $\pi = 1$.

[13] We impose the same supremum of the support for both distributions in order to avoid firms separating types just using a reservation value.

[14] See Shaked and Shanthikumar (1994, Ch. 1.D) for further details.

Experiencing. The output observed in any period is a deviation of the match quality, where the noise ϵ is an iid variable with zero mean. The two parties of the relationship learn simultaneously about the actual quality of the match by experiencing it. We follow Pries (2004) and assume for simplicity that ϵ is uniformly distributed with support $[-\bar{\epsilon}, \bar{\epsilon}]$. For the match quality not to be learned instantaneously, it is required that $y - \bar{\epsilon} < \bar{\epsilon}$. That is, there is an interval of observed productivities for which the parties cannot infer the type of their match, $[y - \bar{\epsilon}, \bar{\epsilon}]$. This implies that the learning process is of the form "all-or-nothing". In other words, a match quality is learned when the observed output falls out of the above interval, which occurs with probability $\alpha \equiv \frac{y}{2\bar{\epsilon}}$. If so, the match quality is revealed to be good with probability π. With the remaining probability, the match is learned to be bad and the job is destroyed. Otherwise, nothing can be inferred and the posterior probability of a good match quality coincides with the prior.

Employment relationships are also subject to exogenous idiosyncratic job-destruction shocks, which arrive with probability λ every period. Notice that otherwise all workers will eventually be employed in the steady state as they will find matches of good quality.

Search Frictions and Contracting Environment. Workers and firms get together via search. Firms commit to a contractual offer to attract candidates. There is perfect information about job offers. Workers direct their search to maximize their expected utility. Search is directed in the sense that firms promising larger values are expected to receive more applications and, hence, are more likely to fill their vacancies. Let x denote a contract and $q(x)$ the associated expected number of applicants. The latter can also be interpreted as the ratio of job-seekers to type x vacancies. We will suppress the dependence of the ratio on the contractual offer hereafter unless necessary for readiness. Vacancies are filled with probability $\eta(q)$ and unemployed workers find jobs with probability $\nu(q)$. Both functions are assumed to be twice continuously differentiable. Furthermore, η (ν) is assumed to be increasing (decreasing) in q to capture the intuition that more candidates in the market competing for the available jobs increase (decrease) the prospects of any given vacancy (worker). Let ϕ denote the elasticity of the job-filling rate, which is assumed to be a decreasing function. Finally, the following limit conditions hold to guarantee existence of equilibrium: $\lim_{q \to 0} \eta(q) = \lim_{q \to 0\infty} \nu(q) = 0$ and $\lim_{q \to 0\infty} \eta(q) = \lim_{q \to 0} \nu(q) = 1$.

The contracting space is restricted as follows. Firms commit to a contract $x \equiv (R, \omega)$. It specifies a reservation value, R, for the probability of the match quality being good. Moreover, it stipulates a two-tier wage schedule $\omega \equiv (w_1, w_2)$, where the first wage is paid while the actual match quality remains unknown and the second wage when it becomes perfectly observed conditional on the match not being destroyed. Because of the information asymmetries, contracts cannot be type contingent.

If wages are backloaded, $w_1 < w_2$, the contract may be interpreted as encompassing promotion or wage bonuses linked to a perceived higher match quality. Notice also the trade-off firms face when setting the reservation value R. The lower R, the more likely firms fill their vacancies, yet the lower the expected returns of the vacancy conditional on being filled.

Value Functions and State Variables. A type i worker derives utility $v(z-T)$ while seeking a job. Conditional on applying to a contract x with reservation probability R, she becomes employed with probability $\nu(q(x))(1-F_i(R))$. Otherwise, she remains unemployed one more period. Her value function satisfies the following functional equation.

$$(1-\beta)U_i = v(z-T) + \beta \max_x \left\{ \nu(q(x)) \int_{\pi \geq R} \Big(E_i(\pi) - U_i \Big) dF_i(\pi) \right\} \tag{1}$$

The employed worker with a type π match obtains a wage w_1 and pays taxes T. She becomes unemployed if either hit by an exogenous shock, which occurs with probability λ, or if the match is learned to be of bad quality, with probability $\alpha(1-\pi)$. Finally, the quality of the match turns out to be good with probability $\alpha\pi$ and the wage w_2 is paid to the worker. Her expected discounted utility is

$$E_i(\pi) = v(w_1 - T) + \beta\Bigg(\lambda U_i + (1-\lambda)\bigg(\alpha\Big((1-\pi)U_i + \pi\frac{v(w_2-T)+\beta\lambda U_i}{1-\beta(1-\lambda)}\Big) + (1-\alpha)E_i(\pi)\bigg)\Bigg) \tag{2}$$

Analogously, the asset value of a filled vacancy with a type π match is

$$J_i(\pi) = \pi y - w_1 + \beta\Bigg(\lambda V + (1-\lambda)\bigg(\alpha\Big((1-\pi)V + \pi\frac{y-w_2}{1-\beta(1-\lambda)}\Big) + (1-\alpha)J_i(\pi)\bigg)\Bigg) \tag{3}$$

A vacant firm incurs cost c when posting a job offer. The job can be filled by either type h or ℓ workers. Let $\rho_i(x)$ denote the believed proportion of workers of type i in submarket x, with the vector $(\rho_\ell(x), \rho_h(x))$ being a point of the simplex Δ^1. The value of a vacant firm is

$$(1-\beta)V = -c + \beta\eta(q(x)) \sum_i \rho_i(x) \int_{\pi \geq R} \Big(J_i(\pi) - V \Big) dF_i(\pi) \tag{4}$$

The expected profits of posting a vacancy must be zero in the steady state because of free entry, $V = 0$. We impose the following condition to ensure positive returns from vacancy-posting.

Assumption 3 *There exists a value $r \in [0,1]$ such that*

$$\mathbb{E}_\ell(\pi|\pi > r)\big(1 - F_\ell(r)\big)y - z\frac{1 - \beta(1-\lambda)(1 - \alpha\mathbb{E}_\ell(\pi|\pi > r)\big(1 - F_\ell(r)\big))}{1 - \beta(1-\lambda)(1-\alpha)} > \frac{c}{\beta}\big(1 - \beta(1-\lambda)\big)$$

Unemployment Insurance and Government. The state variables are the unemployment rate and the employment rate conditional on an unknown match quality for each type of worker. Let u_t^i and e_{1t}^i refer to them at period t, respectively. Note that the mass of employed type i workers at jobs in which the match quality has learned to be good amounts to $1 - u_t^i - e_{1t}^i$.

As stated above, unemployed workers derive utility $v(z - T)$. We primarily point out the consumption related to the period benefits b that unemployment workers collect. The unemployment insurance system provided by the government is funded through a lump sum tax, T. The difference $z - b$ refers to the flow non-employment value related to home production and leisure. Furthermore, we impose that the government's budget balances every period. That is,

$$b\left(\mu_\ell u_t^\ell + \mu_h u_t^h\right) = T, \text{ for every period } t \tag{5}$$

4 Equilibrium

We turn to define and characterize the competitive search equilibrium in the steady sate in an economy with adverse selection. We build upon Guerrieri et al. (2010). We restrict the analysis to limited-commitment equilibrium. That is, either party may walk away from the contract, but cannot renegotiate it. In particular, this limited-commitment feature requires that firms cannot commit to have expected negative profits from any point in time onwards.

Definition 1 *A steady-state competitive search equilibrium with limited commitment consists of a distribution G of vacancies in active submarkets with support $\mathcal{X} \subset X$, value functions U_i, $E_i(\cdot)$ and $J_i(\cdot)$, a queue length function $Q : X \to \mathcal{R}_+$, and a function $\rho : X \to \Delta^1$ such that*

1. *The value functions satisfy the functional equations (1)-(3).*

2. *Firms' profit-maximizing program and free entry:*

$$\forall x = (R, \omega) \in X,\ \ \beta\eta(Q(x)) \sum_i \rho_i(x) \int_{\pi \geq R} J_i(\pi) dF_i(\pi) \leq c,$$

   with equality if $x \in \mathcal{X}$.

3. *Workers' optimal search:*

   the type i worker's value satisfies

$$(1-\beta)U_i = v(z-T) + \beta \max_{x=(R,\omega)\in\mathcal{X}} \left\{ \nu(Q(x)) \int_{\pi\geq R} \Big(E_i(\pi) - U_i \Big) dF_i(\pi) \right\}$$

   Furthermore,

$$\forall x = (R, \omega) \in X, \quad (1-\beta)U_i \geq v(z-T) + \beta\nu(Q(x)) \int_{\pi \geq R} \Big(E_i(\pi) - U_i \Big) dF_i(\pi),$$

   with equality if $Q(x) > 0$ and $\rho_i(x) > 0$. If

$$\int_{\pi\geq R} \Big(E_i(\pi) - U_i \Big) dF_i(\pi) \leq 0,$$

   then either $Q(x) = 0$ or $\rho_i(x) = 0$.

4. *Limited-commitment:*

   If $x \in \mathcal{X}$, $Q(x) > 0$ and $\rho_i(x) > 0$, then

   $E_i(R) \geq U_i$, $v(w_2 - T) \geq (1-\beta)U_\ell$, $J_i(R) \geq 0$, and $y \geq w_2$.

5. *Market-clearing condition:*

$$\int_{\mathcal{X}} \rho_i(x) Q(x) dG(x) = u_i \mu_i, \forall i$$

6. *The government balanced budget condition (5) holds.*

The interpretation of the equilibrium definition is rather standard. First, firms design contracts to maximize profits given the anticipated optimal search behavior of job-seekers. Free entry ensures that the expected profits are zero in the steady-state. Second, after observing all job offers, workers search for jobs that maximize their utility. Third, posted contracts must be consistent with limited commitment. That is, the continuation value of employment exceeds the value of quitting to unemployment at any point in time. Similarly, firms with filled vacancies must have non-negative expected profits. Because of the monotonicity of the value functions E_i and J_i, it suffices to impose that they are above their respective lower bounds at the reservation probability. The fifth equilibrium condition implies that adding up job-seekers across submarkets must amount to the total mass of unemployed of each type. Finally, the government budget balances every period.

As usual, the interest is in the determination of the expectations off-the-equilibrium path that support the equilibrium allocation. Consider a trembling hand kind of argument. Consider an arbitrarily small measure of firms deviating to submarket $x' \notin \mathcal{X}$. Such firms form rational expectations about the optimal search behavior of workers. They may be thought of doing the following thought experiment. If workers were all identical, there would be a flow of workers from an equilibrium contract $x \in \mathcal{X}$ until the point in which they were indifferent between x and x'. Such flows would occur even if different types of workers search in the labor market. If type h workers found optimal to flow in submarket x' at the queue length that made type ℓ workers indifferent, then the ratio $q(x')$ would be determined by the former type and the deviating firms would only receive applications from type h workers, $\rho_h(x') = 1$. This reasoning is captured in the third equilibrium condition.

4.1 Equilibrium Characterization

We now turn to characterize the equilibrium allocation. Two equilibrium results are apparent. First, it cannot be optimal for firms to offer a zero after-tax wage at any production time. This is the case because of the infinite marginal utility at the zero consumption level. Second, it is not optimal either to set the reservation probability at the supremum of the support of the distribution since firms would make expected negative profits. Next, we claim that it cannot be the case that both types of workers apply to the same jobs in equilibrium if wages are strictly below productivity.

Proposition 4.1 *There is no open submarket in equilibrium in which firms receive applications from both types of workers and $w_1, w_2 < y$.*

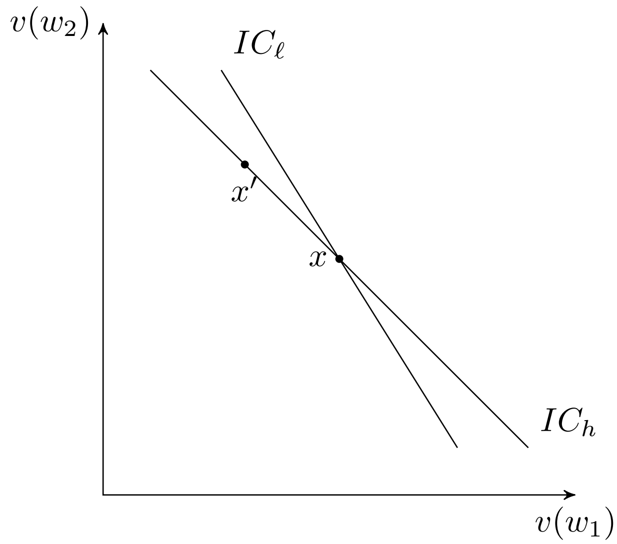

Figure 3: Single-crossing property

To understand the underlying intuition of this result, suppose that there existed a submarket attractive to both types of workers in equilibrium and $T = 0$ for simplicity. The expected value promised to type i applicants in such a submarket amounts to

$$\int_{\pi \geq R} \big(E_i(\pi) - U_i\big) dF_i(\pi) = \big(1 - F_i(R)\big)\bigg(c_1 v(w_1) + c_2 \mathbb{E}_i(\pi|\pi \geq R) v(w_2)\bigg) + c_{3,i}(R),$$

where the positive coefficients c_1 and c_2 are type independent. Figure 3 depicts the locus of wage schedules that provide the same expected value to workers, conditional on employment, in the $(v(w_1), v(w_2))-$space. The submarket x that attracted both types of workers would lay in the intersection point of the two lines. Assumption 1 implies that the line of type h workers is flatter. Consider now the alternative submarket x' with a lower first wage and a higher second one. A deviation to submarket x' would leave type h workers indifferent, while worsening the prospects of the type ℓ applicants. As a result, by deviating to contract x', firms would manage to attract only applicants of type h. In other words, Assumption 1 can be read as a single-crossing property.

To see that such a deviation is profitable, we can rewritte the expected profits as

$$\int_{\pi \geq R} J_i(\pi) dF_i(\pi) = \frac{1 - F_i(R)}{1 - \beta(1-\lambda)(1-\alpha)}\Bigg(-w_1 + \bigg(y + (y - w_2)\frac{\beta(1-\lambda)\alpha}{1 - \beta(1-\lambda)}\bigg)\mathbb{E}_i(\pi|\pi \geq R)\Bigg)$$

Because of Assumption 1, firms anticipate that matches with type h workers last longer in expected terms. This does not suffice, however, to ensure that deviating firms make higher profits from screening out type ℓ workers because jobs might be more difficult to fill with high types. Assumptions 1 and 2 together become a sufficient condition for such deviations to imply a discrete jump in profits, contradicting the equilibrium assumption. Therefore, there cannot be firms receiving applications from both types of workers in equilibrium.

To discourage type ℓ applications, firms could alternatively increase the reservation probability R and compensate type h workers with a higher wage w_1. This deviating strategy would be profitable if and only if

$$\big(E_\ell(R) - U_\ell\big) dF_\ell(R) > \frac{1 - F_\ell(R)}{1 - F_h(R)} \big(E_h(R) - U_h\big) dF_h(R),$$

which does not generally hold.

Note that the above reasoning does not apply if $w_2 = y$ because of limited commitment. Therefore, an equilibrium with both types of workers searching for the same jobs cannot be discarded when $w_2 = y$, particularly when α is small relative to λ or β is sufficiently small. Although an equilibrium with firms offering $w_2 = y$ is theoretically possible, we will focus hereafter on separating equilibrium. A rationale for our focus choice is that firms have incentives to smooth worker's consumption because of risk aversion. In particular, such incentives shorten the wage gap.

We next show existence of a separating equilibrium and characterize it. Consider the following functions $\mathcal{H}_i : \mathcal{R}_+ \to \mathcal{R}_+$.

$$\begin{aligned}
\mathcal{H}_\ell(U) \equiv \sup_{q\in[0,\infty], R\in[0,1], (w_1,w_2)\in\mathcal{R}_+^2} \quad & v(z - T) + \beta\nu(q) \int_{\pi\geq R} \Big(E_\ell(\pi) - U\Big) dF_\ell(\pi) + \beta U \\
\text{s. to} \quad & \beta\eta(q) \int_{\pi\geq R} J_\ell(\pi) dF_\ell(\pi) \geq c \\
& E_\ell(R) \geq U,\ v(w_2 - T) \geq (1-\beta)U,\ J_\ell(R) \geq 0,\ y \geq w_2
\end{aligned}$$

$$
\begin{aligned}
\mathcal{H}_h(U;U_\ell) \equiv \sup_{q\in[0,\infty],R\in[0,1],(w_1,w_2)\in\mathcal{R}^2_+} \quad & v(z-T)+\beta\nu(q)\int_{\pi\geq R}\Big(E_h(\pi)-U\Big)dF_h(\pi)+\beta U \\
\text{s. to} \quad & \beta\eta(q)\int_{\pi\geq R}J_h(\pi)dF_h(\pi)\geq c \\
& E_h(R)\geq U,\ ,v(w_2-T)\geq(1-\beta)U,\ J_h(R)\geq 0,\ y\geq w_2 \\
& v(z-T)+\beta\nu(q)\int_{\pi\geq R}\Big(E_\ell(\pi)-U_\ell\Big)dF_\ell(\pi)\leq(1-\beta)U_\ell
\end{aligned}
$$

As an abuse of language, let us refer to a fixed point of function $\mathcal{H}_i$ as a tuple (U_i, q_i, x_i), with $x_i = (R_i,(w_{1i}, w_{2i}))$. Given the value U_i, the pair (q_i, x_i) maximizes the expected utility of type i workers subject to firms making non-negative profits. The second set of constraints capture the limited-commitment equilibrium feature. In the case of $\mathcal{H}_h$, there is an additional constraint, which is a no-participation condition for type ℓ workers. This additional restriction is necessary to discourage type ℓ workers from applying to type h jobs in a separating allocation. Although an analogous constraint should be written for $\mathcal{H}_\ell$ to ensure the no participation of type h workers, we will show that it is redundant. The following proposition states that an equilibrium allocation is a fixed point of these two functions, and vice versa.

Proposition 4.2 *Let (U_i, q_i, x_i) be a fixed point of function $\mathcal{H}_i$ for $i \in \{\ell, h\}$. Then, it takes part of an equilibrium allocation, where $Q_i(x_i) = q_i$ and $\rho_i(x_i) = 1$.*

Conversely, if $(G, \mathcal{X}, (U_i)_i, Q, \rho)$ is an equilibrium allocation with $\mathcal{X} = \{x_\ell, x_h\}$, then (U_i, q_i, x_i) is a fixed point of $\mathcal{H}_i$.

We had claimed above that type h workers have no incentives to apply to type ℓ jobs. Notice that the equilibrium tuple $(q_\ell,(R_\ell, x_\ell))$ belongs to the domain of the objective function of the maximization problem of function $\mathcal{H}_h$, yet it is not the maximizer. As a result, the maximization problem of function $\mathcal{H}_\ell$ has one constraint less than its counterpart for $\mathcal{H}_h$.

This equivalence result is not surprising as the assumptions in Guerrieri et al. (2010) can be extended to and hold in our dynamic setting. Thus, it can be read as a particular case of their results. The following proposition states the existence of a steady state equilibrium. We first show that the functions $\mathcal{H}_i$ are well-defined. Then, the Berge Maximum Theorem ensures that the functions $\mathcal{H}_i$ are continuous, and the Brower Fixed-point Theorem applies to show that they have a fixed point.

Proposition 4.3 *There exists a competitive search equilibrium with limited commitment.*

Firms commit to a continuation value to attract applicants. They aim to optimize the trade-off between the chances to fill a vacancy and the wage bill. Consider first the submarket for type ℓ workers. Firms have two contractual instruments to minimize the wage bill for any given value because of incomplete markets. First, as said above, firms would benefit from offering perfect consumption smoothing to workers, $w_1 = w_2$. Second, as in Acemoglu and Shimer (1999), risk-neutral firms bear all the risk and find it optimal to provide (limited) insurance to risk-averse workers against income uncertainty. To this aim, a large number of vacancies are created and firms set low reservation probability values. That is, given a wage schedule and the monotonicity of the value function J_ℓ, firms set R_ℓ to make $J_\ell(R_\ell)$ as low as possible. Notice that, due to limited commitment, a low reservation value must be balanced by low wages, what pushes down the employment value $E_\ell(R)$. Indeed, limited commitment establishes lower bounds for these value functions, $J_\ell(R_\ell) \geq 0$ and $E_\ell(R) \geq U_\ell$. These two inequalities cannot be strict at once because firms would benefit from pushing down the reservation value. After some manipulations of the necessary first order conditions, it follows that $w_{1\ell} < w_{2\ell}$. In other words, firms would commit to perfect consumption smoothing if full commitment were imposed.

Firms targeting type h workers also aim to discourage type ℓ applications. This is the case because Assumptions 1 and 2 imply that type h workers deliver higher market returns to firms. First-order stochastic dominance implies that type h workers are more likely to be hired for any reservation value R_h. Likewise, the strict mean residual life order ensures that they are also more likely to stay in the firm longer. Therefore, these firms have an additional motive to backload wages; hence, $w_{1h} < w_{2h}$ also holds. Indeed, such firms implement a composite strategy consisting of further wage-backloading and a higher reservation probability relative to the perfect information case. Since wages increase with job tenure irrespective of the worker's type, so does the average wage.

To sum up, this paper sets a theory of the wage dynamics within a firm. All firms offer a wage premium to risk averse workers in equilibrium because of incomplete markets and the fact that the limited commitment constraint is relaxed once the quality of the match has been learned. For type h workers, the wage premium may also increase because of the optimal strategy of firms to discourage type ℓ applications. Proposition 4.4 states the wage dynamics. This result is consistent with well-known empirical evidence regarding the positive relationship between wages and job tenure. For example, Altonji et al. (2013) finds that most of the wage increase with potential experience is due to general human capital accumulation, and job tenure (as well as job shopping) accounts for over 12% of the mean log wage increase over the first 10 years. See also Topel (1991) and Dustmann and Meghir

(2005). Alternative explanations for the wage growth with job tenure that we do not model are the accumulation of job-specific, sector and general human capital as well as competition between firms because of on-the-job search as in Postel-Vinay and Robin (2002) and Bagger et al. (2014).

Worker turnover reduces with job tenure by construction. This negative relationship results from the job-separation rate for any individual match being higher while the match quality is uncertain. This result is also in line with the empirical literature. See for example Farber (1999). Using French matched employer-employee data, Nagypál (2007) aims to disentangle the effects of learning by doing from learning about the match quality on worker turnover. She finds that the latter dominates for job tenures longer than 6 months.

Proposition 4.4 *The equilibrium wages are such that $w_{1i} < w_{2i}$, for $i \in \{\ell, h\}$. The equilibrium queue length is smaller for type h workers, $q_h < q_\ell$, and $U_\ell < U_h$. Furthermore, the average wage increases with job tenure. So do the individual and average rate of staying employed.*

We also obtain that there are more vacancies in the type h submarket and the continuation value of the unemployed workers of this type is bigger. Firms obtain higher returns from creating jobs that are expected to last longer and, hence, more positions are opened. Although we cannot theoretically compare wages and reservation probabilities across types, we expect them to be higher for type h workers, which is confirmed in our numerical exercises. Our intuition for this result follows our previous reasoning. Given that the unemployment value is lower for type ℓ workers, firms would benefit more from lowering hiring standards to them rather than type h workers. Likewise, wages, and particularly w_1, must be reduced more for those workers for the limited commitment conditions to hold.

The exit rate from unemployment is $\nu_i(q_i)(1 - F_i(R_i))$. Because in equilibrium we have $\nu(q_h) > \nu(q_\ell)$ and expect $R_\ell < R_h$, we cannot assert which type of worker faces a higher the job-finding rate. However, since the job-finding rates are time-invariant for any worker type, we have that provided the exit rates from unemployment differ across types, the average job-finding rate declines with unemployment duration. Furthermore, the survival rate as a function of duration for a given worker type i is driven by $\mathbb{E}_i(\pi | \pi > R_i)$. Because of Assumption 1, if $R_\ell < R_h$, we'd have that the type h workers would stay employed longer. This would be only consistent with an average survival rate falling with unemployment duration if the job-finding rate of type h workers were also higher. We next calibrate our model to the US economy and target the average survival rate as a function of the duration of the previous unemployment spell. The expected results obtain in our calibrated economy.

4.2 Exit Rates and Unemployment Duration

State Variables. For notational convenience, let $\mathbb{E}_i(\pi|R) \equiv \mathbb{E}_i(\pi|\pi > R)$. The state variables are the unemployment rate and the employment rate conditional on an unknown match quality for each type of worker. Let u_t^i and e_{1t}^i refer to them at period t, respectively. Note that the mass of employed type i workers at jobs in which the match quality has learned to be good amounts to $1 - u_t^i - e_{1t}^i$. The dynamics of these two variables are determined by the following laws of motion:

$$u_{t+1}^i = \big(1 - \nu(q_{it})\big(1 - F_i(R_{it})\big)\big)u_t^i + \lambda(1 - u_t^i) + (1-\lambda)\alpha\big(1 - \mathbb{E}_i(\pi|R_{it})\big)e_{1t}^i \tag{6}$$

$$e_{1t+1}^i = \big(1 - \lambda - (1-\lambda)\alpha\big)e_{1t}^i + \nu(q_{it})\big(1 - F_i(R_{it})\big)u_t^i \tag{7}$$

Notice that we allow for the endogenous variables to depend on the type of worker in these expressions. The laws of motion are mostly self-explanatory. The mass of unemployed workers of type i increases because of either exogenous separations or the dissolution of matches when the match quality has realized to be bad, whereas it decreases because of the new matches formed. Likewise, the mass of employed workers increases with the new matches formed, and reduces because of separations due to either exogenous shocks or learning that the match quality is bad.

We now determine the exit rates from unemployment and employment as functions of the duration of the previous unemployment spell.

The rate of unemployed workers of type i who have been looking for jobs for τ periods is determined by

$$u^i(1) = \lambda(1 - u^i) + (1-\lambda)\alpha\big(1 - \mathbb{E}_i(\pi|R_i)\big)e_1^i \tag{8}$$

$$u^i(\tau) = u^i(1)\big(1 - \nu(q_i)\big(1 - F_i(R_i)\big)\big)^{\tau-1} \tag{9}$$

The average job-finding rate at unemployment duration τ is determined by

$$f(\tau) = \frac{\sum_i \mu(i)u^i(\tau)\nu(q_i)\big(1 - F_i(R_i)\big)}{\sum_i \mu(i)u^i(\tau)} \tag{10}$$

Likewise, we define the mass of ongoing matches after t periods and conditional on a previous unemployment spell of duration τ of a type i worker by

$$
\begin{aligned}
e^1_{i\tau}(t) &= \mu(i)u^i(\tau)\nu(q_i)\big(1-F_i(R_i)\big)\big((1-\lambda)(1-\alpha)\big)^{t-1} && (11)\\
e^2_{i\tau}(1) &= 0 && (12)\\
e^2_{i\tau}(t) &= e^2_{i\tau}(t-1)(1-\lambda)+e^1_{i\tau}(t-1)(1-\lambda)\alpha\mathbb{E}_i(\pi|R_i) && (13)
\end{aligned}
$$

The average rate of employment after t periods as a function of the duration of the previous unemployment spell is determined by

$$
es_t(\tau)=\frac{\sum\limits_{i,j}e^j_{i\tau}(t)}{\sum\limits_{i}\mu(i)u^i(\tau)\nu(q_i)\big(1-F_i(R_i)\big)} \tag{14}
$$

5 Calibration

In this Section, we calibrate our model to the US labor market. A period is set to be a week for consistency with our SIPP data. The key parameter values are the total flow value of non-employment, and in particular unemployment benefits, the vacancy costs and those related to the heterogeneity of workers. We first provide the calibration details regarding the remaining parameters.

We normalize the market marginal productivity of labor to one. The discount factor β is consistent with a 5% annual interest rate. We consider CRRA preferences, $v(w)=\frac{w^{1-\sigma}}{1-\sigma}$. We set the coefficient of relative risk aversion to 2. We use the urn-ball matching technology. Petrongolo and Pissarides (2001) argue that this particular technology fails to yield plausible combinations of levels of unemployment and average durations and suggest to introduce a scaling parameter. Therefore, we use the following functional form $\eta(q)=1-e^{-\psi q}$, where ψ is the scaling parameter calibrated to match the average unemployment duration, 15.????.

The cost c firms incur when posting vacancies includes both the recruiting and training expenses. Hall and Milgrom (2008) calibrate the former to be 14% of the average quarterly wage per hire. We benefit from the Abowd and Kramarz (2003) work with French data. They estimate the recruiting and training costs at 13% and 7% of the average quarterly wage per hire, respectively.

Consumption while unemployed z comes from home production and unemployment benefits b, which are thought of as independent of past wages. Following Hall and Milgrom (2008), we calibrate unemployment benefits and the home productivity. First, we set the UI replacement rate for the average wage to 0.25, which lies within the reasonable bounds

Table 2: Calibration

Parameter	**Description**	**Value**	**Target**
Exogenously Set Parameters			
β	Discount factor	0.999	Annual interest rate of 5%
y	Market Productivity	1.0	(Normalization)
σ	Relative risk aversion coefficient	2.0	
Jointly Calibrated Parameters			
λ	Exogenous Job-separation rate		Predicted monthly job-separation rate
ψ	Constant matching technology		average unemp. duration
c	Vacancy cost		20% avg. quarterly wage per hire
b	Unemployment benefits		25% of avg. wage
$z-b$	Home production		46% of avg. productivity
α			
μ	Share of skilled		
σ_h	Standard deviation of normal distribution F_h		
σ_ℓ	Standard deviation of normal distribution F_ℓ		

found in the literature. Hall and Milgrom (2008) estimates that $z-b$ amounts to 0.46 of the average productivity. This would make an overall consumption of 0.71 of the average output for the unemployed, also close to the 0.745 chosen by Costain and Reiter (2008).

There is no direct evidence that could be matched to the parameters related to the heterogeneity of workers in our model and the learning process, namely the proportion of type h workers μ, the standard deviations of the distributions σ_ℓ and σ_h, and the learning probability α. To capture the heterogeneity present in the US economy, in line with Moscarini (2003) and Menzio and Shi (2011), we target the distribution of job separations within the first year over unemployment duration.

[To be completed]

6 Policy Analysis

To be completed

7 Conclusions

To be completed

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8 Appendix

8.1 Data Appendix.

We use data from the 1996 and 2001 panels of the Survey of Income and Program Participation (SIPP). A panel is formed by a number of interviews, called waves. The first panel is formed by 12 waves and the second by 9, covering approximately 4 and 3 years, respectively. Individuals report retrospectively every four months. We are interested in their employment status at a weekly basis, wages, working hours if employed, and a number of demographic and other relevant characteristics.

We define two labor market status in a given week: employment and non-employment. We sort a worker as employed (E) if she reports to have a job, regardless of whether she is working, absent or on temporary layoff. Otherwise, she is classified as non-employed ($\not{E}$). As pointed out in the SIPP technical documentation, this is an important difference with respect to the Current Population Survey (CPS) because the category 'employed' in the latter does not include "those temporarily absent from a job because of layoff and those waiting to begin a new job in 30 days". A job separation occurs when a worker transits from E to $\not{E}$.

We restrict our sample to individuals aged 16 to 65 who lose a job and subsequently find a new one. We do not consider as separations non-employment spells shorter than 2 weeks if the job id is the same before and after. This case amounts to approximately 7% of the observations in the initial dataset. Likewise, we only consider observations with a positive number of working hours reported at re-employment, although this filter does not change the numbers significantly. To investigate the worker turnover within the first year after re-employment, we further restrict our sample to those individuals whose continuation employment histories are observable for at least a year after reemployment. We comment on possible attrition bias below. After implementing these filters to the set of $E\not{E}E$ spells, we end up with 11099 observations. Table 3 sorts observations grouping per individuals. Almost 80% of the observations of our sample correspond to a single individual.

Controlling for seams, observable characteristics and recall. Turnover rates vary across a number of dimensions, such as gender, age, education, and occupation. Furthermore, as in Fujita and Moscarini (2013), there seems to be "seam effects". Whether a new employment spell starts at the very last or the very first month of a wave should have no effect on the job-separation rate. However, we find that the turnover rate for $E\not{E}E$ spells within the first year differs significantly across these cases as Table 4 shows.

Table 3: Number of Observations per Individual

	Freq.	Percent
1	8856	79.79
2	1820	16.40
3	345	3.11
4	68	0.61
5	5	0.05
6	3	0.03
7	2	0.02
Total	11099	100.00

Another important aspect is the empirical relevance of recalls. When studying the 1996 and 2001 panels, Fujita and Moscarini (2013) have 12245 observations of $E\not{E}E$. Out of these, approximately 20% result from a recall. This percentage raises to 32 after an imputation process. SIPP provides a unique job number to identify an employer, with a maximum of two per wave. We identify a recall for a given $E\not{E}E$ spell if either job id number in the first employment spell coincides with either job id in the second spell. We obtain that recall accounts for 21.72% of the observations. That is, the subsample formed by spells not-involving a recall consists of 8688 observations. Indeed, approximately 37% of the recalls follow a less-than-one-month separation. We show in Table 4 that the turnover rates are different depending on whether a recall takes place or not.

To account for these factors, we run a Probit regression and compute the predicted turnover rate at the average worker. We use as regressors national unemployment rate as a business cycle indicator, monthly dummies to capture seasonality effects, and a year linear variable. In addition, we control for age and its square, average accumulated unemployment benefits - deflated using the national CPI- and its square, a quartic polynomial of the length of the previous non-employment spell, and a number of dummy variables for gender, white, black, marital status, education, occupation, industry, firm size, and state as well as seam dummies. We use the longitudinal weights (wpfinwgt) provided by SIPP. A large number of these variables are statistically significant, and particularly the seam dummies. The predicted turnover rate is 43.35, with 95% confidence interval $(42.34, 44.37)$. If we restrict the pro bit regression to observations not involving a recall, we obtain a predicated rate equal 45.56%.

Turnover rate function of non-employment length. To estimate the effects of the length of the previous non-employment spell in the turnover rate, we run a Probit regression using the same controls as above. We replace all variables at their sample averages, and

Table 4: Turnover rate - Seam effects

	Freq.	Percent
Overall	4870	43.88
E spell starting last month of a wave	1012	48.87
E spell starting first month of a wave	1819	39.26
Recall	883	36.62
No recall	3987	45.89
Predicted (whole sample)		43.35
Predicted (no recall)		45.56

make the non-employment duration take values from 1 to 52 weeks.

8.2 Proofs.

Proof of Proposition 4.1.

To save on notation, let us make $T = 0$ and define, for $i \in \{\ell, h\}$,

$$\mathcal{I}_i^w \equiv \int_{\pi \geq R} \Big(E_i(\pi) - U_h \Big) dF_i(\pi)$$

The proof is by contradiction. Suppose that there exists an equilibrium in which both types of workers submit applications to the same job offer $x = (R, \omega)$. Let q denote the equilibrium queue length in that submarket. Then,

$$U_i(1-\beta) = v(z) + \beta\nu(q)\mathcal{I}_i^w, \forall i \in \{\ell, h\}$$

Consider now an arbitrarily small mass ζ of firms deviating to submarket x', which promises an expected value $\mathcal{I'}_i^w = \mathcal{I}_i^w + d\mathcal{I}_i^w$. The equilibrium expectations on the queue length at submarket x' are determined by what type of workers benefits the most. Let q_i denote the queue length that makes type i workers indifferent between submarkets x and x'. Then,

$$dq_i\nu'(q)\mathcal{I}_i^w + \nu(q)d\mathcal{I}_i^w = 0 \tag{15}$$

Consider that the deviating firms offer the same reservation value and a marginally different wage contract which leaves type h workers indifferent. That is, $d\mathcal{I}_h^w = 0$. Then, the total differential $d\mathcal{I}_i^w$ amounts to

$$d\mathcal{I}_i^w = \int_{\pi \geq R} dE_i(\pi) dF_h(\pi) = \frac{1-F_i(R)}{1-\beta(1-\lambda)(1-\alpha)}\Big(v'(w_1)dw_1 + \frac{\beta(1-\lambda)\alpha}{1-\beta(1-\lambda)}\mathbb{E}_i(\pi|\pi \geq R)v'(w_2)dw_2\Big) \tag{16}$$

where the expected value of the truncated distribution is defined as

$$\mathbb{E}_i(\pi|\pi \geq R) \equiv \int_{\pi \geq R} \pi \frac{dF_i(\pi)}{1-F_i(R)}$$

From $d\mathcal{I}_h^w = 0$, it follows that

$$v'(w_2)dw_2 = -v'(w_1)dw_1 \frac{1-\beta(1-\lambda)}{\beta(1-\lambda)\alpha} \frac{1}{\mathbb{E}_h(\pi|\pi \geq R)},$$

Now, we replace $v'(w_2)dw_2$ in expression (16) for type ℓ workers, and obtain

$$\begin{aligned} d\mathcal{I}_\ell^w &= \frac{1-F_\ell(R)}{1-\beta(1-\lambda)(1-\alpha)}\Big(v'(w_1)dw_1 + \frac{\beta(1-\lambda)\alpha}{1-\beta(1-\lambda)}\mathbb{E}_\ell(\pi|\pi \geq R)v'(w_2)dw_2\Big) = \\ &= v'(w_1)dw_1 \frac{1-F_\ell(R)}{1-\beta(1-\lambda)(1-\alpha)}\Big(1 - \frac{\mathbb{E}_\ell(\pi|\pi \geq R)}{\mathbb{E}_h(\pi|\pi \geq R)}\Big) \end{aligned}$$

Assumption 1 tells us that the sign of the total differential $d\mathcal{I}_\ell^w$ is the sign of the differential dw_1. We have argued that the after-tax wages must be strictly positive in equilibrium. Therefore, by reducing w_1, the deviating firms ensure that $d\mathcal{I}_\ell^w < 0$ and, hence, $q_\ell < q_h$ because $\frac{dq_i}{dW_i} > 0$ according to expression (15). This is a contradiction because the deviating firms end up attracting only type h workers while bearing an arbitrarily small increase in their wage bill, implying a discrete jump in profits.

The discrete jump in profits comes from $\int_{\pi \geq R} J_h(\pi)dF_h(\pi) > \int_{\pi \geq R} J_\ell(\pi)dF_\ell(\pi)$. The expected profits can be rewritten as

$$\int_{\pi \geq R} J_i(\pi)dF_i(\pi) = \frac{1-F_i(R)}{1-\beta(1-\lambda)(1-\alpha)}\Big(-w_1 + \Big(y_i + (y_i - w_2)\frac{\beta(1-\lambda)\alpha}{1-\beta(1-\lambda)}\Big)\mathbb{E}_i(\pi|\pi \geq R)\Big)$$

Therefore, the above inequality results from Assumptions 1 and 2.‖

Proof of Proposition 4.2. To save on notation, let us make $T = 0$ and define for $i \in \{\ell, h\}$

$$\mathcal{I}_i^w \equiv \int_{\pi \geq R} \Big(E_i(\pi) - U_h\Big) dF_i(\pi), \text{ and } \mathcal{I}_i^f \equiv \int_{\pi \geq R} J_i(\pi)dF_i(\pi)$$

The proof has two main stages. First, we show that the vectors $(U_i, q_i, x_i)_i$ of the equilibrium allocation constitute a fixed point of functions $\mathcal{H}_\ell$ and $\mathcal{H}_h$. Obviously, the constraints of both maximization problems hold when evaluated at the equilibrium values. To start with, let (U_ℓ, q_ℓ, x_ℓ) be part of the equilibrium allocation. The second equilibrium condition establishes that

$$(1-\beta)U_\ell - v(z) = \beta\nu(q_\ell)\int_{\pi\geq R_\ell}\Big(E_\ell(\pi) - U_\ell\Big)dF_\ell(\pi)$$

Let us now proceed as a proof by contradiction, and assume that it is not a solution of the maximization problem of function $\mathcal{H}_\ell$, given U_ℓ. Then, there must exist a tuple (q', x'), with the contract $x' = (R', (w_1', w_2'))$, such that

$$\nu(q')\mathcal{I'}^w_\ell > \nu(q_\ell)\mathcal{I}^w_\ell, \beta\eta(q')\mathcal{I'}^f_\ell \geq c, E'_\ell(R') \geq U_\ell,\ v(w_2') \geq (1-\beta)U_\ell,\ J'_\ell(R') \geq 0,\ y \geq w_2'$$

where the primes indicate that the integrals and value functions are evaluated at the alternative contract x'. From the definition of the off-the-equilibrium expectations, it follows that $q' < Q(x')$. Then,

$$\beta\eta(Q(x'))\sum\rho_i(x')\mathcal{I'}^f_i > \beta\eta(q')\sum\rho_i(x')\mathcal{I'}^f_i \geq \beta\eta(q')\mathcal{I'}^f_\ell \geq c \tag{17}$$

The second inequality results from Assumptions 1 and 2, which imply that $\mathcal{I'}^f_h \geq \mathcal{I'}^f_\ell$. Expression (17) implies that firms deviating to submarket x' would make strictly positive expected profits, which contradicts the assumption that (U_ℓ, q_ℓ, x_ℓ) was part of an equilibrium allocation. Therefore, the tuple (U_ℓ, q_ℓ, x_ℓ) is a fixed point of function $\mathcal{H}_\ell$.

The proof for type h agents is analogous. Suppose that the equilibrium objects (U_h, q_h, x_h) constitute a fixed problem of function $\mathcal{H}_h$, given U_ℓ. Again, the following equality results from the second equilibrium condition.

$$(1-\beta)U_h - v(z) = \beta\nu(q_h)\int_{\pi\geq R_h}\Big(E_h(\pi) - U_h\Big)dF_h(\pi)$$

If the pair (q_h, x_h) is not a maximizer of the associated maximization problem, there must exist a tuple (q', x') such that

$$\nu(q')\mathcal{I}'^w_h > \nu(q_h)\mathcal{I}^w_h, \beta\eta(q')\mathcal{I}'^f_h \geq c, \nu(q')\mathcal{I}'^w_\ell \leq \nu(q_\ell)\mathcal{I}^w_\ell, \text{ and}$$
$$E'_h(R') \geq U_h,\ v(w'_2) \geq (1-\beta)U_h,\ J'_h(R') \geq 0,\ y \geq w'_2$$

From the first inequality along with the monotonicity of function ν, we obtain that the equilibrium expectations at x' are such that $Q(x') > q'$. Putting this together with the third inequality, we obtain

$$\nu(Q(x'))\mathcal{I}'^w_\ell < \nu(q')\mathcal{I}'^w_\ell \leq \nu(q_\ell)\mathcal{I}^w_\ell \Rightarrow \rho_\ell(x') = 0$$

Then, it follows that

$$\eta(Q(x'))\sum_i \rho_i(x')\mathcal{I}'^f_i = \eta(Q(x'))\mathcal{I}'^f_h > \eta(q')\mathcal{I}'^f_h \geq c$$

That is, the expected profits at submarket x' are strictly positive, which contradicts the assumption of x_h taking part of an equilibrium allocation.

Now, we move to the second stage. Let (U_i, q_i, x_i) be a fixed point of function $\mathcal{H}_i$, for $i \in \{\ell, h\}$. We show that it takes part of an equilibrium allocation. The proof is by construction. The remaining steady-state equilibrium objects are determined as follows: $\mathcal{X} \equiv \{x_\ell, x_h\}$, $dG(x_i) \equiv \frac{\mu_i}{q_i}$, $Q(x_i) \equiv q_i$ and $\rho_i(x_i) \equiv 1$. We still have to define the off-the-equilibrium beliefs. Let $x = (R, w_1, w_2)$ be an arbitrary submarket, and let $\widetilde{q}_i(x)$ be defined as

$$\widetilde{q}_i(x) = \begin{cases} q & \text{, such that } \nu(q)\mathcal{I}^w_i = \frac{(1-\beta)U_i - v(z)}{\beta},\ \text{if } \frac{(1-\beta)U_i - v(z)}{\beta\mathcal{I}^w_i} \in [0,1] \\ 0 & \text{, otherwise.} \end{cases}$$

Then, we define $Q(x) \equiv \max_i \widetilde{q}_i(x)$, and $\rho_h(x) = \begin{cases} 1 & \text{, if } \widetilde{q}_\ell(x) < \widetilde{q}_h(x) \\ 0 & \text{, otherwise.} \end{cases}$

It is obvious that type i workers maximize their expected utility at x_i when searching for jobs among open submarkets. It remains to show that the zero-profit condition holds in both submarkets and that firms maximize profits given their expectations. Because of Proposition 4.1, we can focus on firms targeting one type of workers. We only show the case of skilled workers as the one for the unskilled may be reduced to a particular case of this one. Suppose that firms do not maximize profits at x_h. That is, there exists a submarket $x = (R, (w_1, w_2))$ such that $\eta(Q(x))\mathcal{I}_h^f > c$, $\nu(Q(x))\mathcal{I}_h^w = \frac{(1-\beta)U_h - v(z)}{\beta}$, $\nu(Q(x))\mathcal{I}_\ell^w \leq \frac{(1-\beta)U_\ell - v(z)}{\beta}$ and the limited commitment conditions hold. We then distinguish between two cases.

Case 1: $\nu(Q(x))\mathcal{I}_\ell^w < \frac{(1-\beta)U_\ell - v(z)}{\beta}$**.** Then, there must exist $q < Q(x)$ such that $\eta(q)\mathcal{I}_h^f > c$ and $\nu(q)\mathcal{I}_\ell^w < \frac{(1-\beta)U_\ell - v(b)}{\beta}$. From the monotonicity of function ν, it follows that $\nu(q)\mathcal{I}_h^w > \frac{(1-\beta)U_h - v(z)}{\beta}$, which contradicts the assumption that (U_h, q_h, x_h) is a fixed point of function $\mathcal{H}_h$, given U_ℓ.

Case 2: $\nu(Q(x))\mathcal{I}_\ell^w = \frac{(1-\beta)U_\ell - v(z)}{\beta}$**.** We can assume without loss of generality that $w_1 > T$. Consider the alternative contract $x' = (R', (w_1', w_2'))$ such that $R' = R$, $w_1' = w_1 - \epsilon$, with ϵ arbitrarily small, and w_2' such that type h workers are indifferent between submarkets x and x'. In the proof of Proposition 4.1, we have shown that $\rho_h(x') = 1$, and $\nu(Q(x'))\mathcal{I'}_\ell^w < \frac{(1-\beta)U_\ell - v(z)}{\beta}$. Therefore, we are back to the previous case when considering contract x', which leads to a contradiction.

Finally, notice that this same reasoning applies to show that the expected profits of firms are zero in equilibrium.‖

Proof of Proposition 4.3. We first analyze function $\mathcal{H}_\ell$, and then $\mathcal{H}_h$. Let us refer to the maximization problem associated to function $\mathcal{H}_i$ for some value U as $P_i(U)$.

Let $\mathcal{K} \equiv \left[\frac{v(z)}{1-\beta}, \frac{v(y)}{1-\beta}\right]$. Given some value $U \in \mathcal{K}$, the domain in the maximization problem associated to function $\mathcal{H}_\ell(U)$ results from the intersection of a finite number of compact sets; hence, the objective function of problem $P_\ell(U)$ is defined on a compact set. Assumption 3 ensures that the domain is a non-empty set. Furthermore, the objective function is a continuous real-valued function. Therefore, there exists a solution to problem $P_\ell(U)$, which is attained within the domain, and, hence, the function $\mathcal{H}_\ell$ is well-defined.

Let $\mathcal{C}$ be a correspondence that assigns the set of maximizers of problem $P_\ell(U)$ to a value $U \in \mathcal{K}$, i.e. $\mathcal{C}(U) \equiv \{p = (q, R, w_1, w_2) |\ p \text{ solves } P_\ell(U)\}$. We now show that $\mathcal{C}$ is a compact-valued, continuous correspondence in $\mathcal{K}$. To show upper-hemicontinuity of $\mathcal{C}$ at some value U, consider any sequence $\{U_n\}_n \subset \mathcal{K}$ converging to U and any sequence $\{p_n\}_n$ such that $p_n \in \mathcal{C}(U_n)$ for all n. We need to show that $p \equiv \lim_{n\to\infty} p_n \in \mathcal{C}(U)$. Lower-hemicontinuity requires to show that for any sequence $\{U_n\}_n \subset \mathcal{K}$ converging to U and for any $p \in \mathcal{C}(U)$, there exist a subsequence $\{U_{n_k}\}_k$ and a sequence $\{p_{n_k}\}_k$ such that $p_{n_k} \in \mathcal{C}(U_{n_k})$ and $p \equiv \lim_{k\to\infty} p_{n_k}$. It is easy to see that the continuity of the utility and matching functions ensures that the correspondence $\mathcal{C}$ is both upper- and lower-hemicontinuous, and that $\mathcal{C}(U)$ is a closed and bounded set for any value U.

Then, the Maximum Theorem states that $\mathcal{H}_\ell$ is a continuous function in $\mathcal{K}$. Finally, the Brower fixed-point Theorem applies to ensure existence of a fixed point of function $\mathcal{H}_\ell$.

The proof for the existence of a fixed-point of function $\mathcal{H}_h$ is analogous. We thus only show that the domain of the objective function of problem $P_h(U)$ is non-empty. Notice that the fixed point of function $\mathcal{H}_\ell$ satisfies all the constraints due to Assumptions 1-3. We conclude that there exists an equilibrium allocation.‖

Proof of Proposition 4.4.

To save on notation, we make $T = 0$. The Lagrangian of the maximization problem $P_\ell(U)$ is[15]

$$\mathcal{L} = \nu(q)\int_R \big(E_\ell(\pi)-U_\ell\big)dF_\ell(\pi)+\xi_1\left(\eta(q)\int_R J_\ell(\pi)dF_\ell(\pi)-c/\beta\right)+\xi_2(E_\ell(R)-U_\ell)+\xi_3 J_\ell(R),$$

where ξ_1, ξ_2 and ξ_3 are the lagrangian multipliers. Three of the necessary optimality conditions are

$$\nu(q)\big(1-F_\ell(R)\big)\big(v'(w_1)-\xi_1 q\big)+\xi_2 v'(w_1)-\xi_3 \le 0, \tag{18}$$

$$w_1 \ge 0, \text{ and } w_1\big(\nu(q)\big(1-F_\ell(R)\big)\big(v'(w_1)-\xi_1 q\big)+\xi_2 v'(w_1)-\xi_3\big)=0$$

$$\nu(q)\mathbb{E}_\ell(\pi|R)\big(1-F_\ell(R)\big)\big(v'(w_2)-\xi_1 q\big)+\xi_2 R v'(w_1)-\xi_3 R \le 0, \tag{19}$$

$$w_2 \le y, \text{ and } (y-w_2)\big(\nu(q)\mathbb{E}_\ell(\pi|R)\big(1-F_\ell(R)\big)\big(v'(w_2)-\xi_1 q\big)+\xi_2 R v'(w_2)-\xi_3 R\big)=0$$

$$-\left(\nu(q)\big(E_\ell(R)-U_\ell+\xi_1 q J_\ell(R)\big)dF_\ell(R)-\xi_2\tfrac{\partial E_\ell(R)}{\partial R}-\xi_3\tfrac{\partial J_\ell(R)}{\partial R}\right)\le 0 \tag{20}$$

$$R \ge 0, \text{ and } R\left(\nu(q)\big(E_\ell(R)-U_\ell+\xi_1 q J_\ell(R)\big)dF_\ell(R)-\xi_2\tfrac{\partial E_\ell(R)}{\partial R}-\xi_3\tfrac{\partial J_\ell(R)}{\partial R}\right)=0$$

As $\lim_{w\to 0} v'(w)=\infty$, we conclude from condition (18) and the monotonicity of the matching function ν that the equilibrium $w_{1\ell} > 0$. It cannot be the case that $E_\ell(R) > U_\ell$ and $J_\ell(R) > 0$ at once because then condition (20) would fail. If $E_\ell(R) = U_\ell$ and $J_\ell(R) > 0$, then after some manipulations of conditions (18) and (19) we obtain $v'(w_{1\ell}) > v'(w_{2\ell})$. Due to the concavity of the utility function, this implies $w_{1\ell} < w_{2\ell}$ in equilibrium. The remaining case is analogous.

The Lagrangian of the maximization problem $P_h(U)$ has one more term:

$$\begin{aligned}\mathcal{L} &= \nu(q)\int_R \big(E_h(\pi)-U_h\big)dF_h(\pi)+\xi_1\left(\eta(q)\int_R J_h(\pi)dF_h(\pi)-c/\beta\right)+\xi_2(E_h(R)-U_h)+\xi_3 J_h(R)\\ &\quad -\xi_4\left(\nu(q)\int_R \big(E_\ell(\pi)-U_\ell\big)dF_\ell(\pi)-\frac{(1-\beta)U_\ell-v(z)}{\beta}\right)\end{aligned}$$

[15] For simplicity, we exclude two limited commitment constraints.

The counterparts of conditions (18)-(20) are

$$\nu(q)\big(1-F_h(R)\big)\big(v'(w_1)-\xi_1 q\big)+\xi_2 v'(w_1)-\xi_3-\xi_4\beta\nu(q)\big(1-F_\ell(R)\big)v'(w_1)\leq 0,$$
$$w_1\geq 0,\ \text{and}\ w_1\Big(\nu(q)\big(1-F_h(R)\big)\big(v'(w_1)-\xi_1 q\big)+\xi_2 v'(w_1)-\xi_3-\xi_4\beta\nu(q)\big(1-F_\ell(R)\big)v'(w_1)\Big)=0$$
$$\nu(q)\mathbb{E}_h(\pi|R)\big(1-F_h(R)\big)\big(v'(w_2)-\xi_1 q\big)+\xi_2 Rv'(w_1)-\xi_3 R-\xi_4\beta\nu(q)\mathbb{E}_\ell(\pi|R)\big(1-F_\ell(R)\big)v'(w_2)\leq 0,$$
$$w_2\leq y,\ \text{and}$$
$$(y-w_2)\Big(\nu(q)\mathbb{E}_h(\pi|R)\big(1-F_h(R)\big)\big(v'(w_2)-\xi_1 q\big)+\xi_2 Rv'(w_2)-\xi_3 R-\xi_4\beta\nu(q)\mathbb{E}_\ell(\pi|R)\big(1-F_\ell(R)\big)v'(w_2)\Big)=0$$
$$-\bigg(\nu(q)\big(E_\ell(R)-U_\ell+\xi_1 qJ_\ell(R)\big)dF_h(R)-\xi_2\frac{\partial E_\ell(R)}{\partial R}-\xi_3\frac{\partial J_\ell(R)}{\partial R}-\xi_4\beta\nu(q)\max\{E_\ell(R)-U_\ell,0\}dF_\ell(R)\bigg)\leq 0$$
$$R\geq 0,\ \text{and}$$
$$R\bigg(\nu(q)\big(E_\ell(R)-U_\ell+\xi_1 qJ_\ell(R)\big)-\xi_2\frac{\partial E_\ell(R)}{\partial R}-\xi_3\frac{\partial J_\ell(R)}{\partial R}-\xi_4\beta\nu(q)\max\{E_\ell(R)-U_\ell,0\}dF_\ell(R)\bigg)=0$$

For the same reason, we obtain $w_{1h}>0$. Furthermore, regardless of whether the limited commitment conditions are binding or not, by manipulating the first two necessary conditions we obtain $w_{1h}<w_{2h}$. Since wages and survival rates increase with tenure at the job for any given worker, it is mechanic to show that the averages of these variables are also increasing in job tenure; hence, we omit the proof here.

By construction, a worker separates from the current job with probability $\lambda+(1-\lambda)\alpha\big(1-\mathbb{E}_i(\pi|R)\big)$, while match quality is uncertain, and λ, once it has been revealed. Since learning about match quality takes time, as job tenure increases, newly employed workers either become unemployed or stay at the firm and earn w_2. The composition of the mass of matches changes towards matches of good quality as time passes. Again, the formal proof is obvious and, hence, omitted.

Recall that the equilibrium variables related to the type ℓ market satisfy all the constraints of problem $P_h(U_\ell)$ due to Assumptions 1-3. Therefore, $U_\ell<U_h$.

To show that the expected queue length is smaller in the type h submarket, we can split the firm's problem into two: an application-attracting problem and the design of the optimal contract for any given promised value. The former problem can be written as

$$\max_{q,W}\quad \eta(q)(Y-W)$$
$$\text{s.to}\quad \nu(q)(B(W)-U)\geq U_0,$$

where Y stands for the expected revenue of the firms, W denotes the expected wage costs associated to a promised value $B(W)$ and U is the unemployment value. We can think of function B as increasing and concave in W. Notice that this problem is silent about what the optimal contract looks like. Using the first order conditions and the expression of the unemployment value, it is easy to show that there is a negative relationship between the expected revenue Y and the queue length q. Therefore, because of Assumptions 1 and 2, we obtain that $q_h < q_\ell$.‖